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CMRIT

Internal Assessment Test – II January 2024

Sub:	Mathematics for Computer Science							Code:	BCS301
Date:	17/01/2024	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	AIML/AIDS/ CSE/ISE/ CS DS/CS ML

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Every year a man trades his car for a new car in 3 brands of the popular company ‘Tata motors’. If he has a ‘Nexon’ he trades it for a ‘Punch’. If he has a ‘Punch’ he trades if for a ‘Jaguar Land Rover’. If he has a ‘Jaguar Land Rover’ he is just as likely to trade it for a new ‘Jaguar Land Rover’ or for a ‘Punch’ or a ‘Nexon’ one. In 2020 he bought his first car which was ‘Jaguar Land Rover’. Find the probability that he has i) 2022 ‘Jaguar Land Rover’ ii) 2022 ‘Nexon’ iii) 2023 ‘Punch’ iv) 2023 ‘Jaguar Land Rover’	[8]	CO2	L3
2	Define Probability Vectors, Stochastic matrices, Regular stochastic matrix, stationary distribution and absorbing state of Markov chain.	[7]	CO2	L1
3	Find the unique fixed probability vector [illegible] for the regular stochastic matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$	[7]	CO2	L3

4	Explain the following: Null and Alternative hypothesis, Type I and Type II errors, Critical region, Tests of significance, Two-tailed tests.	[7]	CO3	L1
5	A coin was tossed 1000 times and the head turned up 540 times. Decide on the hypothesis that the coin is unbiased at 1% level of significance.	[7]	CO3	L3
6	In an examination given to students at a large number of different schools the mean grade was 74.5 and S.D grade was 8. At one particular school where 200 students took the examination the mean grade was 75.9. Discuss the significance of this result at 5% and 1% level of significance.	[7]	CO3	L3
7	A random sample for 1000 workers in company has mean wage of Rs. 50 per day and S.D of Rs. 15. Another sample of 1500 workers from another company has mean wage of Rs. 45 per day and S.D of Rs. 20. Does the mean rate of wages varies between the two companies? Find the 95% confidence limits for the difference of the mean wages of the population of the two companies.	[7]	CO3	L3
8	In two large populations there are 30% and 25% respectively of fair-haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations.?	[7]	CO3	L3

1. Given, There are 3 components or cars : Nexon, Punch, Jaguar

So, the State space is = {N, P, J}

Since there are 3 components in the state space, therefore, The order of the Transition probability matrix is 3X3.

According to question :

$$\begin{array}{c|c} & \begin{matrix} N & P & J \end{matrix} \\ \begin{matrix} N \\ P \\ J \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ x & x & x \end{bmatrix} \end{array}$$

we know that, $\sum P_{ij} = 1$

$$\Rightarrow x + x + x = 1$$

$$3x = 1$$

$$x = \frac{1}{3}$$

∴ The Required Transition probability matrix is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

Given, Initially in 2020 he bought his first car which was Jaguar Land Rover. Hence, the initial probability vector is

$$p^{(0)} = [0 \quad 0 \quad 1]$$

(i) To reach year 2022, 2-step transition is required.

i.e $p^{(2)} = p^{(0)} \cdot P^2$

$$= [0 \quad 0 \quad 1] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$p^{(2)} = [0 \quad 0 \quad 1]\begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix}$$

$$p = [1/9, 4/9, 4/9] = [P_N^{(2)}, P_P^{(2)}, P_J^{(2)}]$$

∴ The probability that he has Jaguar Land Rover, In 2022 is

$$\therefore \boxed{P_J^{(2)} = \frac{4}{9}}$$

(ii). From sub question (i).

The probability that he has Nexon, In 2022 is

$$\therefore \boxed{P_N^{(2)} = \frac{1}{9}}$$

(iii). To reach year 2023, 3-step transition is required,

i.e $p^{(3)} = p^{(0)} \cdot p^3$

$$= [0 \quad 0 \quad 1]\begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$= [0 \quad 0 \quad 1]\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/9 & 4/9 & 4/9 \\ 4/27 & 7/27 & 16/27 \end{bmatrix}$$

$$p = [4/27, 7/27, 16/27] = [P_N^{(3)}, P_P^{(3)}, P_J^{(3)}]$$

∴ The probability that he has Punch, In 2023 is

$$\therefore \boxed{P_P^{(3)} = \frac{7}{27}}$$

$\langle \frac{1}{2}{}^{x} \frac{1}{4} \quad \frac{7}{4}$

(iv). From sub question (iii).
The probability that he has Jaguar Land Rover, in 2023 is

$$\therefore \boxed{p_J^{(3)} = \frac{16}{27}}$$

2. Probability Vectors : ~~A vector~~

A vector $v = \{v_1 \ldots v_n\}$ is said to be Probability vector if each of it's components are non-negative and their sum is equal to unity.

