USN

Internal Assessment Test II – December 2023

Sub:	Mathematics - 1 for EEE Stream					Sub Code:	BMATE101
Date:	01/12/2023	Duration:	90 mins	Max Marks:	50	Sem / Sec:	1 / M,N,O,P (C- CYCLE)

Answer All questions

		MARKS	OBE CO	OBE RBT
1.	Find the dominant eigenvalue and the corresponding eigenvector of the matrix $$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$ by power method taking the initial vector as $[1,0,0]^T$ **(perform only 6 iterations).**	[08]	CO5	L3
2.	Find the rank of the matrix: $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$	[07]	CO5	L3
3.	Investigate the values of λ and μ such that the following system may have (i) unique solution, (ii) infinitely many solutions and (iii) no solution. $$x + y + z = 6$$ $$x + 2y + 3z = 10$$ $$x + 2y + \lambda z = \mu$$	[07]	CO5	L3

4. Solve using Gauss Jordan Method: [07] CO5 L3

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

5. Solve using Gauss Seidel method (perform only 4 iterations): [07] CO5 L3

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

6. Find the extreme values of the function [07] CO2 L3

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

7. If $u = \tan^{-1}\left(\frac{y}{x}\right)$, where $x = e^t - e^{-t}$, $y = e^t + {}^{-}e^t$, then find $\frac{du}{dt}$ [07] CO2 L3

IAT-2 (Dec 2023)

1) Find the dominant eigenvalue and the corresponding eigen vector of the matrix, $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$, by power method taking the initial vector as $[1,0,0]^T$.

Sol: Here $X^{0} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is the initial eigen vector.

$$A X^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$A X^{(1)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 2 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$A X^{(2)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 0 \\ 2.6 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$A X^{(3)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix} = \begin{bmatrix} 2.93 \\ 0 \\ 2.86 \end{bmatrix} = 2.93 \begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

$$A X^{(4)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix} = \begin{bmatrix} 2.98 \\ 0 \\ 2.96 \end{bmatrix} = 2.98 \begin{bmatrix} 1 \\ 0 \\ 0.99 \end{bmatrix} = \lambda^{(5)} X^{(5)}$$

$$A x^{(5)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.99 \end{bmatrix} = \begin{bmatrix} 2.99 \\ 0 \\ 2.98 \end{bmatrix} = 2.99 \begin{bmatrix} 1 \\ 0 \\ 0.997 \end{bmatrix} = \lambda^{(6)} x^{(6)}$$

$$A x^{(6)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.997 \end{bmatrix} = \begin{bmatrix} 2.997 \\ 0 \\ 2.994 \end{bmatrix} = 2.997 \begin{bmatrix} 1 \\ 0 \\ 0.999 \end{bmatrix}$$

$$= \lambda^{(7)} x^{(7)}$$

Thus the largest eigen value is approximately 3 & the corresponding eigen vector is $[1 \ 0 \ 1]^T$

2) Find the rank of the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

Sol^n:- $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_2 \leftrightarrow R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$ & $R_4 \to R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$ & $R_4 \to R_4 - R_2$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow \rho(A) = \text{rank}(A) = 2.$

3) Investigate the values of λ and μ such that the following system may have (i) unique sol^n, (ii) infinitely many sol^{n}s & (iii) no sol^n.

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

Sol^n:- $[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 1 & 2 & 3 & : & 10 \\ 1 & 2 & \lambda & : & \mu \end{bmatrix}$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 1 & \lambda-1 & : & \mu-6 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

(i) unique sol^n :- $\lambda \neq 3$.

(ii) Infinitely many sol^n :- $\lambda = 3$ & $\mu = 10$.

(iii) No sol^n :- $\lambda = 3$ & $\mu \neq 10$.

4) Solve using Gauss Jordan method:-

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

Sol:- $[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right]$

$R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array}\right]$$

$R_1 \to R_1 + 2R_2$ & $R_3 \to R_3 - 7R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$R_3 \to \frac{-1}{8} R_3$

$$\sim \begin{bmatrix} 1 & 0 & 3 & : & 11 \\ 0 & -1 & 1 & : & 4 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

$R_1 \to R_1 - 3R_3$ & $R_2 \to R_2 - R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & 2 \\ 0 & -1 & 0 & : & +1 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

$$\Rightarrow \boxed{x = 2, \ y = -1, \ z = 3}$$

5) Solve using Gauss Seidel method (perform only 4 iterations):

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25.$$

Sol: The eqⁿs are diagonally dominant and hence we first write them in the following form:

$$x = \frac{1}{20}\left[17 - y + 2z\right]$$
$$y = \frac{1}{20}\left[-18 - 3x + z\right]$$
$$z = \frac{1}{20}\left[25 - 2x + 3y\right]$$

We start with the trial soln $x=0,\ y=0,\ z=0$.

