Solutions to IAT-3 PHYSICS FOR EEE

1A

Expression for Fermi Level in Intrinsic Semiconductor

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

(2 marks)

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

(2 marks)

If m_e and m_h are equal,

$E_f = (Ec + Ev)/2$

So, Fermi level is said to be at the centre of energy gap

(2 marks)

1B

FOUR PROBE METHOD

Four Probe method permits measurements of resistivity in samples having a wide variety of shapes, including the resistivity of small volumes within bigger pieces of semiconductor. In this manner the resistivity of both sides of p-n junction can be determined with good accuracy before the material is cut into bars for making devices. This method of measurement is also applicable to silicon and other semiconductor materials.

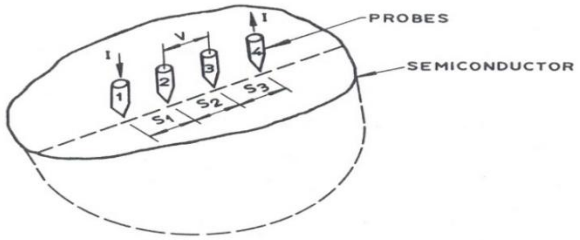


FIG. 5 MODEL FOR THE FOUR PROBE RESISTIVITY MEASUREMENTS

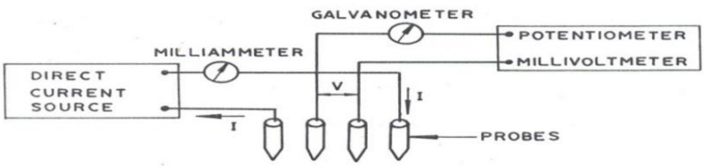


FIG. 6 CIRCUIT USED FOR RESISTIVITY MEASUREMENTS

(2 marks)

The basic model for all these measurements is indicated in Figure. Four sharp probes are placed on a flat surface of the material to be measured, current is passed through the two outer electrodes, and the floating potential is measured across the inner pair. If the flat surface on which the probes rest is adequately large and the crystal is big the semiconductor may be considered to be a semi-infinite volume. To prevent minority carrier injection and make good contacts, the surface on which the probes rest, maybe mechanically lapped.

The experimental circuit used for measurement is illustrated schematically in Fig A nominal value of probe spacing which has been found satisfactory is an equal distance of 2.0 mm between adjacent probes. This permit measurement with reasonable current of n-type or p-type semiconductor from 0.001 to 50 ohm. cm

(2 marks)

PROCEDURE:

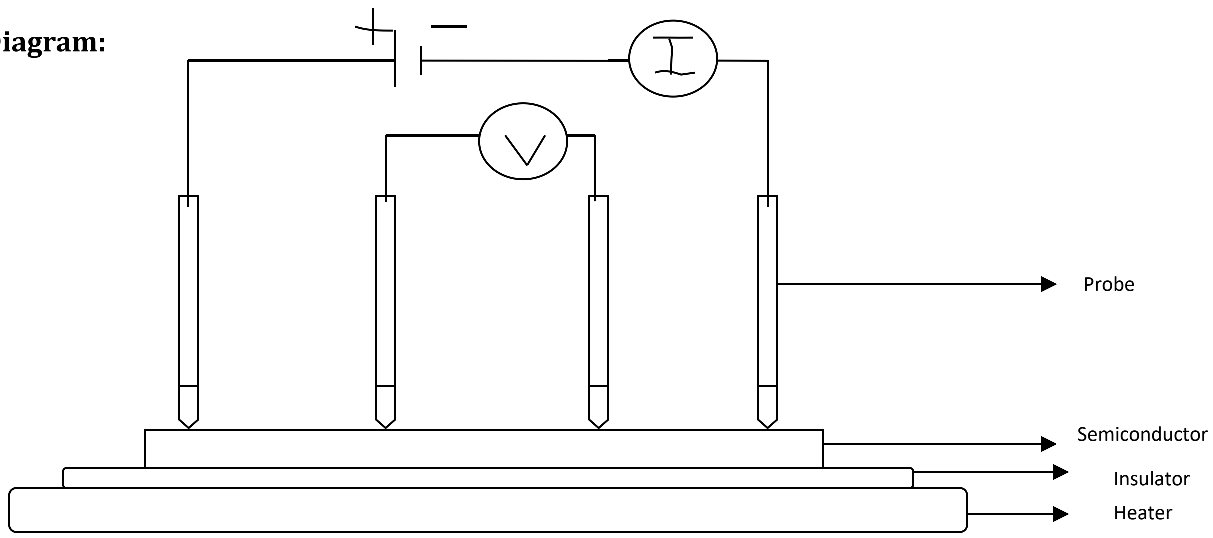
1. The semiconductor sample is placed in the four probe fixture and the probes are in contact with the semiconductor sample. Turn on the instrument and set the constant current to flow through the semiconductor. Note down the current reading (I).
2. Turn on the oven. The temperature begins to increase.
1. Note down the probe output voltage at regular intervals of say 5^0C beginning from 320K to 380K.
2. The resistivity of the semiconductor is found by the expression