Ex : $v = \{1/2 \;\; 1/2\}$

$\Sigma v_i = 1$ and $v_i \geq 0$

• Stochastic matrices :

A square matrix $P = P_{ij}$ is said to be stochastic matrix if every row of the matrix is of the form of Probability vector.

Ex : ①. A Identity matrix of any order.

②. $A = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$

• Regular stochastic matrix :

A stochastic matrix P is said to be a Regular stochastic matrix ~~is~~ if all the entries of some power p^n are positive.

Ex : $A = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$ consider $A^2 = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/4 & 3/4 \end{bmatrix}$

$\therefore A$ is a Regular stochastic matrix.

• Stationary distribution:

A stationary distribution ~~is~~ of a Markov chain is a probability distribution which remains unchanged in a Markov chain as time progresses.

Mathematically $vP = v$.

• Absorbing state of Markov chain:

A state in Markov chain is said to be absorbing if and only if rows of the transition matrix corresponding to the state has 1 in the main diagonal and zeroes elsewhere.

Ex:

$$A = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cc} & \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ & \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}\begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} \end{array}$$

Here a_2 is an absorbing state (row-wise)

3. Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$ is a regular stochastic matrix.

Let $v = [x \;\; y \;\; z]$ be the fixed probability vector, such that

$x + y + z = 1 \Rightarrow z = 1 - x - y \rightarrow (1)$.

To find unique fixed probability vector, we have to solve

$$vA = v$$

$$[x \;\; y \;\; z]\begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} = [x \;\; y \;\; z]$$

$$\left[\frac{y}{6},\ x+\frac{y}{2}+\frac{2z}{3},\ \frac{y}{3}+\frac{z}{3}\right]=[x\ y\ z]$$

$$\Rightarrow \frac{y}{6}=x \rightarrow (2)$$

$$x+\frac{y}{2}+\frac{2z}{3}=y \Rightarrow 6x+3y+4z=6y \rightarrow (3)$$

$$\frac{y}{3}+\frac{z}{3}=z \Rightarrow y+z=3z \Rightarrow y=2z \rightarrow (4).$$

Substitute eq^n (1) in (3)

we get, $6x+3y+4(1-x-y)=6y$

$$6x+4-4x-4y=3y$$

$$2x+4=7y$$

Substitute eq^n (2) in above eq^n.

$$\frac{2y}{\cancel{6}\,3}+4=7y$$

$$\frac{y}{3}+4=7y$$

$$7y-\frac{y}{3}=4$$

$$21y-y=12$$

$$20y=12$$

$$y=\frac{\cancel{12}\,3}{\cancel{20}\,5}$$

$$\therefore \boxed{y=\frac{3}{5}}$$

Substitute $y=\frac{3}{5}$ in eq^n (2)

$$\frac{\cancel{3}\,1}{5\times\cancel{6}\,2}=x \Rightarrow x=\frac{1}{10}$$

$$\therefore \boxed{x=\frac{1}{10}}$$

Substitute $y = \frac{3}{5}$ in eqⁿ (4), we get

$$\frac{3}{5} = 2z$$

$$\therefore \boxed{z = \frac{3}{10}}$$

$\therefore$ $\boxed{v = [x \; y \; z] = \left[\frac{1}{10} \; \frac{3}{5} \; \frac{3}{10}\right]}$ is the required unique fixed probability vector.

4. • **Null hypothesis :**

It is a hypothesis which test for the possible rej-ection under the assumption that ~~is~~ it is True.

• **Alternative hypothesis :**

Any hypothesis that is complamentary of Null hypo-thesis is known as Alternative hypothesis.

• **Critical region :**

A region the amounts to the rejection of Null hypothesis is called Critical region.

• **Tests of Significance :**

The process which helps us ~~do~~ to decide whether the hypothesis is Accepted and/or Rejected is known as Tests of significance.

• **Two-tailed tests :**

A statistical test in which the critical area of a distribution is two-sided and tests whether the sample is greater or less than a range of values.

Test	Critical value of Z	
	5% level	1% level
Two-tailed	-1.96 & 1.96	-2.58 & 2.58

• **Type-I error :**

If the hypothesis is rejected ~~under the~~ when the hypothesis is actually True is called Type-I error.

