Solutions to IAT-3 PHYSICS FOR CSE

1. A

Two vectors are orthogonal if their inner product is zero.

(3 marks)

$$\langle\alpha|\beta\rangle \equiv \langle\beta|\alpha\rangle = 0$$

$$Examples:$$

$$|0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

EX: $|1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\langle 1\|0\rangle = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

If the Vectors are orthogonal and each vector has unit length, then they are said to be orthonormal

(3 marks)

$$\langle 0|0\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1$$

$$\langle 1|1\rangle = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

1B

HADAMARD MATRIX $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (1 mark)

It is Hermitian and also satisfies unitary condition.

$$H[H^{+}]^{T} = I$$

$$[H^{+}] = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$[H^{+}]^{T} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(3 marks)

$$H[H^{+}]^{T} = \frac{1}{2}\begin{bmatrix} (1X1)+(1X1) & (1X1)+(1X-1) \\ (1X1)+(-1X1) & (1X1)+(-1X-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2A

PHASE GATE S gate: It is P gate with ϕ = π/2 . It represents rotation of 90^0 about Z axis

$$S = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$ (2 marks)

Truth Table .

(2 marks) (2 marks)

INPUT	OUTPUT
$\alpha\|0\rangle + \beta\|1\rangle$	$\alpha\|0\rangle + \beta e^{i\theta}\|1\rangle$
0	$\frac{1}{\sqrt{2}}\|0\rangle$
1	$\frac{1}{\sqrt{2}}e^{-i\theta}\|1\rangle$

2B .

(Min. 4 differences- 1mark each)

Classical computer	Quantum computer
1.Data is stored 0's and 1's represented by capacitor / transistor/etc at LOW / HIGH voltage	1.Data is stored as 0's (lower energy state), 1's (upper energy state) and linear combination of these states occupied by photons/electrons/ atoms/ nuclei / ions
2.Processing performed through logic gates	2. superposition allows for exponentially many quantum states at once
3. A bit is either in 0 or 1 state	QUBIT can 0 or 1 or superposed state
4. Quantum entanglement not possible	3. Quantum entanglement applicable Two particles that are too far apart can be strongly correlated.
5.Classical computer conduct sequential operations	5. Quantum computers can do 2^n operations at a time
6 Classical gates are irreversible	6. Quantum gates are reversible
7. Number of BITS needed for memory is linear function of number of numbers	7 Number of QUBITS needed for memory is logarithmic function of number of numbers

3A

PAULI MATRICES

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

σ_x is a classical not gate. When operated on a state vector say $|0\rangle$, it flips to $|1\rangle$

$$\sigma_x|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + 1x0 \\ 1x1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\sigma_x|1\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + 1x1 \\ 1x0 + 0x1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(2 marks)

$$\sigma_y|0\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0x1 + (-ix0) \\ ix1 + 0x0 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix}$$

$$\sigma_y|1\rangle = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0x0 + (-ix1) \\ ix0 + 0x1 \end{bmatrix} = \begin{bmatrix} -i \\ 0 \end{bmatrix}.$$

(2 marks)

$$\sigma_z|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1x1 + 0x0 \\ 0x1 + -1x0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_z|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1x0 + 0x1 \\ 0x0 + -1x1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

(2 marks)

3B

(Condition- 1 mark, Cheching- 3 marks)

$$A = \begin{bmatrix} 2i & 1-4i \\ 1+4i & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2i & 1+4i \\ 1-4i & 4 \end{bmatrix}$$

$$A^{+} = [A *]^{T} \quad = \begin{bmatrix} -2i & 1-4i \\ 1+4i & 4 \end{bmatrix}$$

$A \neq A^{+}$

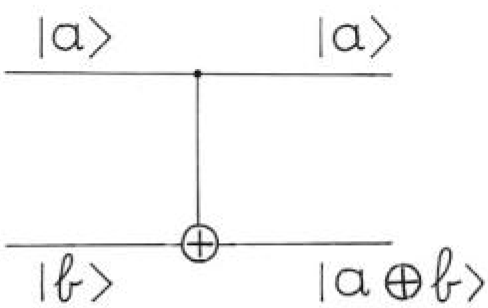
Hence it is not Hermitian

4A

CONTROLLED NOT GATE

a and b are two inputs. a is called control qubit and b the target qubit. Target qubit flips if and only if a= 1. If a = 0, the second qubit remains unchanged.