$$\rho = \frac{V \times 2\pi \times S}{I \times F}$$

5. Plot the graph of ln ρ against 1/T and find the slope.
6. Energy gap of the semiconductor is calculated using the relation

$$E_g = 2 \times k \times slope \quad eV$$

Diagram:



TABULATION & OBSERVATION:

Temperature T ( )	1/T ( )	Probe output voltage	Resistivity ρ( )	ln ρ

Observations:

Average distance between the probes S =0.2cm

Correction factor F =5.89

The resistivity of the semiconductor is found by the expression

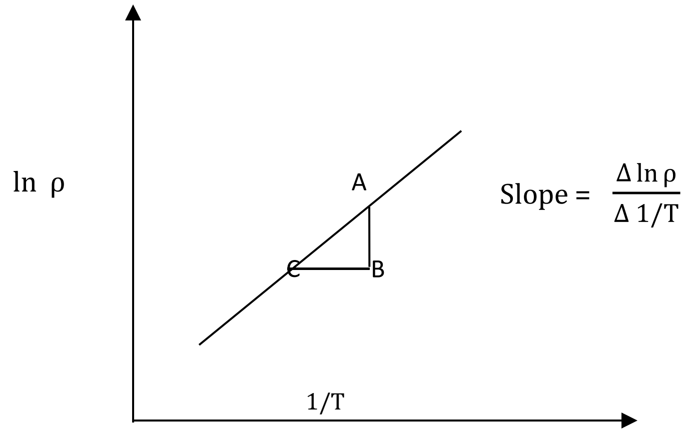
$$\rho = \frac{V \times 2\pi \times S}{I \times F}$$

V is the voltage measured between the inner probes
S is the distance between the probes in cm.
F is the correction factor (0.89)
I is the current through the semiconductor

Energy gap of the semiconductor is calculated using the relation

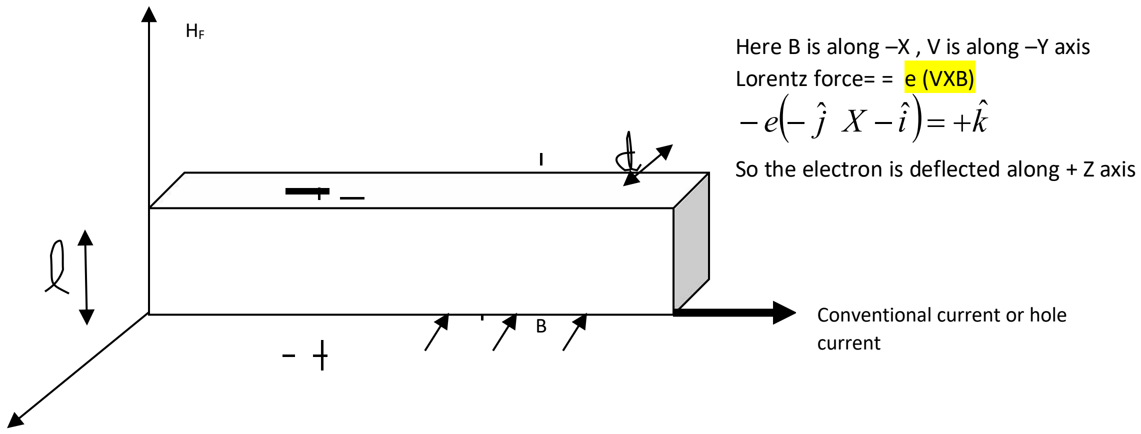
$$E_g = 2 \times k \times slope \quad eV$$

NATURE OF GRAPH:



2A

Hall effect: When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.



