Model Question Paper-II (CBCS Scheme)

USN										

Third Semester B.E Degree Examination

AV-MATHEMATICS FOR EC ENGINEERING STREAM (BMATEC301)

TIME: 03 Hours **Max.Marks:100**

Note: (i) Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE.**
(ii) Statistical tables and Mathematics Formula handbooks are allowed**.**

<table>
<tr><th colspan="3">Module -1</th><th>M</th><th>L</th><th>C</th></tr>
<tr><td>Q.01</td><td>a</td><td>Find the Fourier series expansion of $f(x)=\begin{cases} x & :0\le x\le \pi \\ 2\pi - x & :\pi \le x\le 2\pi \end{cases}$.</td><td>6</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>b</td><td>Expand $f(x)=2x-1$ as a Cosine half range Fourier series in $0 < x < 1$.</td><td>7</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>c</td><td>Obtain the constant term and the first coefficients of cosine and sine terms in the Fourier series expansion for the following data:
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>y</td><td>9</td><td>18</td><td>24</td><td>28</td><td>26</td><td>20</td></tr>
</table></td><td>7</td><td>L3</td><td>CO1</td></tr>
<tr><td colspan="3">OR</td><td></td><td></td><td></td></tr>
<tr><td>Q.02</td><td>a</td><td>Obtain the Fourier series for the triangular wave function
$f(x)=\begin{cases} \pi + x, & \text{for } -\pi \le x\le 0 \\ \pi - x, & \text{for } \ 0\le x\le \pi \end{cases}$</td><td>6</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>b</td><td>Obtain a half range sine series for $f(x)=(x-1)^2$ in (0, 1)</td><td>7</td><td>L2</td><td>CO1</td></tr>
<tr><td></td><td>c</td><td>Obtain the constant term and coefficients of first cosine and sine terms in the expansion of y from the following table
<table>
<tr><td>x</td><td>0</td><td>60°</td><td>120°</td><td>180°</td><td>240°</td><td>300°</td><td>360°</td></tr>
<tr><td>y</td><td>7.9</td><td>7.2</td><td>3.6</td><td>0.5</td><td>0.9</td><td>6.8</td><td>7.9</td></tr>
</table></td><td>7</td><td>L3</td><td>CO1</td></tr>
<tr><td colspan="3">Module-2</td><td></td><td></td><td></td></tr>
<tr><td>Q. 03</td><td>a</td><td>Find the Fourier Transform of the function $f(x)=\begin{cases} 1, & \text{for } \lvert x\rvert \le a \\ 0, & \text{for } \lvert x\rvert > a \end{cases}$
Hence evaluate (i) $\int_0^\infty \frac{\sin ax\cos sx}{x}\,dx$, (ii) $\int_0^\infty \frac{\sin ax}{x}\,dx$.</td><td>6</td><td>L2</td><td>CO2</td></tr>
</table>

	b	Find the Fourier cosine Transform of the function $f(x)=e^{-a\lvert x\rvert}$. Hence evaluate $\int_0^\infty \frac{\cos mx}{a^2+x^2}\,dx$.	7	L2	CO2
	c	Find the Transform of the sequence $\boldsymbol{f}=[0,\ 1,\ 4,\ 9]^{\boldsymbol{T}}$ of four values using Fast Fourier Transform.	7	L3	CO2
OR					
Q.04	a	Find the complex Fourier Transform of the function $f(x)=e^{-a^2x^2},\ a>0$	6	L2	CO2
	b	Find the Fourier sine Transform of the function $f(x)=\begin{cases}4x, & \text{for } 0<x<1\\ 4-x, & \text{for } 1<x<4\\ 0, & \text{for } x>4\end{cases}$	7	L1	CO2
	c	Obtain the inverse Fourier cosine Transform of the function $F_c(s)=\frac{\sin as}{s},\ a>0$	7	L3	CO2
Module-3					
Q. 05	a	Find the Z-transform of: $i)\ \sin hn\theta;\ \ ii)\ \cos n\theta$	6	L2	CO3
	b	Find the inverse Z-transform of $\frac{z^2+z}{z^3+6z^2+11z+6}$	7	L3	CO3
	c	Solve the difference equation using Z-transform $u_{n+2}+4u_{n+1}+3u_n=3^n$, with $u_0=0\ \&\ u_1=1$	7	L3	CO3
OR					
Q. 06	a	Find the Z-transform of $a^n n^2+4\sin\left(\frac{n\pi}{4}\right)+5$	6	L2	CO3
	b	Find the Inverse Z-transform of $\frac{z^3-20z}{(z-2)^3(z-4)}$	7	L3	CO3
	c	If $Z(u_n)=\frac{2z^2+3z+4}{(z-3)^3},\ \lvert z\rvert>3$. Find $u_1, u_2\ \&\ u_3$	7	L3	CO3
Module-4					
Q. 07	a	Solve $4D^4-4D^3-23D^2+12D+36=0$.	6	L2	CO4
	b	Solve $\frac{d^2y}{dx^2}-4y=\cosh(2x-1)+3^x$.	7	L2	CO4
	c	Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$	7	L3	CO4
OR					
Q. 08	a	Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=3\sin x+4\cos x$	6	L2	CO4

<table>
<tr><td></td><td>b</td><td>Solve $(2x+3)^2\dfrac{d^2y}{dx^2}-(2x+3)\dfrac{dy}{dx}-12y=6x.$</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c</td><td>An alternating E.M.F. $E\sin pt$ is applied to a circuit at $t=0$. Given the equation for the current i as $L\frac{d^2i}{dt^2}+R\frac{di}{dt}+\frac{i}{c}=pE\sin pt$, find i when (i) $CR^2>4L$ (ii) $CR^2>4L$</td><td>7</td><td>L3</td><td>CO4</td></tr>
<tr><td colspan="3">Module-5</td><td></td><td></td><td></td></tr>
<tr><td>Q. 09</td><td>a</td><td>Find a least square straight line for the following data
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>6</td><td>4</td><td>3</td><td>5</td><td>4</td><td>2</td></tr>
</table></td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>If the coefficient of correlation between the variables x and y is 0.5 and the acute angle between their lines of regression is $tan^{-1}\left(\frac{3}{8}\right)$, show that $\sigma_x=\frac{1}{2}\sigma_y$</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>Determine rank correlation for the following data which shows the marks obtained in two quizzes in mathematics
<table>
<tr><td>Marks in first quiz X</td><td>6</td><td>5</td><td>8</td><td>8</td><td>7</td><td>6</td><td>10</td><td>4</td><td>9</td><td>7</td></tr>
<tr><td>Marks in second quiz Y</td><td>8</td><td>7</td><td>7</td><td>10</td><td>5</td><td>8</td><td>10</td><td>6</td><td>8</td><td>6</td></tr>
</table></td><td>7</td><td>L3</td><td>CO5</td></tr>
<tr><td colspan="3">OR</td><td></td><td></td><td></td></tr>
<tr><td>Q. 10</td><td>a</td><td>Find a least square quadratic curve for the following data
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>y</td><td>12</td><td>10.5</td><td>10</td><td>8</td><td>7</td><td>2</td><td>8.5</td><td>9</td></tr>
</table>
Estimate $y\ at\ x=\ 6.5$</td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b</td><td>For the following data find the correlation coefficient between x and y.
<table>
<tr><td>x</td><td>6</td><td>5</td><td>8</td><td>8</td><td>7</td><td>6</td></tr>
<tr><td>y</td><td>8</td><td>7</td><td>7</td><td>10</td><td>5</td><td>8</td></tr>
</table>
Also find the standard error estimates.</td><td>7</td><td>L3</td><td>CO5</td></tr>
<tr><td></td><td>c</td><td>Find the two regression lines from the following data
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>y</td><td>14</td><td>12</td><td>16</td><td>11</td><td>14</td><td>12</td></tr>
</table></td><td>7</td><td>L3</td><td>CO5</td></tr>
</table>

