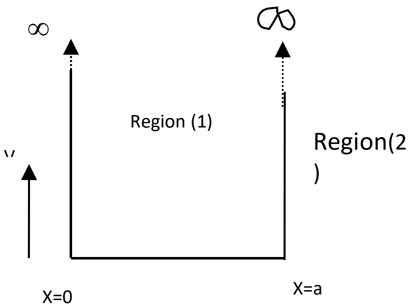
IAT -1 PHYSICS SOLUTION

1A

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X-axis in the region from X= 0 to X = a in a one dimensional potential well as shown in the diagram. The potential energy is assumed to be zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1)

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \because V = 0$$ for region (1)

But $k^2 = \frac{8\Pi^2 mE}{h^2}$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

Auxiliary equation is $(D^2 + k^2)x = 0$

Roots are D = +ik and D = -ik

The general solution is

$$x = Ae^{ikx} + Be^{-ikx}$$
$$= A(\cos kx + i\sin kx) + B(\cos kx - i\sin kx)$$
$$= (A+B)\cos kx + i(A-B)\sin kx$$
$$= C\cos kx + D\sin kx$$

The boundary conditions are (3 marks)

1. At x=0, $\Psi = 0 \therefore C = 0$

2. At x=a, $\Psi = 0$

D sin ka = 0 $\Longrightarrow$ ka = nΠ ………(2)

where n = 1, 2 3…

$$\therefore \Psi = D\sin\left(n\frac{\Pi}{a}\right)x$$

From (1) and (2) $E = \frac{n^2h^2}{8ma^2}$ (2 marks)

To evaluate the constant D:

Normalisation: For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2(\frac{n\Pi}{a})x dx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}(1 - \cos 2(\frac{n\Pi}{a})x)dx = 1$$

$$\int_0^a \frac{D^2}{2}dx - \int_0^a \frac{1}{2}\cos 2(\frac{n\Pi}{a})x)dx = 1$$

$$\frac{D^2 a}{2} - \quad [\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2\frac{a}{2} - 0 = 1$$

$$D = \sqrt{\frac{2}{a}}$$

$$\therefore \Psi_n = \sqrt{\frac{2}{a}}\sin\left(n\frac{\Pi}{a}\right)x$$ (2 marks)

1B Formula 1mark, Substitution 1 mark, Answer 1 mark

$$E = \frac{n^2 h^2}{8mL^2}$$

For 1st excited state

$$n = 2, E_2 = \frac{2^2 h^2}{8mL^2} = 140eV$$

For ground state, $n = 1$, *also* $L^1 = \frac{L}{2}$

$$For\ n = 1\ E_1 = \frac{1^2 h^2}{8m\left(\frac{L}{2}\right)^2} = \frac{4E_2}{4} = 140eV$$

2A

HEISENBERG'S UNCERTAINTY PRINCIPLE:

The position and momentum of a particle cannot be determined accurately and simultaneously. The product of uncertainty in the measurement of position (Δx) and momentum is always greater than or equal to $\frac{h}{2\Pi}$.

(2 marks)

$$(\Delta x).(\Delta p) \geq \frac{h}{4\Pi}$$

TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:

We know that the diameter of the nucleus is of the order of 10^{-15}m.If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\Delta x = 5x10^{-15} m$$

Now the uncertainty in momentum is

$$\Delta x = 5x10^{-15} m$$

$$\Delta P = \frac{h}{4\pi x \Delta x} = 0.1x10^{-19} kg.m/s$$

Then the momentum of the electron can atleast be equal to the uncertainty in momentum.

$$P \approx \Delta P = 0.1x10^{-19} kg.m/s$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities -non-relativistic-case)

$$E = \frac{p^2}{2m} = 5.5x10^{-11} J = 343MeV$$ (4 marks)

The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus ($n \rightarrow p + e^{-} + \nu$ / $p \rightarrow n + e^{+} + \nu$). (1 mark)

2B Formula 1mark, Substitution 1 mark, Answer 1 mark

$$\lambda = \frac{h}{\sqrt{2mE}} = 1.2x10^{-10} m$$

$$E = \frac{h^2}{2m\lambda^2} = 1.67x10^{-17} J$$

3A

Time independent Schrödinger equation

A matter wave can be represented in complex form as

$$\Psi = A \sin kx (\cos wt + i \sin wt)$$

$$\Psi = A \sin kx e^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA \cos kx e^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A \sin kx e^{iwt} = -k^2 \Psi \text{ .... (1)}$$

From Debroglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$ (3 marks)

$$k = \frac{2\pi}{\lambda} = \frac{2\Pi p}{h}$$

$$k^2 = 4\Pi^2 \frac{p^2}{h^2} \text{ .... (2)}$$

Total energy of a particle

E = Kinetic energy + Potential Energy

$$E = \frac{1}{2} m v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

(3 marks)