(2 marks)

Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

(2 marks)

Expression for electron concentration:

At equilibrium, Lorentz force is equal to force due to applied electric field

Bev =- e E_H

Hall Field E_H = Bv

Lorentz force (F_L)

$$Current\ density\ J = I/A = neAV/A = -n_e ev$$

$$v = \frac{J}{n_e e}$$

Hence

$$E_H = B\frac{-J}{n_e e}$$

$$\frac{E_H}{JB} = -\frac{1}{n_e e} = R_H$$

Electric force (eE)

R_H is known as Hall coefficient. It is negative for n type and p type for

Electron concentration $n_e = \frac{BI}{V_H d}$

Hall voltage V_H = $E_H . l = Bvl$ E = dv/dx

(2 marks)

2B

Law of mass action:

The product of electron and hole concentration is constant at any given temperature independent of Fermi energy.This principle is used to calculate hole and electron densities.

$$n_e n_h = 4\left(\frac{kT}{2\pi h^2}\right)^3 \left(m_e^* m_h^*\right)^{\frac{3}{2}} e^{-\frac{E_g}{kT}} = a\ \ cons\ tan\ t$$

Electron concentration $n_e = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_c - E_F}{kT}\right)}$

(2 marks)

Hole concentration $n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\left(\frac{E_F - E_{VF}}{kT}\right)}$

For Intrinsic semiconductor $n_e = n_h = n_i$

hence

$$n_i^2 = n_e n_h = 4\left(\frac{kT}{2\pi h^2}\right)^3 \left(m_e^* m_h^*\right)^{\frac{3}{2}} e^{-\frac{E_g}{kT}} = a \;\; cons \tan t$$

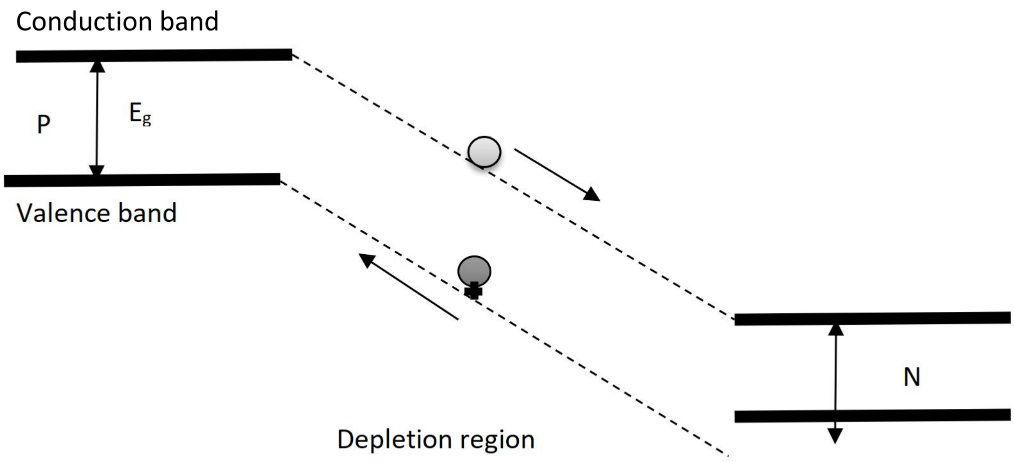
(2 marks)

3A

PHTODIODE

It is a light sensitive PN Junction operating in reverse bias. When a light photon of energy greater than the bandgap of the PN junction is incident on the junction, electrons are excited from the valence band to conduction band creating free electron – hole pairs. The reverse voltage applied produces high electric field. The minority charge carriers (electrons on P side and holes on N side) diffuse through the junction and constitute reverse current

(2 marks)



The wavelength of light radiation $\lambda = \frac{hc}{E_g}$

Quantum efficiency = No. of electron hole pairs produced/no. of Photons incident

Photo responsivity = Photo current / Incident energy

(2 marks)

Applications of Photo diode

(2 marks)

1. **As a Photodetector**: A PIN diode can change the absorbed light into electrical energy. The placement of intrinsic region between the p and n region increases the region for radiation absorption. With the increased radiation absorption region, the efficiency of the device to produce electrical energy also increases. Thus, it can be used as a photodiode.
2. **As a radio frequency switch**: The intrinsic region isolates the p and n region of the diode due to which capacitance decreases. The capacitance of the device should almost negligible in order to operate it as a switch.
3. **As a voltage rectifier**: PIN diode is able to bear high reverse voltage due to the intrinsic layer. This leads to an increase in the breakdown voltage of the diode. Hence, due to this, the device allows the rectification of high input voltage.