$n=1$

1a) ► Here, the Fourier coefficients are

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\left\{\int_0^{\pi} x\,dx + \int_{\pi}^{2\pi}(2\pi - x)\,dx\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{x^2}{2}\right]_0^{\pi} + \left[2\pi x - \frac{x^2}{2}\right]_{\pi}^{2\pi}\right\} = \frac{1}{\pi}\left\{\frac{\pi^2}{2} + (2\pi)^2 - \frac{(2\pi)^2}{2} - 2\pi^2 + \frac{\pi^2}{2}\right\} = \pi, \qquad \text{(i)}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left\{\int_0^{\pi} x\cos nx\,dx + \int_{\pi}^{2\pi}(2\pi - x)\cos nx\,dx\right\}$$

$$= \frac{1}{\pi}\left\{\left[x\left(\frac{\sin nx}{n}\right) - 1\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi} + \left[(2\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{1}{n^2}(\cos n\pi - 1) - \frac{1}{n^2}(\cos 2n\pi - \cos n\pi)\right\} = \frac{2}{\pi n^2}\{(-1)^n - 1\}, \qquad \text{(ii)}$$

and $$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\left\{\int_0^{\pi} x\sin nx\,dx + \int_{\pi}^{2\pi}(2\pi - x)\sin nx\,dx\right\}$$

$$= \frac{1}{\pi}\left\{\left[x\left(-\frac{\cos nx}{n}\right) - 1\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi} + \left[(2\pi - x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{-\pi\left(\frac{\cos n\pi}{n}\right) - \pi\left(-\cos\frac{n\pi}{n}\right)\right\} = 0 \qquad \text{(iii)}$$

Using expressions (i), (ii) and (iii) we obtain the required Fourier expansion as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}\cos nx$$

Q1
b)

Expand $f(x) = 2x - 1$ *as a cosine half range Fourier series in* $0 < x < 1$

>> Comparing the given interval $(0, 1)$ with $(0, l)$ we have $l = 1$. The corresponding cosine half range Fourier series is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$$

where $a_0 = \frac{2}{1}\int_0^1 f(x)\,dx$, $a_n = \frac{2}{1}\int_0^1 f(x)\cos n\pi x\,dx$

$$a_0 = 2\int_0^1 (2x-1)\,dx = 2\Big[x^2 - x\Big]_0^1 = 0$$

$$a_n = 2\int_0^1 (2x-1)\cos n\pi x\,dx.$$

$$= 2\left[(2x-1)\frac{\sin n\pi x}{n\pi} - (2)\cdot\frac{-\cos n\pi x}{n^2\pi^2}\right]_0^1$$

$$= \frac{4}{n^2\pi^2}\Big[\cos n\pi x\Big]_0^1 = \frac{4}{n^2\pi^2}(\cos n\pi - 1)$$

$$a_n = \frac{-4}{n^2\pi^2}\left\{1 - (-1)^n\right\}$$

Thus the required cosine half range Fourier series is given by

$$f(x) = \sum_{1}^{\infty} \frac{-4}{n^2\pi^2}\left\{1 - (-1)^n\right\}\cos n\pi x$$

Q1) c) Obtain the const term and the coefficients of the first cosine and sine terms in the Fourier expansion of y from the table

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Here $N=6$ & interval of x should be $0 \le x < 6$

$\therefore$ length of the interval $= 6-0 = 6$

$2l = 6 \Rightarrow l = 3$

FS of period $2l$ is ;

$$y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

Here $l = 3$ Let $\frac{\pi x}{3} = \theta$

$$f(x) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos n\theta + \sum_{1}^{\infty} b_n \sin n\theta$$

~~Since $l=3$ Series~~

x	$\theta = \pi x/3$ in deg	y	$y \cos\theta$	$y \sin\theta$
0	0	9	9	0
1	60°	18	9	15.588
2	120°	24	−12	20.784
3	180°	28	−28	0
4	240°	26	−13	−22.516
5	300°	20	10	−17.32
		125	−25	−3.464

$a_0 = \frac{2}{N} \Sigma y = \frac{1}{3}(125) = 41.67$ → const term

$a_1 = \frac{2}{N} \Sigma y \cos\theta = \frac{1}{3}(-25) = -8.333$ → coeff' of fir
cosine term

$b_1 = \frac{2}{N} \Sigma y \sin\theta = \frac{1}{3}(-3.464) = -1.155$ → coeff' of
first sine tem

2a) $f(x) = \begin{cases} \pi + x, & -\pi \le x \le 0 \\ \pi - x, & 0 \le x \le \pi \end{cases}$

Consider $f(-x) = \pi - x = f(x)$

$\therefore f(x)$ is an even function. $\Rightarrow b_n = 0$

The Fourier series of $f(x)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{———} ①$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi} (\pi - x)\,dx$$

$$= \frac{2}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \frac{2}{\pi}\left[\pi^2 - \frac{\pi^2}{2}\right] = \frac{2}{\pi}\left(\frac{\pi^2}{2}\right) = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} (\pi - x)\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(\pi - x)\frac{\sin nx}{n} - (-1)\frac{-\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[(0-0) - \frac{1}{n^2}(\cos n\pi - \cos 0)\right]$$

$$= \frac{-2}{\pi n^2}\left[(-1)^n - 1\right] = \frac{2}{\pi n^2}\left[1 - (-1)^n\right]$$

From ①, required FS is:

