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20MCA14

First Semester MCA Degree Examination, Dec.2023/Jan.2024

Mathematical Foundation for Computer Application

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of Normal Distribution table is permitted.

Module-1

1 a. Define powerset, union of two sets, complement of a set with an example to each. **(06 Marks)**

b. If A, B, C are any three sets then prove,

(i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(07 Marks)**

c. Find eigen value and corresponding eigen vectors for, $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **(07 Marks)**

OR

2 a. Let $A = \{1,2,3,4,5\}$, $B = \{1,2,3,4,5,6,7\}$. Find $A \cup B$, $A \cap B$, $A \Delta B$, where Δ is symmetric difference. **(06 Marks)**

b. In a class of a 52 students 30 study C++, 28 study Java, 13 study both. How many study at least one language? How many study none of these. **(07 Marks)**

c. Define pigeonhole principle. If 5 colours are used to paint 26 doors then prove that at least 6 doors will have same colour? **(07 Marks)**

Module-2

3 a. For any two propositions p and q prove that $p \to p \vee q$ is a tautology and $p \wedge (\sim p \wedge q)$ is a contradiction. **(06 Marks)**

b. Using laws of logic prove that,

(i) $(p \vee q) \wedge [\sim \{(\sim p) \wedge q\}] \Leftrightarrow p$

(ii) $p \to (q \to r) \Leftrightarrow p \wedge q \to r$ **(07 Marks)**

c. If m is an odd integer then prove that m + 7 is an even integer by direct proof. **(07 Marks)**

OR

4 a. Define : (i) Duality of a logical expression.
(ii) Contra positive of a logical expression.
(iii) Logical equivalence with an example to each one. **(06 Marks)**

b. Check validity.
If Ravi goes out with friends he will not study.
If he does not study his father becomes angry
His father is not angry
____
∴ Ravi has not gone out with friends **(07 Marks)**

c. Suppose universe contains set of all positive integers consider following open statements :
$P(x): x \le 3$, $g(x): x+1$ is odd
$r(x): x > 0$ Then find truth values of,
(i) $P(2)$ (ii) $P(-1) \wedge q(1)$ (iii) $p(4) \vee (q(1) \wedge r(2))$ **(07 Marks)**

Module-3

5 a. Define (i) Reflexive (ii) Antisymmetric and transitive relations with an example to each. **(06 Marks)**

b. Let $A = \{1,2,3,4\}$, define a relation R on A by $(a,b) \in R$ if and only if $a \le b$. Then write R as ordered pairs write matrix and digraph of R. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 6, 12}, define a relation R on A by a R b if and only if a decides b. Then prove that R is partial order on A. Draw Hasse diagram of R. **(07 Marks)**

OR

6 a. If R_1 and R_2 be relations from set A to B where A = {a, b, c}, B = {a, b, c, d} defined by,
$R_1 = \{(a, a) (a, c) (b, d) (c, a) (c, b) (c, c)\}$
$R_2 = \{(a, a) (a, b) (a, c) (a, d) (b, d) (c, b) (c, d)\}$
Then find
(i) Complement of R_1 and R_2
(ii) $R_1 \cup R_2$ and $R_1 \cap R_2$ **(06 Marks)**

b. For any non-empty sets A, B, C, prove that
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **(07 Marks)**

c. Let R be a relation on set of integers Z defined by xRy if and only if x – y is an even integer. Then prove that R is an equivalence relation. **(07 Marks)**

Module-4

7 a. If a coin is tossed twice. A random variable X represents the number of heads turning up. Find discrete probability distribution of X. Find its mean and variance. **(06 Marks)**

b. Probability that a pen manufactured by a factory to be defective is $\frac{1}{10}$. If 12 such pens are manufactured, what is the probability that,
(i) exactly 2 are defective
(ii) at least 2 are defective. **(07 Marks)**

c. If marks of 100 students in an examination follows normal distribution with mean 70 and standard deviation 5 then find number of students who's marks,
(i) Less than 65 (ii) More than 75. **(07 Marks)**

OR

8 a. Find values of K such that following is a probability distribution. Hence find mean and variance. **(06 Marks)**

x_i	-3	-2	-1	0	1	2	3
$P(x_i)$	k	2k	3k	4k	3k	2k	k

b. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation of the distribution. Given than $\phi(0.5) = 0.19$, $\phi(0.42) = 1.4$ **(07Marks)**

c. Find C such that $f(x) = \begin{cases} \frac{x}{6} + C & 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$

is a probability dencity function also find $P(1 \le x \le 2)$. **(07 Marks)**

Module-5

9 a. Define with examples to each one,

(i) Sub-graph

(ii) Complement of a graph.

(iii) Planar graph. **(06 Marks)**

b. Define iso morphism. Verify following graphs are iso morphic or not.

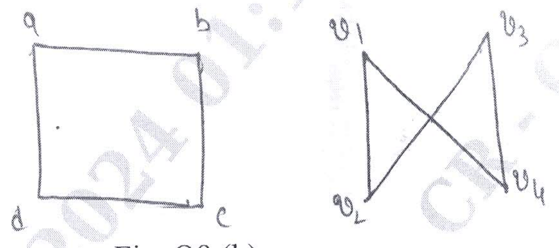

Fig. Q9 (b) **(07 Marks)**

c. Show that K_5 is non-planar graph. **(07 Marks)**

OR

10 a. Explain Konigsbridge problem. **(06 Marks)**

b. Define chromatic number. Find chromatic number of following graph :

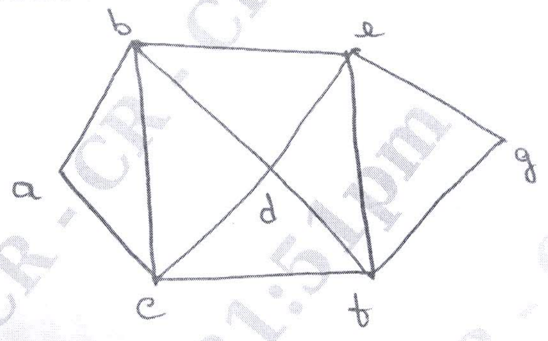

Fig. Q10 (b) **(07 Marks)**

c. Prove that in every graph the number of vertices of odd degrees is even. **(07 Marks)**

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