3B

<u>Conductivity of Intrinsic semiconductors:</u>

Electron current $I_e = n_h eAv_d(e)$

Hole current $I_h = n_h eAv_d(h)$

current density $J = \frac{I}{A} = \frac{I_e + I_h}{A} = n_h ev_d(h) + n_e ev_d(e) = \sigma E$

But drift velocity $V_d = \mu E = \mu . J/\sigma$ $\mu = V_d/E$ $J = \sigma . E$

(2 marks)

Using (1), $\sigma = n_e e \mu_e + n_h e \mu_h$

In an intrinsic semiconductor, number of holes is equal to number of electrons.

$\sigma_{int} = n_e e[\mu_e + \mu_{hole}]$

n_e is the electron concentartion

n_p is the hole concentration

μ_e is the mobility of electrons

μ_h is the mobility of holes

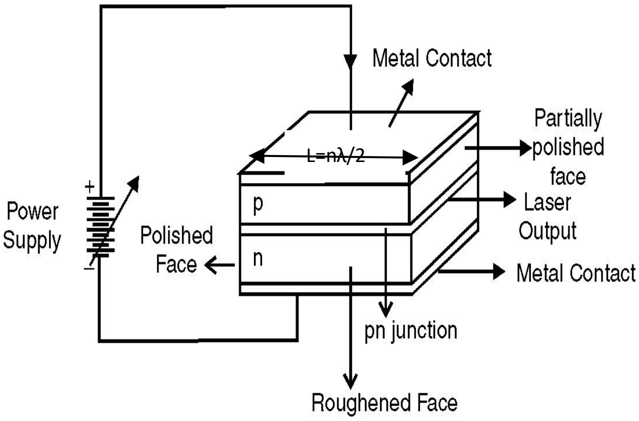
(2 marks)

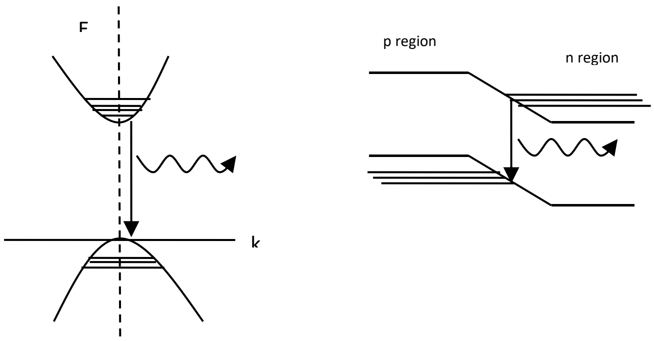
4A

Construction- 3 marks, Working- 3 marks

Gallium – Arsenide Semiconductor laser :

It is the only device which can be used for amplification in the infrared and optical ranges. Amplification is possible if the population of the valence and conduction bands could be inverted as shown in the diagram.





The first laser action was observed in a GaAs junction(8400Å) which is a direct gap semiconductor.

When a heavily doped junction is forward biased, electrons from n side are injected into p side causing population inversion. They combine with holes on the P side releasing photons. The junction region is the active region .The optical cavity is formed by the faces of the crystal itself which are taken on the cleavage plane and are then polished. The wavelength of the radiation depends on temperature. The wavelength of laser increases as the energy gap decreases. The frequency can be increased to the optical region by alloying with phosphor according to the relation $Ga(As)_{1-x}P_x$.