(2 marks)



Input	Output
$\|00\rangle$	$\|00\rangle$
$\|01\rangle$	$\|01\rangle$
$\|10\rangle$	$\|11\rangle$
$\|11\rangle$	$\|10\rangle$

Truth table (2 marks)

INPUT CONTROL BIT	INPUT TARGET BIT	OUTPUT CONTROL BIT	OUTPUT TARGET BIT
X_1	X_o	Y_1	y_o
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

(2 marks)

4B

A Linear Operator 'X' operates such that $A|0\rangle = i|1\rangle$ and $A|1\rangle = -i|0\rangle$ Find the matrix representation of 'X'.

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(2 marks)

$$A|0\rangle = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = i|1\rangle = i\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$a \times 1 + b \times 0 = 0 \Rightarrow a = 0$$

$$c \times 1 + d \times 0 = i \Rightarrow c = i$$

$$A|1\rangle = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -i|0\rangle = -i\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$a \times 0 + b \times 1 = -i \Rightarrow b = -i$$
$$c \times 0 + d \times 1 = 0 \Rightarrow d = 0$$
$$\therefore A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

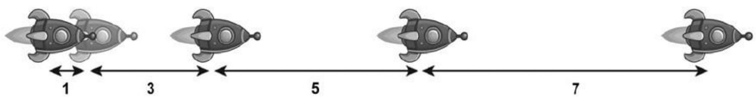
(2 marks)

5A

ODD RULE

(2 marks)

When acceleration is constant, one can use the Odd Rule to time the frames. With this method, one calculate the distance the object moves between frames using a simple pattern of odd numbers. Between consecutive frames, the distance the object moves is a multiple of an odd number. For acceleration, the distance between frames increases by multiples of 1, 3, 5, 7, etc.



Rocket speeding up using the Odd Rule.

For deceleration, the multiples start at a higher odd number and decrease, for example 7, 5, 3, 1.

Sled coming to a stop using the Odd Rule.

The Odd Rule is a multiplying system based on the smallest distance traveled between two frames in the sequence. For a slow-out, this is the distance between the first two frames; for a slow-in, it's the distance between the last two frames. This distance, the base distance, is used in all Odd Rule calculations.

Odd Rule Scenarios .

(4 marks)

Here are a few different scenarios for calculating the distance an object travels between keys in a slow-in or slow-out.

Base Distance Known Speeding up

If the object is speeding up, the first frame distance is the base distance. If one knows the base distance, figuring out the distance the object travels at each frame is pretty straightforward. Just multiply the base distance by 3, 5, 7, etc. to get the distances between consecutive frames, or use squares to multiply the base distance to get the total distance traveled on each frame.

Base Distance Known Slowing Down

Suppose one wants an object to slow down, and one knows the distance between the last two frames before it stops. For slow-ins, the base distance is the distance between the last two frames. The solution is to work backward, as if the object were speeding up in the opposite direction. Working
backward, multiply the base distance by 3, 5, 7, etc. to get the distances between each previous frame in the sequence.

Using the base distance, one can calculate the distances between each frame. If one adds up the distances traveled, one will find that they add up to exactly 0.4m.
In the charts above, note that the distances in the last column are squared numbers: 4 = 2^2, 9 = 3^2, 16 = 4^2,and so on. One of the benefits of the Odd Rule is one can calculate the total distance traveled from the start point to thecurrent frame with the following formula:
Odd number multiplier for consecutive frames = ((frame # – 1) X 2) – 1

Total Distance and Number of Frames Known, Speeding Up
If one knows the total distance and the total number of frames, one can find the base distance with this formula:Base distance = Total distance/(Last frame number – 1) ^2
Suppose there is a jump push (takeoff) with constant acceleration over 5 frames, and the total distance traveled is 0.4m. Using the formula above, we find the base distance.
Base distance = 0.4m/(5 – 1) ^2 = 0.4m/16 = 0.025m

First Key Distance Known Slowing Down
Suppose one has a moving object that one wants to slow down, and one has set the first frame of the slow-in to give an idea of the pacing for the sequence. In this case, one can consider that the distance the object moved between the last two frames before the slow-in is part of the calculation— the distance between them becomes the first frame distance, and the first slow-in frame becomes the second frame in the sequence..