• **Type-II error :**

If the hypothesis is accepted when the hypothesis is actually False is called Type-II error.

5. Given, number of tosses, $n = 1000$

no. of head turned up, $x = 540$. (i.e the probability of getting head out of 1000 trials)

Hypothesis : The coin is unbiased at 1% level of significance.

Thus, The probability of getting head in one toss is

$p = \frac{1}{2}$ => $q = 1 - p = \frac{1}{2}$

Expected value of getting head out of 1000 trials is

$\mu = np = \cancel{1000}^{500} \times \frac{1}{\cancel{2}} = 500$

The standard deviation, $\sigma = \sqrt{npq} = \sqrt{1000 \times \frac{1}{2} \times \frac{1}{2}}$

$\sigma = \sqrt{250} = 15.81$

we know that,

$$z = \frac{x - \mu}{\sigma} = \frac{540 - 500}{15.81}$$

$$\boxed{z = 2.53 < 2.58}$$

$\therefore$ In two-tailed test, As $z = 2.53 < 2.58$, the hypothesis is accepted at 1% level of significance and we conclude that the coin is unbiased at 1% level of significance.

6. Given, Mean grade, $\mu = 74.5$

st. deviation, $\sigma = 8$.

No. of students at one particular school, $n = 200$

Define the hypothesis:

$H_0 : \mu = 74.5$, there is no change in mean grade.

$H_1 : \mu \neq 74.5$, being $\mu > 74.5$, $\mu < 74.5$.

The mean grade at one particular school is, $\bar{x} = 75.9$

We know that,

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{75.9 - 74.5}{8 / \sqrt{200}}$$

$\therefore z = 2.48$

(i) ~~At 5%~~

(ii) In One-tailed test:

As $z = 2.48 > 1.645$ and $z = 2.48 > 2.33$, the hypothesis is significant at both 5% and 1% level of significance respectively.

(ii) In two-tailed test :

- As $z = 2.48 > 1.96$, the hypothesis is ~~not~~ significant at 5% level of significance
- As $z = 2.48 < 2.58$, the hypothesis is not significant at 1% level of significance.

7. Given, $n_1 = 1000$; $\bar{x}_1 = 50$; $\sigma_1 = 15$

$n_2 = 1500$; $\bar{x}_2 = 45$; $\sigma_2 = 20$.

Hypothesis : There is no change in mean rate of wages between two companies.

We know that,

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{50-45}{\sqrt{\frac{225}{1000} + \frac{400}{1500}}}$$

$$\boxed{Z = 7.13}$$

$\therefore$ As $z = 7.13 \begin{cases} > 1.645 \\ > 2.33 \end{cases}$ } In one-tailed test
$\begin{cases} > 1.96 \\ > 2.58 \end{cases}$ } In two-tailed test } hypothesis rejected.

$\therefore$ We conclude that, hypothesis is rejected and there the mean rate of wages varies between the two companies.

To find 95% confidence limits in two-tailed test :

we know that

$$C.L = (\bar{x}_1 - \bar{x}_2) \pm Z_{0.05} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$C.L = (50-45) \pm 1.96 \sqrt{\frac{225}{1000} + \frac{400}{1500}}$$

$$= 5 + 1.96 \sqrt{\frac{225}{1000} + \frac{400}{1500}} \text{ and } 5 - 1.96 \sqrt{\frac{225}{1000} + \frac{400}{1500}}$$

$$= 6.37 \text{ and } 3.63 \text{ approx.}$$

$\therefore$ The 95% confidence limits are $\boxed{6.37}$ and $\boxed{3.63}$ approx.

8. Given, Proportion, $P_1 = 30\% = 0.3$

and Proportion, $P_2 = 25\% = 0.25$

Total population in 1st sample, $n_1 = 1200$

Total population in 2nd sample, $n_2 = 900$

We know that,

Compined probability, $P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{1200(0.3) + 900(0.25)}{1200 + 900}$

$$P = 0.279 \Rightarrow q = 1 - p = 0.721$$

We have,

$$\cancel{z = \frac{P_1 - P_2}{\sqrt{pq}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}} \qquad z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$z = \frac{0.3 - 0.25}{\sqrt{0.279(0.721)\left(\frac{1}{1200} + \frac{1}{900}\right)}}$$

$$\boxed{z = 2.53}$$

In two-tailed test:

$\therefore$ $z = 2.53$ $\begin{cases} > 1.96 & \text{at 5\% level of significance} \\ < 2.58 & \text{at 1\% level of significance.} \end{cases}$