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left[1 - (-1)^n\right]\cos nx$$

26) ► **(a)** The *half-range sine series* for the given function $f(x) = (x-1)^2$ over the interval $(0, 1)$ is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right) \text{ with } l = 1 \qquad \text{(i)}$$

Here,

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx = 2\int_0^1 (x-1)^2 \sin n\pi x\, dx$$

$$= 2\left[(x-1)^2\left(-\frac{\cos n\pi x}{n\pi}\right) - 2(x-1)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right) + 2\left(\frac{\cos n\pi x}{n^3\pi^3}\right)\right]_0^1$$

$$= 2\left\{\frac{1}{n\pi} + \frac{2}{n^3\pi^3}(\cos n\pi - 1)\right\} = \frac{2}{n\pi}\left\{1 + \frac{2}{n^2\pi^2}[(-1)^n - 1]\right\}$$

Substituting this in (i), we get

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[1 + \frac{2}{n^2\pi^2}\{(-1)^n - 1\}\right]\sin n\pi x \qquad \text{(ii)}$$

as the required half-range sine series.

Q2) C Here the interval of x is 0^0 to 360^0. That is $0 \leq x < 2\pi$.

We are required to find a_0, a_1, b_1 only.

x^0	y	$\cos x$	$y \cos x$	$\sin x$	$y \sin x$
0	7.9	1	7.9	0	0
60	7.2	0.5	3.6	0.866	6.2352
120	3.6	– 0.5	– 1.8	0.866	3.1176
180	0.5	– 1	– 0.5	0	0
240	0.9	– 0.5	– 0.45	– 0.866	– 0.7794
300	6.8	0.5	3.4	– 0.866	– 5.8888
Totals	**26.9**		**12.15**		**2.6846**

Here $N = 6$; $2/N = 1/3$

$$a_0 = \frac{2}{N} \Sigma y \quad = \frac{1}{3} (26.9) = 8.9667 \quad ; \quad \frac{a_0}{2} = 4.48335$$

$$a_1 = \frac{2}{N} \Sigma y \cos x \ = \frac{1}{3} (12.15) = 4.05$$

$$b_1 = \frac{2}{N} \Sigma y \sin x \ = \frac{1}{3} (2.6846) = 0.8949$$

The Fourier series upto the first harmonic is given by

$$y = a_0/2 + (a_1 \cos x + b_1 \sin x)$$

Thus $\quad y = 4.48335 + (4.05 \cos x + 0.8949 \sin x)$

Q3 (a) Complex F T of $f(x)$ is given by

$$F(u) = \int_{x=-\infty}^{\infty} f(x)\, e^{iux}\, dx$$

$$= \int_{-a}^{a} 1 \cdot e^{iux}\, dx \qquad \because f(x) = \begin{cases} 1 & \text{for } -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$$

$$= \left[\frac{e^{iux}}{iu}\right]_{x=-a}^{a} = \frac{1}{iu}\left[e^{iua} - e^{-iua}\right]$$

$$= \frac{1}{iu}\left[(\cancel{\cos au} + i \sin au) - (\cancel{\cos au} - i \sin au)\right]$$

$$= \frac{2 i \sin au}{iu} = \frac{2 \sin au}{u}$$

Now,

To evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$

we have $F(u) = \frac{2 \sin au}{u}$

$$f(x) = F^{-1}[F(u)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \cancel{\frac{\sin au}{u}}\, F(u)\, e^{-iux}\, du$$

$$= \frac{1}{\cancel{2}\pi}\int_{-\infty}^{\infty} \cancel{2}\, \frac{\sin au}{u}\, e^{-iux}\, du$$

Put $x = 0$.

~~Since~~ $x = 0$ lies in $|x| \leq a$. Here $f(x) = 1$

Hence $\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin au}{u}\, du = 1$

$\Rightarrow \frac{2}{\pi}\int_{0}^{\infty} \frac{\sin au}{u}\, du = 1$ $\qquad \because \frac{\sin au}{u}$ is even

$\Rightarrow \int_{0}^{\infty} \frac{\sin au}{u}\, du = \frac{\pi}{2}$

$\therefore \int_{0}^{\infty} \frac{\sin u}{u}\, du = \frac{\pi}{2}$ putting $a = 1$

Hence we have $\int_{0}^{\infty} \frac{\sin x}{x}\, dx = \frac{\pi}{2}$

3b) $f(x) = e^{-a|x|}$

Fourier cosine transform is given by $F_c(u) = \int_0^\infty f(x) \cos ux \, dx$

$$F_c(u) = \int_0^\infty e^{-a|x|} \cos ux \, dx$$

$$= \int_0^\infty e^{-ax} \cos ux \quad \text{as } |x| = x \text{ for } x > 0$$

$$= \left[\frac{e^{-ax}}{(-a)^2+u^2} \{-a \cos ux + u \sin ux)\right]_0^\infty$$

$$= \frac{1}{a^2+u^2}\left[0 - 1(-a+0)\right]$$

$$= \frac{a}{a^2+u^2}$$

Now Inverse Fourier cosine Transform is given by

$$f(x) = \frac{2}{\pi}\int_0^\infty F_c(u) \cos ux \, du = \frac{2}{\pi}\int_0^\infty \frac{a}{a^2+u^2} \cos ux \, du$$

In $(0, \infty)$, $f(x) = e^{-ax}$

$$\therefore e^{-ax} = \frac{2}{\pi}\int_0^\infty \frac{a}{a^2+u^2} \cos ux \, du$$

Now put $u = x$ & $x = m$

$$\therefore e^{-am} \times \frac{\pi}{2} = \int_0^\infty \frac{a}{a^2+x^2} \cos mx \, dx$$

$$\Rightarrow \int_0^\infty \frac{\cos mx}{a^2+x^2} = \frac{\pi e^{-am}}{2a}$$

Q4 (a) $F(u) = F[f(x)] = \int_{-\infty}^{\infty} f(x) e^{iux} dx$

$$F(u) = \int_{-\infty}^{\infty} e^{-a^2x^2} e^{iux} dx$$

$$= \int_{-\infty}^{\infty} e^{-a^2\left(x^2 - \frac{iux}{a^2}\right)} dx$$

$$= \int_{-\infty}^{\infty} e^{-a^2\left[x^2 - \frac{2iux}{2a^2} + \left(\frac{iu}{2a^2}\right)^2 - \left(\frac{iu}{2a^2}\right)^2\right]} dx$$

$$= \int_{-\infty}^{\infty} e^{-a^2\left(x - \frac{iu}{2a^2}\right)^2} e^{-a^2\left(\frac{u^2}{4a^4}\right)} dx$$