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\Pi^2 (E - V)2m}{h^2}$$

∴ From (1) (1 mark)

$$\frac{d^2\Psi}{dx^2}+\frac{8\Pi^2m(E-V)\Psi}{h^2}=0$$

For 3D

$$\frac{d^2\Psi}{dx^2}+\frac{d^2\psi}{dy^2}+\frac{d^2\psi}{dx^2}+\frac{8\Pi^2m(E-V)\Psi}{h^2}=0$$

3B Formula 1mark, Substitution 1 mark, Answer 1 mark

$$\Delta v=\frac{0.009}{100}\times 8.5\times 10^5=76.5m/s$$

$$\Delta x=\frac{h}{4\pi\Delta p}=\frac{h}{4\pi m\Delta v}=7.57x10^{-7}m$$

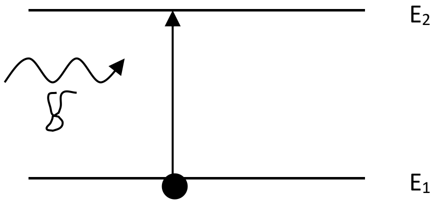
4A

Expression for energy density: (3 marks)

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N1 and N_2 be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident..



$$\gamma=\frac{E_2-E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_γ.

Rate of absorption = $B_{12}\ N_1U_\gamma$

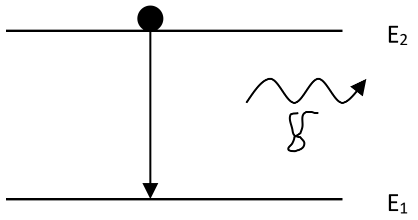
Here B_{12} is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

Rate of spontaneous emission = $A_{21}\ N_2$

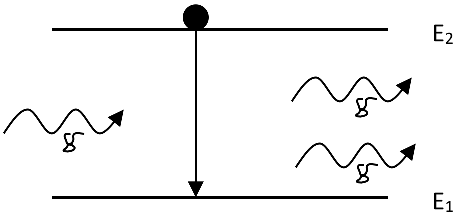
Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



$$\gamma = \frac{E_2 - E_1}{h}$$

Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_γ.

Rate of stimulated emission = $B_{21}\ N_2U_\gamma$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12}\ N_1U_\gamma = A_{21}\ N_2 + B_{21}\ N_2\ U_\gamma$$

$$U_\gamma = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

Rearranging this, we get

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}N_1}{B_{21}N_2} - 1}\right]$$

From Boltzmann's law , $\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_\gamma = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}e^{\frac{h\gamma}{kT}} - 1}\right]$$

From Planck's radiation law,

$$U_\gamma = \frac{8\pi h\gamma^3}{c^3}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]} - 1}\right]$$

Comparing these expressions, we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3} \text{ and } \frac{B_{12}}{B_{21}} = 1$$ (3 marks)