5B

Walking

(1 mark)

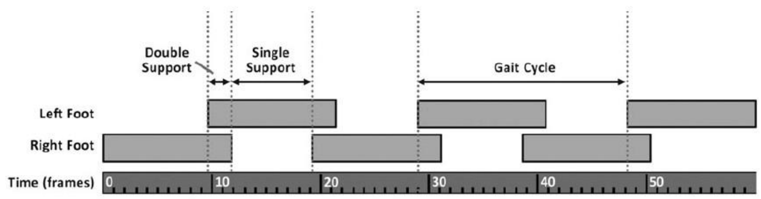
Walks feature all the basics of mechanics while including personality. The ability to animate walk cycles is one of the most important skills a character animator needs to master.

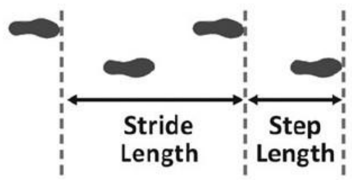
Strides and Steps

(3 marks)

A step is one step with one foot. A stride is two steps, one with each foot. Stride length is the distance the character travels in a stride, measured from the same part of the foot. Step and stride length indicate lengthwise spacing for the feet during a walk.

Gait is the timing of the motion for each foot, including how long each foot is on the ground or in the air. During a walk, the number of feet the character has on the ground changes from one foot (single support) to two feet (double support) and then back to one foot. You can plot the time each foot is on the ground to see the single and double support times over time. A normal walking gait ranges from 1/ 3 to 2/3 of a second per step, with 1/2 second being average.





Walk Timing

Walking is sometimes called “controlled falling.” Right after you move past the passing position, your body’s center of gravity is no longer over your base of support, and you begin to tip. Your passing leg moves forward to stop the fall, creating your next step. Then the cycle begins again. The horizontal timing for between the four walk poses is not uniform. The CG slows in going from the contact to passing position, then slows out from passing to contact. The CG also rises and falls, rising to the highest position during passing and the lowest during contact. The head is in the highest position during passing.

6A

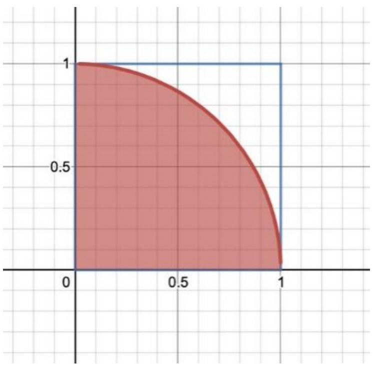
Monte carlo method:

Monte Carlo methods vary, but tend to follow a particular pattern: (3 marks)

1. Define a domain of possible inputs
2. Generate inputs randomly from a probability distribution over the domain
3. Perform a deterministic computation on the inputs
4. Aggregate the results

Monte Carlo method applied to approximating the value of π. For example, consider a quadrant inscribed in a unit square. Given that the ratio of their areas is $\pi/4$, the value of π can be approximated using a Monte Carlo method:

(3 marks)



1. Draw a square, then Inscribe a quadrant within it
2. Uniformly scatter a given number of points over the square OR find the total number of full squares
3. Count the number of points inside the quadrant, i.e. having a distance from the origin of < 1
4. The ratio of the inside-count and the total-sample-count is an estimate of the ratio of the two areas, $\pi/4$.

   Multiply the result by 4 to estimate π/

   Number of points = 78

   Total number of points = 100

   Π = 4 x 78/100 =3.12

6B (Min. 4 differences- 1 mark each)

The field of statistics is divided into two major divisions: descriptive and inferential. Each of these segments is important, offering different techniques that accomplish different objectives. Descriptive statistics describe what is going on in a population or data set. Inferential statistics, by contrast, allow scientists to take findings from a sample group and generalize them to a larger population. The two types of statistics have some important differences.