Put $a\left(x - \frac{iu}{2a^2}\right) = t$

$\Rightarrow a\,dx = dt$

$\Rightarrow dx = \frac{dt}{a}$

when $x = \infty$, $t = \infty$

when $x = -\infty$, $t = -\infty$

$$\therefore F(u) = \int_{-\infty}^{\infty} e^{-t^2} e^{-u^2/4a^2} \frac{dt}{a}$$

WKT

$$\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi}$$

$$F(u) = e^{-u^2/4a^2} \frac{\sqrt{\pi}}{a}$$

Q4 (b) Fourier Cosine Transform is given by

$$F_c(u) = \int_0^\infty f(x) \cos ux \, dx$$

$$= \int_0^1 f(x) \cos ux \, dx + \int_1^4 f(x) \cos ux \, dx + \int_4^\infty f(x) \cos ux \, dx$$

$$\therefore F_c(u) = \int_0^1 4x \cos ux \, dx + \int_1^4 (4-x) \cos ux \, dx + \int_4^\infty 0 \cdot \cos ux \, dx$$

Applying Bernoulli's rule,

$$= \left[4x \frac{\sin ux}{u} - 4\left(-\frac{\cos ux}{u^2}\right)\right]_0^1 + \left[(4-x)\frac{\sin ux}{u} - (-1)\left(\frac{-\cos ux}{u^2}\right)\right]_1^4$$

$$= \frac{4}{u}(\sin u - 0) + \frac{4}{u^2}(\cos u - 1) + \frac{1}{u}(0 - 3\sin u) - \frac{1}{u^2}(\cos 4u - \cos u)$$

$$= \underset{\checkmark}{\frac{4}{u}\sin u} + \underset{\checkmark\checkmark}{\frac{4}{u^2}\cos u} - \frac{4}{u^2} - \underset{\checkmark}{\frac{3}{u}\sin u} - \frac{1}{u^2}\cos 4u + \underset{\checkmark\checkmark}{\frac{1}{u^2}\cos u}$$

$$= \frac{1}{u}\sin u + \frac{5}{u^2}\cos u - \frac{4}{u^2} - \frac{1}{u^2}\cos 4u$$

4c) $$F_c(\alpha) = \frac{\sin a\alpha}{\alpha}, \quad a > 0$$

4c) ▶ Using the Fourier cosine inversion formula, we find that

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_c(\alpha) \cos \alpha x \, d\alpha = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin a\alpha}{\alpha} \cos \alpha x \, d\alpha$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{2} \int_0^\infty \frac{\sin \alpha(a+x) + \sin \alpha(a-x)}{\alpha} \, d\alpha$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \int_0^\infty \frac{\sin (a+x)\alpha}{\alpha} \, d\alpha + \int_0^\infty \frac{\sin (a-x)\alpha}{\alpha} \, d\alpha \right\} \qquad \text{(i)}$$

On using the standard result

$$\int_0^\infty \frac{\sin kt}{t}\,dt = \begin{cases} \pi/2 & \text{for } k > 0 \\ -\pi/2 & \text{for } k < 0, \end{cases}$$

we note that

$$\int_0^\infty \frac{\sin (a+x)\alpha}{\alpha}\,d\alpha = \frac{\pi}{2} \quad \text{for all } \; x > 0, a > 0$$

and

$$\int_0^\infty \frac{\sin (a-x)\alpha}{\alpha}\,d\alpha = \begin{cases} \pi/2 & \text{for } x < a \\ -\pi/2 & \text{for } x > a \end{cases}$$

We also have

$$\int_0^\infty \frac{\sin (a-x)\alpha}{\alpha}\,d\alpha = 0 \quad \text{for } \; x = a.$$

Hence (i) yields

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi}}\left(\dfrac{\pi}{2} + \dfrac{\pi}{2}\right) & \text{for } x < a \\ \dfrac{1}{\sqrt{2\pi}}\left(\dfrac{\pi}{2} + 0\right) & \text{for } x = a \\ \dfrac{1}{\sqrt{2\pi}}\left(\dfrac{\pi}{2} - \dfrac{\pi}{2}\right) & \text{for } x > a \end{cases}$$

$$= \begin{cases} \sqrt{(\pi/2)} & \text{for} \quad x < a \\ \sqrt{\pi}/2\,\sqrt{2} & \text{for} \quad x = a \\ 0 & \text{for} \quad x > a \end{cases}$$

This is the inverse Fourier cosine transform of the given function. ■

Module-3

5)a. Find the Z-transform of

i) Sinh nθ.

$\Rightarrow \sinh\theta = \frac{e^{\theta}-e^{-\theta}}{2}$

Soln: $\sinh n\theta = \frac{e^{n\theta}-e^{-n\theta}}{2}$

Apply Z_T on both side

$$Z_T[\sinh n\theta] = Z_T\left[\frac{e^{n\theta}-e^{-n\theta}}{2}\right]$$

$$Z_T\left[\frac{e^{n\theta}-e^{-n\theta}}{2}\right] = \frac{1}{2}\left[Z_T[e^{n\theta}] - Z_T[e^{-n\theta}]\right]$$

$$= \frac{1}{2}\left[Z_T[k_1^n] - Z_T[k_2^n]\right]$$

Where $k_1 = e^{\theta}$, $k_2 = e^{-\theta}$

$$= \frac{1}{2}\left[\frac{z}{z-k_1} - \frac{z}{z-k_2}\right]$$

$$= \frac{z}{2}\left[\frac{1}{z-k_1} - \frac{1}{z-k_2}\right]$$

$$= \frac{z}{2}\left[\frac{\cancel{z}-k_2-\cancel{z}+k_1}{z^2-z(k_1+k_2)+k_1k_2}\right]$$

$k_1 = e^{\theta}$

$k_2 = e^{-\theta}$

$$= \frac{z}{2}\left[\frac{e^{\theta}-e^{-\theta}}{z^2-z[e^{\theta}+e^{-\theta}]+e^{\theta}e^{-\theta}}\right]$$

$$= \frac{z}{\cancel{2}}\left[\frac{\cancel{2}\sinh\theta}{z^2-2z\cosh\theta+1}\right]$$

$$Z_T[\sinh n\theta] = \frac{z\sinh\theta}{z^2-2z\cosh\theta+1} \;//$$

ii) $\cos n\theta$

Soln: $Z_T[\cos n\theta]$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{in\theta} = \cos n\theta + i\sin n\theta$