First Iteration :-

$$x^{(1)} = \frac{1}{20}\left[17-0+0\right] = \frac{17}{20} = 0.85$$

$$y^{(1)} = \frac{1}{20}\left[-18-3(0.85)+0\right] = -1.0275$$

$$z^{(1)} = \frac{1}{20}\left[25-2(0.85)+3(-1.0275)\right] = 1.0109$$

2nd Itr :-

$$x^{(2)} = \frac{1}{20}\left[17-(-1.0275)+2(1.0109)\right] = 1.0025$$

$$y^{(2)} = \frac{1}{20}\left[-18-3(1.0025)+1.0109\right] = 0.9998$$

$$z^{(2)} = \frac{1}{20}\left[25-2(1.0025)+3(-0.9998)\right] = 0.9998$$

3rd Itr :-

$$x^{(3)} = \frac{1}{20}\left[17-(-0.9998)+2(0.9998)\right] = 0.9999$$

$$y^{(3)} = \frac{1}{20}\left[-18-3(0.9999)+0.9998\right] = -1.000005$$

$$z^{(3)} = \frac{1}{20}\left[25-2(0.9999)+3(-1.000005)\right] = 1.000002$$

4th Itr:-

$$x^{(4)} = \frac{1}{20}\left[17 - (-1.000005) + 2(1.000002)\right]$$

$$= 1$$

$$y^{(4)} = \frac{1}{20}\left[-18 - 3(1) + 1.000002\right]$$

$$= -1$$

$$z^{(4)} = \frac{1}{20}\left[25 - 2(1) + 3(-1)\right] = 1$$

Thus, $x=1$, $y=-1$, $z=1$ is the required solution.

6) Find the extreme values of the function

$$f(x,y) = x^3 + y^3 - 3x - 12y + 20.$$

Soln:- $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

$f_x = 3x^2 - 3$, $f_y = 3y^2 - 12$

$f_x = 0$ & $f_y = 0$

$\Rightarrow 3x^2 - 3 = 0$ & $3y^2 - 12 = 0$

$\Rightarrow 3(x^2 - 1) = 0$ & $3(y^2 - 4) = 0$

$\Rightarrow (x^2-1)=0 \quad \& \quad y^2-4=0$

$\Rightarrow (x+1)(x-1)=0 \quad \& \quad (y-2)(y+2)=0$

$\Rightarrow x=\pm 1 \quad \& \quad y=\pm 2.$

Therefore $(1,2),(1,-2),(-1,2),(-1,2)$ are the stationary points.

$A = f_{xx} = 6x$

$B = f_{xy} = 0$

$C = f_{yy} = 6y$

	$(1,2)$	$(1,-2)$	$(-1,2)$	$(-1,-2)$
$A=6x$	$6>0$	6	-6	$-6<0$
$B=0$	0	0	0	0
$C=6y$	12	-12	12	-12
$AC-B^2$	$72>0$	$-72<0$	$-72<0$	$72>0$
Conclusion	Minm pt.	Saddle pt.	Saddle pt.	Maxm pt.

Minimum value of $f(x,y)$ at $(1,2)$

$\Rightarrow f(1,2) = 1+8-3-24+20 = 2.$

Maximum value of $f(x,y)$ is at $(-1,-2)$,

$f(-1,-2) = -1-8+3+24+20 = 38.$

7) If $u = \tan^{-1}(y/x)$, where $x = e^t - e^{-t}$, $y = e^t + e^{-t}$, then find $\frac{du}{dt}$.

Sol:- $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$ — (1)

Diff 'u' partially w.r.t 'x', we get

$\frac{\partial u}{\partial x} = \frac{1}{1+(y/x)^2} \times y\left(\frac{-1}{x^2}\right) = \frac{x^2}{x^2+y^2} \times \frac{-y}{x^2}$

$\frac{\partial u}{\partial x} = \frac{-y}{x^2+y^2}$ — (2)

Diff 'u' partially w.r.t. 'y', we get

$\frac{\partial u}{\partial y} = \frac{1}{1+(y/x)^2} \times \frac{1}{x} = \frac{x^2}{x^2+y^2} \times \frac{1}{x}$

$\frac{\partial u}{\partial y} = \frac{x}{x^2+y^2}$ — (3)

$x = e^t - e^{-t}$

$\frac{dx}{dt} = e^t + e^{-t}$ — 4

$y = e^t + e^{-t}$

$\frac{dy}{dt} = e^t - e^{-t}$ — 5

Sub. 2, 3, 4 & 5 in 1, we get

$$\frac{du}{dt} = \left(\frac{-y}{x^2+y^2}\right)\left(e^t + e^{-t}\right) + \left(\frac{x}{x^2+y^2}\right)\left(e^t - e^{-t}\right)$$

$$= -\frac{(e^t + e^{-t})(e^t + e^{-t})}{(e^t - e^{-t})^2 + (e^t + e^{-t})^2} + \frac{(e^t - e^{-t})(e^t - e^{-t})}{(e^t - e^{-t})^2 + (e^t + e^{-t})^2}$$

$$= \frac{-(e^t + e^{-t})^2 + (e^t - e^{-t})^2}{(e^t - e^{-t})^2 + (e^t + e^{-t})^2}$$

$$= \frac{\cancel{-e^{2t}} \cancel{- e^{-2t}} - 2e^t e^{-t} \cancel{+ e^{2t}} \cancel{+ e^{-2t}} - 2e^t e^{-t}}{e^{2t} + e^{-2t} \cancel{- 2e^t e^{-t}} + e^{2t} + e^{-2t} \cancel{+ 2e^t e^{-t}}}$$

(12)

$$\Rightarrow \frac{du}{dt} = \frac{-4}{2e^{2t}+2e^{-2t}} = \frac{-2}{e^{2t}+e^{-2t}}$$

X