Descriptive Statistics

Descriptive statistics is the type of statistics that probably springs to most people's minds when they hear the word "statistics." In this branch of statistics, the goal is to describe. Numerical measures are used to tell about features of a set of data. There are a number of items that belong in this portion of statistics, such as:

- The average, or measure of the center of a data set, consisting of the mean, median, mode, or midrange
- The spread of a data set, which can be measured with the range or standard deviation
- Overall descriptions of data such as the five number summary
- Measurements such as skewness and kurtosis
- The exploration of relationships and correlation between paired data
- The presentation of statistical results in graphical form

These measures are important and useful because they allow scientists to see patterns among data, and thus to make sense of that data. Descriptive statistics can only be used to describe the population or data set under study: The results cannot be generalized to any other group or population.

Types of Descriptive Statistics

There are two kinds of descriptive statistics that social scientists use:

Measures of central tendency capture general trends within the data and are calculated and expressed as the mean, median, and mode. A mean tells scientists the mathematical average of all of a data set, such as the average age at first marriage; the median represents the middle of the data distribution, like the age that sits in the middle of the range of ages at which people first marry; and, the mode might be the most common age at which people first marry.

Measures of spread describe how the data are distributed and relate to each other, including:

- The range, the entire range of values present in a data set
- The frequency distribution, which defines how many times a particular value occurs within a data set
- Quartiles, subgroups formed within a data set when all values are divided into four equal parts across the range
- Mean absolute deviation, the average of how much each value deviates from the mean
- Variance, which illustrates how much of a spread exists in the data
- Standard deviation, which illustrates the spread of data relative to the mean

Measures of spread are often visually represented in tables, pie and bar charts, and histograms to aid in the understanding of the trends within the data.

Inferential Statistics

Inferential statistics are produced through complex mathematical calculations that allow scientists to infer trends about a larger population based on a study of a sample taken from it. Scientists use inferential statistics to examine the relationships between variables within a sample and then make generalizations or predictions about how those variables will relate to a larger population.

It is usually impossible to examine each member of the population individually. So scientists choose a representative subset of the population, called a statistical sample, and from this analysis, they are able to say something about the population from which the sample came. There are two major divisions of inferential statistics:

- A confidence interval gives a range of values for an unknown parameter of the population by measuring a statistical sample. This is expressed in terms of an interval and the degree of confidence that the parameter is within the interval.
- Tests of significance or hypothesis testing where scientists make a claim about the population by analyzing a statistical sample. By design, there is some uncertainty in this process. This can be expressed in terms of a level of significance.

Techniques that social scientists use to examine the relationships between variables, and thereby to create inferential statistics, include linear regression analyses, logistic regression analyses, ANOVA, correlation analyses, structural equation modeling, and survival analysis. When conducting research using inferential statistics, scientists conduct a test of significance to determine whether they can generalize their results to a larger population. Common tests of significance include the chi-square and t-test. These tell scientists the probability that the results of their analysis of the sample are representative of the population as a whole.

7A

Modeling the Probability for Proton Decay

The experimental search for Proton Decay was undertaken because of the implications of the Grand unification Theories. The lower bound for the lifetime is now projected to be on the order of $\tau = 10^{33}$ Years. The probability for observing a proton decay can be estimated from the nature of particle decay and the application of Poisson Statistics. The number of protons N can be modeled by the decay equation

$$N_0 = N\ e^{-\lambda t}$$

Here $\lambda = 1/t = 10^{-33}/$ year is the probability that any given proton will decay in a year. Since the decay constant λ is so small, the exponential can be represented by the first two terms of the Exponential Series.