$(e^{i\theta})^n = k^n$

$k = e^{i\theta}$

$$Z_T[e^{in\theta}] = Z_T[k^n]$$

$$= \frac{z}{z-k}$$

$$= \frac{z}{z-e^{i\theta}}$$

$$= \frac{z}{z-e^{i\theta}} \times \frac{z-e^{-i\theta}}{z-e^{-i\theta}}$$

$$= \frac{z(z-e^{-i\theta})}{z^2 - ze^{i\theta} - ze^{i\theta}+1}$$

$$Z_T[e^{in\theta}] = \frac{z^2 - z(\cos\theta + i\sin\theta)}{z^2 - z(e^{i\theta}+e^{-i\theta})+1}$$

$$= \frac{z^2 - z\cos\theta + iz\sin\theta}{z^2 - 2z\cos\theta+1}$$

$$= \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta+1} + \frac{iz\sin\theta}{z^2-2z\cos\theta+1}$$

~~therefore equal Rp Imag~~

$$Z_T[\cos n\theta] = \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta+1}$$

Similarly,

$$Z_T[\sin n\theta] = \frac{z\sin\theta}{z^2 - 2z\cos\theta+1}$$

b) Find the inverse Z-transform of $\frac{z^2+z}{z^3+6z^2+11z+6}$

Sol^n

let $\bar{u}(z) = \frac{z^2+z}{z^3+6z^2+11z+6}$

To factorize the denominator,

$z^3+6z^2+11z+6$

$z=-1$

$-1+6-11+6=0$

By synthetic division,

$$\begin{array}{r|rrrr} -1 & 1 & 6 & 11 & 6 \\ & 0 & -1 & -5 & -6 \\ \hline & 1 & 5 & 6 & \boxed{0} \end{array}$$

$z^2+5z+6=0$

$z^2+3z+2z+6=0$

$z(z+3)+2(z+3)=0$

$(z+2)(z+3)=0$

$\bar{u}(z) = \frac{z^2+z}{z^3+6z^2+11z+6} \Rightarrow \frac{\bar{u}(z)}{z} = \frac{z+1}{z^3+6z^2+11z+6}$

$\frac{z+1}{z^3+6z^2+11z+6} = A$

$$\frac{\bar{u}(z)}{z} = \frac{z+1}{(z+1)(z+2)(z+3)} = \frac{A}{z+1} + \frac{B}{z+2} + \frac{C}{z+3}$$

$$\frac{z+1}{(z+1)(z+2)(z+3)} = \frac{A(z+2)(z+3)+B(z+1)(z+3)+C(z+1)(z+2)}{(z+1)(z+2)(z+3)}$$

$z+1 = A(z+2)(z+3)+B(z+1)(z+3)+C(z+1)(z+2)$

$z+1 = A(z^2+5z+6)+B(z^2+4z+3)+C(z^2+3z+2)$

$z+1 = Az^2+5Az+6A+Bz^2+4Bz+3B+Cz^2+3Cz+2C$

Equating coefficients,

c> Solve the difference equation using Z-transform

$u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, with $u_0 = 0$ & $u_1 = 1$

→ Take z-tranform on b/s of given equation,

$$Z_T[u_{n+2}] + 4Z_T[u_{n+1}] + 3Z_T[u_n] = Z_T[3^n]$$

$$z^2[\bar{u}(z) - u_0 - u_1 z^{-1}] + 4z[\bar{u}(z) - u_0] + 3\bar{u}(z) = \frac{z}{z-3}$$

$$z^2\bar{u}(z) - z^2 u_0 - z u_1 + 4z\bar{u}(z) - 4z u_0 + 3\bar{u}(z) = \frac{z}{z-3}$$

$$\bar{u}(z)[z^2+4z+3] - u_0\cancelto{0}{[z^2+4z]} \cancelto{0}{- u_1 z} = \frac{z}{z-3}$$

$$\bar{u}(z) = \frac{z}{(z-3)(z^2+4z+3)}$$

$$\bar{u}(z) = \frac{z}{(z+1)(z+3)(z-3)}$$

$$\frac{\bar{u}(z)}{z} = \frac{1}{(z+1)(z+3)(z-3)} = \frac{A}{z+1} + \frac{B}{z+3} + \frac{C}{z-3}$$

$$\frac{1}{(z+1)\cancel{(z+3)}(z-3)} = \frac{A(z+3)(z-3) + B(z+1)(z-3) + C(z+1)(z+3)}{\cancel{(z+1)(z+3)(z-3)}}$$

$$1 = A(z^2-9) + B(z^2-2z-3) + C(z^2+4z+3)$$

$$1 = Az^2 - 9A + Bz^2 - 2Bz - 3B + Cz^2 + 4Cz + 3C$$

Equating coefficients,

$$A + B + C = 0$$

$$-2B + 4C = 0$$

$$-9A - 3B + 3C = 1$$

$$A = \frac{-1}{8}, \quad B = \frac{1}{12}, \quad C = \frac{1}{24}$$

$$\frac{\bar{u}(z)}{z} = \frac{-1}{8}\cdot\frac{1}{z+1} + \frac{1}{12}\,\frac{1}{z+3} + \frac{1}{24}\,\frac{1}{z-3}$$

$$\bar{u}(z) = \frac{-1}{8}\cdot\frac{z}{z+1} + \frac{1}{12}\cdot\frac{z}{z+3} + \frac{1}{24}\cdot\frac{z}{z-3}$$

Take inverse z-transform on both sides,

$$Z_T^{-1}[\bar{u}(z)] = \frac{-1}{8} Z_T^{-1}\left[\frac{z}{z+1}\right] + \frac{1}{12} Z_T^{-1}\left[\frac{z}{z+3}\right] + \frac{1}{24} Z_T^{-1}\left[\frac{z}{z-3}\right]$$

$$Z_T^{-1}[\bar{u}(z)] = \frac{-1}{8} Z_T^{-1}\left[\frac{z}{z-(-1)}\right] + \frac{1}{12} Z_T^{-1}\left[\frac{z}{z-(-3)}\right] + \frac{1}{24} Z_T^{-1}\left[\frac{z}{z-3}\right]$$

$$\boxed{Z_T^{-1}[\bar{u}(z)] = \frac{-1}{8}(-1)^n + \frac{1}{12}(-3)^n + \frac{1}{24}3^n}$$

$$A + B + C = 0$$

$$5A + 4B + 3C = 1$$

$$6A + 3B + 2C = 1$$

Solving above equations,

$$\boxed{A = 0 \ , B = 1 \ , C = -1}$$

$$\frac{\bar{u}(z)}{z} = \frac{A}{z+1} + \frac{B}{z+2} + \frac{C}{z+3}$$

$$\bar{u}(z) = 0 \cdot \frac{z}{z+1} + 1 \cdot \frac{z}{z+2} - 1 \cdot \frac{z}{z+3}$$