Conclusions 1 mark

1.Rate of stimulated emission is directly proportional to wavelength

2. Rate of Induced absorption is equal to rate of Stimulated emission

4B **Formula - 1mark, Substitution - 1 Mark, Answer- 1 mark**

$$\frac{N_g}{N_e} = e^{\frac{hc}{\lambda kT}}$$

$$T = 27 = 273 = 300K$$

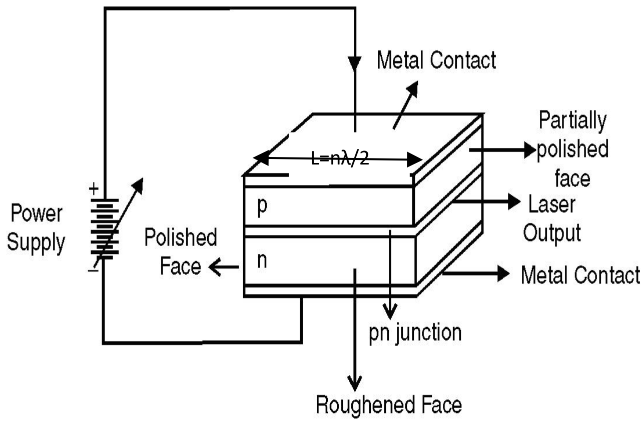
$$\frac{N_{ex}}{N_g} = 1.095x10^{-29}$$

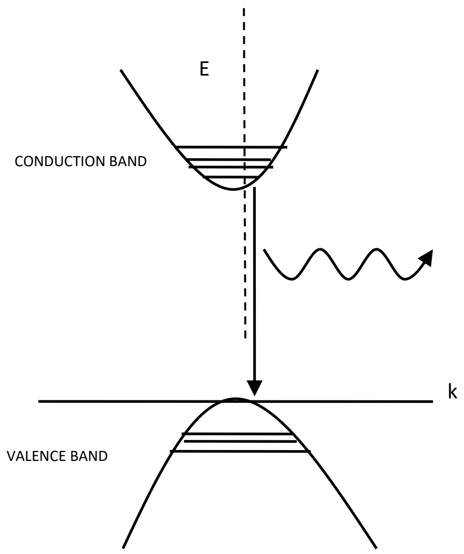
$$\lambda = 7205x10^{-10}m$$

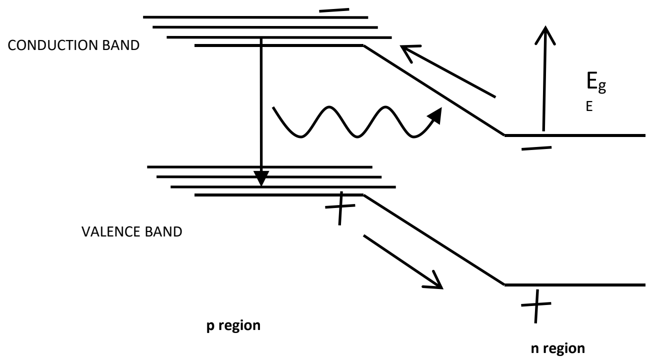
5A

Semiconductor Laser : Construction- 3 marks

It is a cheaper , portable low power efficient Laser.







Working- 3 marks

The first laser action was observed in a GaAs junction(8400Å) which is a direct gap semiconductor.

When a heavily doped junction is forward biased, electrons from n side are injected into p side causing population inversion. They combine with holes on the p side releasing photons. The junction region is the active region .The optical cavity is formed by the faces of the crystal itself which are taken on the cleavage plane and are then polished. The wavelength of laser increases as the temperature increases as the energy gap decreases.

If E_g is the energy gap, then $E_g = eV_{forward} = \frac{hc}{\lambda}$

5B

Formula 1mark, Substitution 1 mark, Answer 2 marks

$$P = 0.006W$$
$$\lambda = 650x10^{-9}m$$
$$P = \frac{E}{t} = \frac{nhc}{\lambda t}$$
$$n = \frac{P\lambda t}{hc} = 1.96x10^{16}$$

6A

BAR CODE SCANNER: (3 marks)

Barcodes are codes that contain data or information about a particular product. Different letters and numbers are encoded using bars of varying lengths and widths. These barcodes are significant for accurate inventory management. For example, the barcode seen on the tag of apparel in a retail store will have some black and white spaces with a product number, which is detected and scanned by a computer.

It gets tallied with the data representing this barcode in the computer, and it will help in the billing process. A handheld scanner at the hands of the cashiers at almost all organised retail outfits is a familiar sight for every customer. In a store, an employee scans the barcode that is attached to a product, and that adds up quickly to a customer's order. Today barcodes are found almost in every store. We see them on courier packages and also in print advertisements. Quick Response or (QR) codes are a more advanced form of barcodes with a mobile-friendly scanning process.

How Does A Barcode Scanner Work?

A **barcode scanner** is an input device that uses light beams to scan and digitally convert printed barcodes. It then decodes the data and sends the data to a computer. It consists of a lens, a source of light, and a light sensor that can translate optical impulses into electrical signals. A **barcode scanner** contains a decoder that analyses the image data provided by the sensor and sends it to the output port. After scanning an image, it links to a host computer to pass along the captured information.

The decoder recognises the barcode symbols, translates the bar and space content, and transmits the data to a computer in such a format that we can read. The decoder can also drop data into Accel or Excel databases. Special application software can put that data into inventory records for monitoring the work-in-progress, or for receiving and sending files. It reduces the need for manual data collection and thereby helps mitigate chances of human error. It also quickens tasks such as managing assets, monitoring point-of-sale transactions, and tracking inventory.