$N \approx N_o(1-\lambda t)$ (2 marks)

For a small sample, the observation of a proton decay is infinitesmal, but suppose we consider the volume of protons represented by the Super Kameokande neutrino detector in Japan. The number of protons in the detector volume is reported by Ed Kearns of Boston University to be 7.5 x 10^{33}protons. For one year of observation, the number of expected proton decays is then

$N - N_o = N_o \lambda t = 7.5x10^{33} x10^{-33} x10^{-33} / year x1 year = 7.5$ (2 marks)

About 40% of the area around the detector tank is covered by photo-detector tubes, and if we take that to be the nominal efficiency of detection, we expect about three observations of proton decay events per year based on a 10^{33}year lifetime.

So far, no convincing proton decay events have been seen. Poisson statistics provides a convenient means for assessing the implications of the absence of these observations. If we presume that $\lambda = 3$ observed decays per year is the mean,

then the Poisson distribution function tells us that the probability for zero observations of a decay is

$p(k) = \frac{\lambda^k e^{-\lambda}}{k!}$ $p(k) = \frac{3^0 e^{-3}}{o!} = 0.05$ (2 marks)

7B

The number of particles emitted randomly by a radio active sample obeys Poisson distribution with λ =4. Calculate p(x=0), P(x =1)

$$P(X = k = 0) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(X = k = 0) = \frac{4^0 e^{-4}}{0!}$$

$P(X = k = 0) = 0.0183$ (2 marks)

$$P(X = k = 1) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$P(X = k = 1) = \frac{4^1 e^{-4}}{1!}$$

$P(X = k = 1) = 0.0732$ (2 marks)

8A

Size and Scale(3 marks)

The size and scale of characters often play a central role in a story's plot. What would Superman be without his height and bulging biceps? Some characters, like the Incredible Hulk, are even named after their body types.

e often equate large characters with weight and strength, and smaller characters with agility and speed. There is a reason for this. In real life, larger people and animals do have a larger capacity for strength, while smaller critters can move and maneuver faster than their large counterparts. When designing characters, you can run into different situations having to do with size and scale, such as:

1. Human or animal-based characters that are much larger than we see in our everyday experience. Superheroes, Greek gods, monsters,
2. Human or animal-based characters that are much smaller than we are accustomed to, such as fairies and elves.
3. Characters that need to be noticeably larger, smaller, older, heavier, lighter, or more energetic than other characters.
4. Characters that are child versions of older characters. An example would be an animation featuring a mother cat and her kittens. If the kittens are created and animated with the same proportions and

timing as the mother cat, they won’t look like kittens; they’ll just look like very small adult cats.

Proportion and Scale

Creating a larger or smaller character is not just a matter of scaling everything about the character uniformly. To understand this, let’s look at a simple cube. When you scale a cube, its volume changes much more dramatically than its surface area. Let us say each edge of the cube is 1 unit length. The area of one side of the cube is 1 square unit, and the volume of the cube is 1 cubed unit. If you double the size of the cube along each dimension, its height increases by 2 times, the surface area increases by 4 times, and its volume increases by 8 times. While the area increases by squares as you scale the object, the volume changes by cubes.

Wight and strength(3 marks)

Body weight is proportional to volume. The abilities of your muscles and bones, however, increase by area because their abilities depend more on cross-sectional area than volume. To increase a muscle or bone’s strength, you need to increase its cross- sectional area. To double a muscle’s strength, for example, you would multiply its width by $\sqrt{2}$. To triple the strength, multiply the width by $\sqrt{3}$. Since strength increases by squares and weight increases by cubes, the proportion of a character’s weight that it can lift does not scale proportionally to its size.

Let us look at an example of a somewhat average human man. At 6 feet tall, he weighs 180 pounds and can lift 90 pounds. In other words, he can lift half his body weight. If you scale up the body size by a factor of 2, the weight increases by a factor of 8. Such a character could then lift more weight. But since he weighs more than 8 times more than he did before, he can not lift his arms and legs as easily as a normal man. Such a giant gains strength, but loses agility.

8B

In the case of Jump action, push height is 0.5m and Jump magnification is 5.5. Calculate the jump height, push acceleration. Gravitation acceleration = 10m/s^2

Jump magnification = Jump height / push height

Jump height = 5.5 x 0.5 =2.75m(2 marks)

Jump magnification = Push acceleration / gravitational acceleration

Push acceleration = 5.5 x 10 =55 m/s^2(2 marks)