$$\bar{u}(z) = \frac{z}{z-(-2)} - \frac{z}{z-(-3)}$$

$$Z_T^{-1}\left[\bar{u}(z)\right] = Z_T^{-1}\left[\frac{z}{z-(-2)}\right] - Z_T^{-1}\left[\frac{z}{z-(-3)}\right]$$

$$\boxed{Z_T^{-1}\left[\bar{u}(z)\right] = (-2)^n - (-3)^n}$$

6a) Find the z-transform of $a^n n^2 + 4\sin\left(\frac{n\pi}{4}\right) + 5$

Sol: Let $a^n n^2 + 4\sin\left(\frac{n\pi}{4}\right) + 5 = u_n$

$$\therefore Z_T[u_n] = \bar{u}(z)$$

$$\bar{u}[z] = \underset{\underset{Z_1}{\downarrow}}{Z_T\left[a^n n^2\right]} + 4\underset{\underset{Z_2}{\downarrow}}{Z_T\left[\sin\left(\frac{n\pi}{4}\right)\right]} + 5\underset{\underset{Z_3}{\downarrow}}{Z_T[1]}$$

Z_1: $Z_T\left[a^n n^2\right]$

$$Z_T[n^2] = \frac{z^2+z}{(z-1)^3}$$

Using damping rule; $Z_T[a^n u_n] = \bar{u}(z/a)$

$$\Rightarrow Z_T[a^n u_n] = \left[\frac{z^2+z}{(z-1)^3}\right]_{z \to z/a} = \frac{(z/a)^2 + (z/a)}{(z/a-1)^3}$$

$$\therefore Z_T[a^n u_n] = \frac{az^2 + a^2 z}{(z-a)^3}$$

$$\boxed{\therefore Z_T[a^n u_n] = \frac{az(z+a)}{(z-a)^3}} \text{ —— ①}$$

Z_2: $Z_T\left[\sin\left(\frac{n\pi}{4}\right)\right]$

We know that $e^{in\pi/4} = \cos\left(\frac{n\pi}{4}\right) + i\sin\left(\frac{n\pi}{4}\right)$ $\left\{\because e^{n\theta} = \cos n\theta + i\sin n\theta\right\}$

Let $e^{(i\pi/4)^n} = k^n$

$$Z_T[k^n] = \frac{z}{z-k} \Rightarrow Z_T\left[\left(e^{i\pi/4}\right)^n\right] = \frac{z}{z - e^{i\pi/4}}$$

$$\Rightarrow Z_T\left[\cos\left(\frac{n\pi}{4}\right) + i\sin\left(\frac{n\pi}{4}\right)\right] = \frac{z}{z-\{\cos(\pi/4) + i\sin(\pi/4)\}}$$

$$= \frac{z}{z-\left[\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right]}$$

$$Z_T\left[\cos\left(\frac{n\pi}{4}\right) + i\sin\left(\frac{n\pi}{4}\right)\right] = \frac{z}{z-\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}}$$

$$= \frac{z}{\left\{\left(z-\frac{1}{\sqrt{2}}\right)-\frac{i}{\sqrt{2}}\right\} \times \left\{\left(z-\frac{1}{\sqrt{2}}\right)+\frac{i}{\sqrt{2}}\right\}} \times \left\{\left(z-\frac{1}{\sqrt{2}}\right)+\frac{i}{\sqrt{2}}\right\}$$

$$Z_T\left[\cos\left(\frac{n\pi}{4}\right) + i\sin\left(\frac{n\pi}{4}\right)\right] = \frac{z\cdot\left[\left(z-\frac{1}{\sqrt{2}}\right)+\frac{i}{\sqrt{2}}\right]}{z^2-\sqrt{2}z+1} \qquad \{(a+b)(a-b) = a^2-b^2\}$$

$$= \frac{z^2-\frac{z}{\sqrt{2}} + i\left(\frac{z}{\sqrt{2}}\right)}{z^2-\sqrt{2}z+1}$$

Equating real and imaginary parts, we get;

$$\Rightarrow \boxed{Z_T\left[\cos\left(\frac{n\pi}{4}\right)\right] = \frac{z^2-\frac{z}{\sqrt{2}}}{z^2-\sqrt{2}z+1}} \quad \& \quad \boxed{Z_T\left[\sin\left(\frac{n\pi}{4}\right)\right] = \frac{z}{\sqrt{2}(z^2-\sqrt{2}z+1)}} \text{ — ②}$$

$$\underline{\underline{Z_3}}: Z_T[1] = \frac{z}{z-1} \text{ —— ③}$$

$$\therefore \boxed{Z_T[u_n] = \frac{az(z+a)}{(z-a)^3} + \frac{4z}{\sqrt{2}(z^2-\sqrt{2}z+1)} + \frac{5z}{z-1}} \quad \text{where}$$

$$u_n = a^n n^2 + 4\sin\left(\frac{n\pi}{4}\right) + 5.$$

6b) *Find the inverse Z-transform of* $\dfrac{z^3 - 20z}{(z-2)^3(z-4)}$

☞ Let $\overline{u}(z) = \dfrac{z^3 - 20z}{(z-2)^3(z-4)}$

Ve have,

$$Z_T^{-1}\left[\frac{z}{z-2}\right] = 2^n, \quad Z_T^{-1}\left[\frac{2z}{(z-2)^2}\right] = 2^n n$$

$$Z_T^{-1}\left[\frac{2z^2+4z}{(z-2)^3}\right] = 2^n \cdot n^2\,, \quad Z_T^{-1}\left[\frac{z}{z-4}\right] = 4^n$$

We resolve $\bar{u}\,(z)$ as follows.

$$\bar{u}(z) = \frac{z^3-20z}{(z-2)^3\,(z-4)} = A\cdot\frac{z}{z-2} + B\cdot\frac{2z}{(z-2)^2} + C\cdot\frac{2z^2+4z}{(z-2)^3} + D\cdot\frac{z}{z-4} \cdots \quad ...(1)$$

$$\frac{z^3-20z}{(z-2)^3\,(z-4)} = \frac{Az(z-2)^2\,(z-4)+2Bz\,(z-2)(z-4)+C\,(2z^2+4z)(z-4)+Dz\,(z-2)^3}{(z-2)^3\,(z-4)}$$

or $z^2-20 = A(z-2)^2\,(z-4)+2B(z-2)(z-4)+C(2z+4)(z-4)+D(z-2)^3$

Put $z=2 \;:\; -16 = -16C \quad \therefore\; C=1$

Put $z=4 \;:\; -4 = D(8) \quad \therefore\; D = -1/2$

Equating the coefficient of z^3 on both sides we have,

$A+D=0 \quad \therefore\; A = 1/2$

Put $z=0 : -20 = A(4)(-4)+2B(8)+C(-16)+D(-8)$

ie, $-20 = -8+16B-16+4 \;\therefore\; B=0$

Substituting the values of A, B, C, D in (1) and taking inverse we have,

$$Z_T^{-1}[\,\bar{u}(z)\,] = \frac{1}{2}Z_T^{-1}\left[\frac{z}{z-2}\right] + Z_T^{-1}\left[\frac{2z^2+4z}{(z-2)^3}\right] - \frac{1}{2}Z_T^{-1}\left[\frac{z}{z-4}\right]$$

$$= \frac{1}{2}\cdot 2^n + 2^n\cdot n^2 - \frac{1}{2}\cdot 4^n$$

Thus, $\boxed{Z_T^{-1}[\,\bar{u}(z)\,] = u_n = 2^{n-1} + 2^n\cdot n^2 - 2^{2n-1}}$