If E_g is the energy gap, then $E_g = eV_{forward} = \frac{hc}{\lambda}$

4B

Formula - 1 mark, Substitution- 1 mark, Answer- 2 marks

$$\sigma_{\text{int}} = n_e e(\mu_e + \mu_h) = n \times 1.6x10^{-19}(0.36 + 0.14)$$

$$\rho = \frac{1}{\sigma}$$

$$n = \frac{1}{\rho e(\mu_e + \mu_h)} = 5x10^{18} m^{-3}$$

5A

Wave equation for electric field:

Consider the equation $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$= -\mu \frac{\partial \vec{H}}{\partial t}$ $\quad \because \vec{B} = \mu \vec{H}$

Taking curl on both sides

$$\nabla x(\nabla x \vec{E}) = -\mu \frac{\partial}{\partial t}\left(\nabla x \vec{H}\right) \qquad \text{............(1)}$$

(2 marks)

Also $\nabla x(\nabla x \vec{E}) = \nabla\left(\nabla \bullet \vec{E}\right) - \nabla^2 \vec{E} = \nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E}$ ....(2)

From (1) and (2) ,

$$\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\nabla x \vec{H}\right)$$

$$\because \nabla \bullet \vec{D} = \rho$$

$$\nabla \bullet \varepsilon \vec{E} = \rho$$

$$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon}$$

$$\nabla\left(\frac{\rho}{\varepsilon}\right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

(2 marks)

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu \frac{\partial J}{\partial t} + \nabla\left(\frac{\rho}{\varepsilon}\right)$$

For a free space where there are no charges (ρ=0), no currents (J=0).

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0 \qquad \text{................(3)}$$

This is the characteristic form of a wave equation .The solution to this equation represents a wave .

(2 marks)

5B

GAUSS LAW OF MAGNETOSTATICS

$$\nabla \bullet \vec{B} = 0$$

Gauss law for magnetic fields:

(2 marks)

Magnetic flux lines always form a closed loop. If a closed surface is imagined in a magnetic field, for every flux line that enters the surface, there must always be a flux line emerging out of the surface.

Therefore total flux over the closed surface = 0.

$$\phi = \int \vec{B} \bullet \vec{ds} = 0$$

$$\Rightarrow \nabla . \vec{B} = 0$$

Divergence of magnetic field is zero. [There is no source or sink for magnetic field].They form closed loops. Hence magnetic fields are said to be solenoidal.

FARADAYS LAW (2 marks)

$$\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \textbf{(Faradays law)}$$

Faradays law of electromagnetic induction:

According to this law, the magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through it. The induced emf will be in a direction which opposes the change which causes it.

Induced emf e = - $\frac{d\varphi}{dt}$ where Φ is the flux linking with the circuit.

Differential form of Faradays Law:

From Stokes theorem $\oint E.dl = \int\int_s \left(\nabla x \vec{E}\right).ds$

The above equation becomes

$$\int\int_s \left(\nabla x \vec{E}\right).ds = -\int\int \frac{d\vec{B}}{dt}.\vec{ds}$$

$$\therefore \nabla x\vec{E} = -\frac{d\vec{B}}{dt}$$

6A

Gauss divergence theorem:

Statement: The volume integral of the divergence of a vector function 'F' over a volume 'V' is equal to the surface integral of the normal component of the vector function 'F' over the surface enclosing the volume V.

(2 marks)

Explanation:

Consider a Gaussian surface enclosing a charge Q with a charge density ρ_v.

Then $Q = \int\int\int_v \rho dv$

From Gauss law, total charge enclosed is

$$\therefore Q = \int\int \vec{D} \bullet ds$$

(2 marks)

From differential form of Gauss law, $\nabla \bullet \vec{D} = \rho_v$

$$\therefore \int_v \nabla \bullet \vec{D}\, dv = \int_s \vec{D} \bullet ds$$

$$\therefore Q = \iint \int_v \nabla \bullet \vec{D} dv = \int_s \vec{D} \bullet ds$$

(2 marks)

6B

Formula – 2 marks, Answer-2 marks

$$curl\ \ of\ \ \vec{A} = \nabla X \vec{A} = \begin{vmatrix} \ddot{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$curl\ \ of\ \ \vec{A} = \nabla X \vec{A} = \begin{vmatrix} \ddot{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 + yz^2 & xy^2 & x^2 y \end{vmatrix}$$

$$= \hat{i}(x^2 - 0) - \hat{j}(2xy \ \ - 2yz) + \hat{k}(y^2 - z^2)$$

7A

EQUATION OF CONTINUITY:

Consider a closed surface enclosing a volume of charge density ρ. If there is any charge flow through this surface, then charge flow through a small area ds is $\vec{J} \bullet \vec{ds}$

Total amount of charge flowing outwards = $\int \int_s \vec{J} \bullet \vec{ds}$

$$\Rightarrow \int\int\int_V \nabla \bullet \vec{J}\ dv = -\frac{\partial}{\partial t}\int\int\int_V \rho dv$$

(2 marks)

$$\Rightarrow \nabla \bullet \vec{J} = -\frac{\partial \rho}{\partial t}$$

Consider the expression for amperes law ,

$$\nabla x \vec{H} = \vec{J}$$

Taking divergence on both sides

$$\nabla \bullet (\nabla x \vec{H}) = \nabla \bullet \vec{J}$$

Since the divergence of a curl is zero

$$\nabla \bullet \vec{J} = 0$$

This means that divergence of current is zero or there is no net accumulation or loss of charge.[There are no sources or sinks of current].This is true in the case of steady ,direct currents which are continuous through out the circuit.

(2 marks)

However in a circuit containing a capacitor, during discharging, the current starts at the positively charged plate and the charge diminishes as the current flows to the negatively charged plate where it accumulates. Hence we can look upon the condenser plates as sources or sinks of currents. So Maxwell concluded that the equation $\nabla \bullet \vec{H} = \vec{J}$ is incomplete and besides the term $\vec{J}$, there must be a term added such that the sum of the two will give zero divergence.

From the equation of continuity

$$\nabla \bullet \vec{J} = -\frac{\partial \rho}{\partial t} \text{ where } \rho \text{ is the charge density}$$

$$\nabla \bullet \vec{J} = -\frac{\partial}{\partial t}\left(\nabla \bullet \vec{D}\right) = \nabla \bullet \frac{\partial \vec{D}}{dt}$$

$$\Rightarrow \nabla \bullet \left(\vec{J} + \frac{\partial \vec{D}}{dt}\right) = 0$$

Thus although $\nabla \bullet \vec{J}$ is not zero but the divergence of $\left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right)$ is always zero .Hence Maxwell made the assumption that the term $\vec{J}$ in amperes law must be replaced by $\left(\vec{J}+\frac{\partial \vec{D}}{\partial t}\right)$.

$\therefore$ **For time varying fields** $\nabla x \vec{H} = \vec{J}+\frac{\partial \vec{D}}{\partial t}$ (2 marks)

The term $\left(\frac{\partial \vec{D}}{\partial t}\right)$ is known as displacement current density .It is clear from the above expression that the rate of change of electric displacement produces magnetic field just as the conduction current ($\vec{J}$)does.

7B

Formula - 1 mark, Substitution- 1 mark, Answer- 2 marks

Along the axis of the coil

$$Radius = 1m$$
$$n = 100$$
$$x = 0.2m$$
$$I = 0.4A$$
$$\vec{B} = \mu \frac{nIR^2}{2(R^2 + x^2)^{\frac{3}{2}}} = \frac{4\pi x 10^{-7} x 100 \times 0.4 x 1^2}{2(1^2 + 0.2^2)^{\frac{3}{2}}} = 2.36 x 10^{-5} T$$

8A

Divergence:

(2 marks)

It represents the magnitude of a physical quantity emerging or converging at a point. For example tip of a fountain head is a source of divergence. Electric fields are said to be divergent in nature. Mathematically it is obtained by differentiating components of a vector function F (F_x ,F_y, F_z) with respect to position coordinates x,y,z respectively.

$$\nabla \bullet \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Ex: Volume charge density enclosed in a closed surface is expressed as

$$\nabla \bullet \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Right hand side in the above expression is a scalar. Divergence operation on vector yields a scalar function. Divergence of vector is zero if there is no outflow or inflow. Magnetic fields form closed loops and their divergence is zero. $\nabla \bullet B = 0$

Diverging electric field lines at a positive chargeis an example for Positive divergence.

converging electric field lines at a negative charge is an example of negative divergence.