LASER PRINTER (3 marks)

- To begin the laser printer process, the document is broken down into digital data and sent from the respective computer to the printer.
- The process called conditioning involves applying a charge to the drum unit and the paper as it passes through the corona wire. Adding a static charge to the paper allows an image to be electrostatically transferred to the laser printer page.
- The primary charge roller springs to life, spinning the adjacent organic photoconductor (OPC) drum. Ions on the corona wire coat the drum with static electricity. The electro-photographic process begins at the molecular level. The drum completes its revolution, slathered with a negative charge.
- the photosensitive drum is exposed to a laser beam. Every area of the drum exposed to the laser has its surface charge reduced to about 100 volts DC.
- An invisible latent print is generated as the printer's drum turns. The image that will ultimately be printed exists for the first time as a thin layer of electrons on the OPC drum.
- The darkness within the printer cartridge is broken by the glow of the laser. The beam bounces off a spinning, multi-sided mirror and breaks into countless rays of information, spraying the OPC drum with its knowledge, turning the negative charges positive.
- Line-by-line, the laser speaks to the revolving surface of the drum unit, describing a page with the language of charged toner particles. This part is black, this part is yellow, and this part...yes, this part...is wonderfully magenta. The drum wears a positively charged image on its surface, ready to transfer onto the paper.
- In the developing stage, toner is applied to the latent image on the drum. Toner is composed of negatively charged powdered plastics — black, cyan, magenta, and yellow. The drum is held at a microscopic distance from the toner by a control blade.
- Toner is **85-95%** finely ground plastic. Other toner ingredients used in printers include colored pigments, fumed silica, and control agents. Silica keeps the toner particles from clumping and sticking together. It also helps the toner flow smoothly from the

cartridge to the printer. Bits of zinc, iron, and chromium are used as control agents to retain the negative electrostatic charge of the toner particles.

- The secondary corona wire, or transfer roller, applies a positive charge onto the paper. The agitator unit inside the toner cartridge hopper spins, and the toner begins to heat up. The toner adder spins, pulling toner in, gathering toner dust on its surface. A doctor blade sweeps over the adjacent developer roller, leveling the toner to a precise height. All the spinning and commotion has left the magenta particles on its surface with a negative charge, and when it comes in contact with the positively charged image on the OPC drum, the laws of attraction take over. The negatively charged toner on the surface of the drum is magnetically attracted to the positively charged areas on the paper.The magenta toner particles are pulled from the developer onto the drum according to the precise instructions left by the laser. A few magenta toner particles here, several there, and a bunch more that will blend with black, yellow, and blue to form a rainbow of beautiful colors. The sheet of paper passes over each color cartridge -- magenta, yellow, cyan, and finally black — as the image is transferred onto the paper.
- The final phase is fusing. Heat and pressure are applied to the toner by the fuser unit. The toner generates a permanent bond as it is pressed and melted into the paper. Teflon covers the fuser unit as a light silicon oil is applied in order to remove any possibility of the sheet of paper sticking to them.
- The fuser unit essentially melts the toner powder onto the page, creating the image. A wiper blade cleans any remaining particles off the OPC drum and deposits them into a waste bin. Any latent charge left on areas of the drum surface is erased, restored, refreshed, and ready for the laser printer to sing again.

6B

Formula 1mark, Substitution 1 mark, Answer 2 marks

$$\Delta E \times \Delta t = \frac{h}{4\pi}$$

$$lifetime\ \ \Delta t = \frac{h}{4\pi.\Delta E}$$

$$E = \frac{hc}{\lambda}$$

$$\Delta E = -\frac{hc}{\lambda^2}\Delta\lambda = \frac{6.62x10^{-34}x3x10^{8}}{(6328x10^{-10})^2}x5x10^{-15}$$

$$\Delta t = 2.13x10^{-8}s$$

7A

Matter waves are the probability waves associated with matter. The wavelength of the group of waves associated with particle of mass m moving with a velocity v is given by the expression (1mark)

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2mE}}$$

where his the Planck's constant

Properties of Matter waves: (2marks)

1. Matter waves represent the probability density variation in a region.
2. A matter wave in complex form is written as $\Psi = A\sin kx(\cos wt + i\sin wt)$. It is obtained as general solution to Schrodinger's equation.
3. Matter waves are neither transverse nor longitudinal and their velocity is equal to that of Particle.
4. They propagate as group of waves.

Derivation: (3marks)

$$E = mc^2$$

$$E = hf$$

$$hf = mc^2$$

$$p = mc = \frac{hf}{c} = \frac{h}{\frac{c}{f}} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv}$$

7B

Formula 1mark, Substitution 1 mark, Answer 2 marks

n =2

$$\Pr obability = \int_{3L/2}^{3L/4} |\psi^2| dx = \int \frac{2}{L}\sin^2\frac{n\pi}{L}x.dx = \frac{2}{L}\int (\frac{1-\cos 2n\pi x/L}{2}dx$$

$$= \frac{2}{L}\int\frac{1}{2}dx - \frac{2}{2L}\int\frac{\cos 2n\pi x/L}{}dx = \frac{1}{L}[x]_{L/3}^{3L/4} - \frac{1}{L}\left[\frac{\sin 2n\pi x/L}{2n\pi/L}\right]_{L/3}^{3L/4}$$

$$= 0.347$$