Power series Method

6C Given, $Z_T(u_n) = \dfrac{2z^2+3z+4}{(z-3)^3}$, $|z|>3$ show that $u_1 = 2$, $u_2 = 21$, $u_3 = 139$

☞ *[As in Problem-53, results (1) to (4) need to be given]*

$$u_0 = \lim_{z\to\infty} \frac{2z^2+3z+4}{(z-3)^3}$$

$$= \lim_{z\to\infty} \frac{z^2(2+3/z+4/z^2)}{z^3(1-3/z)^3}$$

$$= \lim_{z\to\infty} \frac{1}{z}\cdot\frac{(2+3/z+4/z^2)}{(1-3/z)^3} = 0$$

$$\therefore \quad u_0 = 0$$

$$u_1 = \lim_{z\to\infty} z\left[\frac{2z^2+3z+4}{(z-3)^3}\right] \text{ since } u_0 = 0.$$

$$= \lim_{z\to\infty} z^3\cdot\frac{(2+3/z+4/z^2)}{z^3(1-3/z)^3} = 2$$

$$\therefore \quad u_1 = 2$$

$$u_2 = \lim_{z\to\infty} z^2\left[\frac{2z^2+3z+4}{(z-3)^3} - 0 - \frac{2}{z}\right]$$

$$= \lim_{z\to\infty} z^2\left[\frac{2z^3+3z^2+4z-2(z^3-9z^2+27z-27)}{z(z-3)^3}\right]$$

$$u_2 = \underset{z \to \infty}{lt} \frac{z(21z^2 - 50z + 54)}{(z-3)^3}$$

$$= \underset{z \to \infty}{lt} \frac{z^3(21 - 50/z + 54/z^2)}{z^3(1-3/z)^3} = 21$$

$$\therefore \quad u_2 = 21$$

$$u_3 = \underset{z \to \infty}{lt}\, z^3 \left[\frac{2z^2 + 3z + 4}{(z-3)^3} - \frac{2}{z} - \frac{21}{z^2} \right]$$

$$= \underset{z \to \infty}{lt}\, z^3 \left[\frac{z^3(2z^2 + 3z + 4) - 2z^2(z-3)^3 - 21z(z-3)^3}{z^3(z-3)^3} \right]$$

$$= \underset{z \to \infty}{lt} \left[\frac{(2z^5 + 3z^4 + 4z^3) - (2z^2 + 21z)(z^3 - 9z^2 + 27z - 27)}{(z-3)^3} \right]$$

$$= \underset{z \to \infty}{lt} \left[\frac{(2z^5 + 3z^4 + 4z^3) - (2z^5 + 3z^4 - 135z^3 + 513z^2 - 567z)}{(z-3)^3} \right]$$

$$= \underset{z \to \infty}{lt} \left[\frac{139z^3 - 513z^2 + 567z}{(z-3)^3} \right]$$

$$= \underset{z \to \infty}{lt} \frac{z^3(139 - 513/z + 567/z^2)}{z^3(1-3/z)^3} = 139$$

$$\therefore \quad u_3 = 139$$

Thus we have proved that $\boxed{u_1 = 2,\ u_2 = 21 \text{ and } u_3 = 139}$

7a ven,

$(4D^4-4D^3-23D^2+12D+36)y=0$

Given an Homogeneous equation in the form of $f(D)y=0$

$\therefore (4D^4-4D^3-23D^2+12D+36)y=0$ —①

∴ The Auxiallary eqⁿ of ① is:-

$(4m^4-4m^3-23m^2+12m+36)y=0$

$\therefore \quad 4m^4-4m^3-23m^2+12m+36=0$

let $m=2$

$4(2)^4-4(2)^3-23(2)^2+12(2)+36=0$

$64-32-92+24+36=0$

$124-124=0$

$0=0$ (✓)

∴ By synthetic Division method

$$\begin{array}{c|ccccc} 2 & 4 & -4 & -23 & 12 & 36 \\ & 0 & 8 & 8 & -30 & -36 \\ \hline & 4 & 4 & -15 & -18 & \boxed{0} \end{array}$$

$4m^3+4m^2-15m-18=0$

let $m=2$

$4(2)^3+4(2)^2-15(2)-18=0$

$4(8)+4(4)-30-18=0$

$32+16-48=0$

$48-48=0$

$0=0(R)$

$\therefore \boxed{m=2}$

$$\therefore\ 2\ \begin{array}{|cccc}
4 & 4 & -15 & -18 \\
0 & 8 & 24 & 18 \\
\hline
4 & 12 & 9 & \boxed{0}
\end{array}$$

$4m^2+12m+9=0$

$4m^2+6m+6m+9=0$

$2m(2m+3)+3(2m+3)=0$

$(2m+3)(2m+3)=0$

$m=-\frac{3}{2}, -\frac{3}{2}$

$\therefore m=2,2,-\frac{3}{2},-\frac{3}{2}$

$\therefore m_1=2,2, \quad m_2=-\frac{3}{2},-\frac{3}{2}$

$\therefore$ The complementary function is:-

$$y=(c_1+c_2x)e^{m_1x}+(c_3+c_4x)e^{m_2x}$$

$$y=(c_1+c_2x)e^{2x}+(c_3+c_4x)e^{\left(-\frac{3}{2}\right)x}$$

$\therefore$ solution of $(4D^4-4D^3-23D^2+12D+36)y=0$ is

$$y=(c_1+c_2x)e^{2x}+(c_3+c_4x)e^{\left(-\frac{3}{2}\right)x}$$

b) $\frac{d^2y}{dx^2} - 4y = \cos h\,(2x-1) + 3^x$

$(D^2-4)y = \cos h\,(2x-1) + 3^x$

$D^2 - 4 = 0$

$m^2 - 4 = 0$

$m^2 = \pm 4$

$m = 2, -2$

$Cf = C_1 e^{2x} + C_2 e^{-2x}$

$PI = \frac{\cos h\,(2x-1) + 3^x}{D^2-4}$

$= \frac{e^{2x-1} + e^{-(2x-1)}}{2(D^2-4)} + \frac{3x}{D^2-4}$

$= \frac{1}{2} \cdot \frac{e^{2x-1}}{D^2-4} + \frac{1}{2}\frac{e^{-(2x-1)}}{D^2-4} + \frac{3x}{D^2-4}$