Gradient :

(2 marks)

This operation is performed to study spatial variation of a scalar variable. It yields a vector function**.**

Ex: Variation of temperature in a room is represented as

$$\nabla T = \frac{\partial T}{\partial x}\hat{a}_x + \frac{\partial T}{\partial y}\hat{a}_y + \frac{\partial T}{\partial z}\hat{a}_z$$

The right hand side of this expression is a vector. It is in the direction of **maximum** increase/decrease of the scalar variable.

Electric field is represented as gradient of scalar vector potential

$$E = -\nabla V = (-)\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

Curl :

(2 marks)

This operation is a measure of degree of rotation per unit area. It yields a vector.

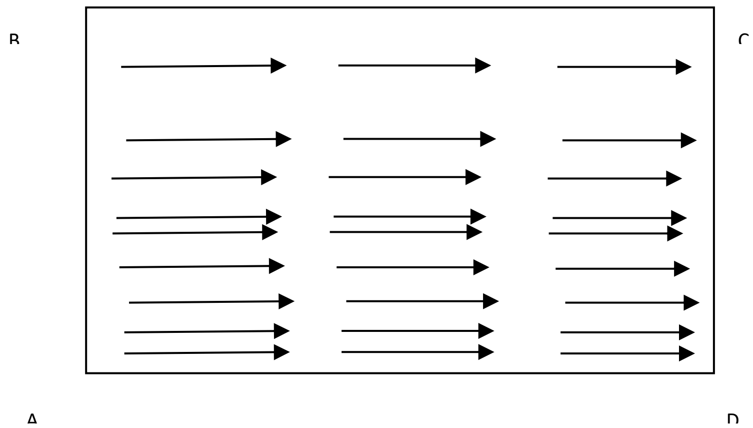
$$\nabla X\vec{a} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix}$$

Ex: Magnetic field around a straight conductor carrying current is expressed as

$$\nabla XH = J$$

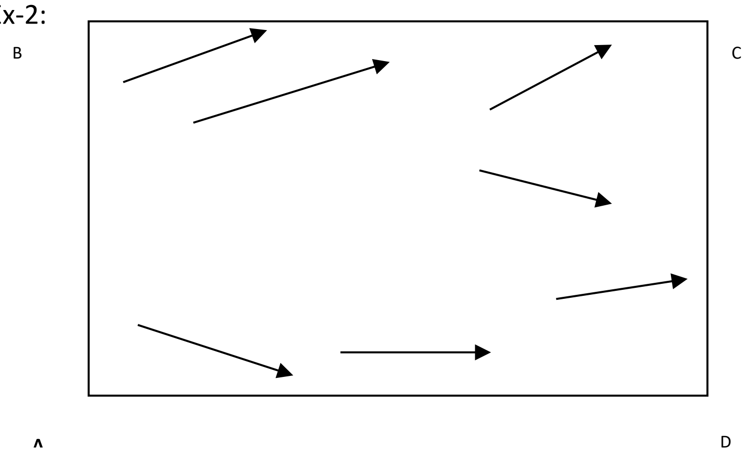
Curl of a irrotational vector is zero. Static electric fields possesses no curl. $\nabla XE = 0$

Ex-1:



In the above diagram, net rotation of a vector over the area ABCD is zero. So curl is zero.

Ex-2:



In the above diagram, net anticlockwise rotation is observed for a vector over the area ABCD. So curl is finite and positive.

8B

TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES:

Consider a uniform plane wave propagating along X direction in a medium where there are no free charges.

From Maxwell's equation $\nabla.\vec{D} = \rho = 0$

For free space

$$\nabla.\varepsilon\vec{E} = \rho = 0$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \qquad \text{...........(1)}$$

As the wave is propagating along X direction .So E_y and E_z remain constant along Y and Z direction respectively.

$$\frac{\partial E_y}{\partial y} = 0 = \frac{\partial E_z}{\partial z}$$

(2 marks)

So from (1) $\frac{\partial E_x}{\partial x} = 0$ .....(2)

$\therefore \frac{\partial E_x}{\partial x} = 0 \Rightarrow E_x =$ Constant or E_x =0.

For a plane wave E_x cannot be a constant as it is periodic.

$$\therefore E_x = 0$$

This means that a uniform plane wave progressing in the x direction has no X component of electric field. It indicates that electric field is perpendicular to the direction of propagation.

(2 marks)