$= P_1 + P_2 + P_3$

$P_1 = \frac{x}{2} \cdot \frac{e^{2x-1}}{2D} = \frac{x}{2}\frac{e^{2x-1}}{4} = \frac{x}{8}e^{2x-1}$ Put $x = 2$

$P_2 = \frac{1}{2}\frac{e^{-(2x-1)}}{D^2-4} = \frac{1}{2}\frac{e^{-(2x-1)}}{(-2)^2-4}$ $x = -2$

$= \frac{x}{2} \cdot \frac{e^{-(2x-1)}}{2D} = \frac{x}{2}\frac{e^{-(2x-1)}}{-4} = \frac{-x}{8}e^{-(2x-1)}$

$P_3 = \frac{3^x}{D^2-4} = \frac{(e^{\log 3})^x}{D^2-4} = \frac{e^{\log 3 \cdot x}}{D^2-4} = \frac{e^{\log 3 \cdot x}}{(\log 3)^2-4} = \frac{3^x}{(\log 3)^2-4}$ //

$y = C_1 e^{2x} + C_2 e^{-2x} + \frac{x}{8}e^{2x-1}$

$-\frac{x}{8}e^{-(2x-1)} + \frac{3^x}{(\log 3)^2-4}$

$= \sim + \frac{x}{4}\sin h(2x-1)$

$+ \frac{3^x}{(\log 3)^2-4}$

7c (i) $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

Given that

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right) \rightarrow ①$$

put $x = e^t \Rightarrow t = \log x$

$$\left[D(D-1)(D-2) + 2[D(D-1)] + 2\right]y = 10\left(e^t + \frac{1}{e^t}\right)$$

$$\left[D(D^2-2D-D+2) + 2D^2 - 2D + 2\right]y = 10(e^t + e^{-t})$$

$$\left[D(D^2-3D+2) + 2D^2 - 2D + 2\right]y = 10(e^t + e^{-t})$$

$$\left[D^3 - \cancel{3D^2} + \cancel{2D} + \cancel{2D^2} - \cancel{2D} + 2\right]y \quad 10(e^t + e^{-t})$$

$$[D^3 - D^2 + 2)y = 10(e^t - e^{-t})$$

Consider

$D^3 - D^2 + 2 = 0$

The A.E is $m^3 - m^2 + 2 = 0$, put $\boxed{m = -1}$

$(-1)^3 - (-1)^2 + 2 = 0$

$-1 - 1 + 2 = 0 \Rightarrow -2 + 2 = 0$

-1	1	-1	0	2
	0	-1	2	-2
	1	-2	2	$\boxed{0}$

$m^2 - 2m + 2 = 0$

$$m = \frac{+2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$m = \frac{+2 \pm \sqrt{4-8}}{2}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{+2 \pm \sqrt{-4}}{2}$$

$$m = \frac{+\cancel{2} \pm 2i}{\cancel{2}}$$

$m = -1, 1 + i,$

$CF = C_1e^{-t} + [C_2\cos t + C_3\sin t]e^t$

$P.I = 10[e^t + e^{-t}]$

$$= 10\left\{\left[\frac{e^t}{D^3 - D^2 + 2}\right] + \left[\frac{e^{-t}}{D^3 - D^2 + 2}\right]\right\}$$

} replace D by -1

Replace D by 1(A)

$$PI = 10\left[\frac{e^t}{(1)^3 - (1)^2 + 2}\right] + \left[\frac{e^{-t}}{(-1)^3 - (-1)^2 + 2}\right]$$

$$= \overset{5}{\cancel{10}}\left[\frac{e^t}{\cancel{2}} + \frac{e^{-t}}{2}\right]$$

$3(-1)^2 - 2(-1)$

$3 + 2$

$= 5[e^t + e^{-t}]$

$$= 10\left[\frac{e^t}{2}\right] + t\frac{1}{3D^2 - 2D}e^{-t}$$

$$= 10\left[\frac{e^t}{2}\right] + t\frac{1}{5}e^{-t}$$

$$= 5e^t + \overset{2}{\cancel{10}}t \cdot \frac{e^{-t}}{\cancel{5}}$$

$= 5e^t + 2t \cdot e^{-t}$

$y = CF + PI$

$= c_1e^{-t} + [c_2\cos t + c_3\sin t]e^t + 5e^t + 2te^{-t}.$

$$= \frac{c_1}{x} + x\left[c_2\cos\log x + c_3\sin\log x\right] + 5x + 2\log x\frac{1}{x}$$ //

ॐ

⑧ Solve $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=3\sin x+4\cos x$
ⓐ

Ⓐ Given,

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=3\sin x+4\cos x \quad - ①$$

As WKT,

$$D=\frac{d}{dx}$$

∴ Equation ① can be written as:-

$$D^2y+4Dy+4y=3\sin x+4\cos x$$

$$(D^2+4D+4)y=3\sin x+4\cos x \quad - ②$$

∴ Consider Homogenous part eqn ② then,

$$(D^2+4D+4)y=0$$

The Auxillary eqn is:-

$$m^2+4m+4=0$$

$$m^2+2m+2m+4=0$$

$$m(m+2)+2(m+2)=0$$

$$(m+2)(m+2)=0$$

$$\boxed{m=-2,-2}$$

∴ The complementary Function is:-

~~C.F~~ $$\boxed{C\cdot F=(C_1+C_2x)e^{-2x}}$$

The Particular Integral of eqⁿ ② is:-

$$PI = \frac{\phi(x)}{f(D)}$$

$$PI = \frac{3\sin x + 4\cos x}{D^2+4D+4}$$

$$PI = \frac{3\sin x}{D^2+4D+4} + \frac{4\cos x}{D^2+4D+4}$$

[∵ Repalace 'D^2' with $-a^2$:
$a=1,\ a^2=1,\ D^2=-1$]

~~PI = 3sinx / [illegible]~~

$$PI = \frac{3\sin x}{-1+4D+4} + \frac{4\cos x}{-1+4D+4}$$

$$PI = \frac{3\sin x}{4D+3} \times \frac{4D-3}{4D-3} + \frac{4\cos x}{4D+3} \times \frac{4D-3}{4D-3}$$

$$PI = \frac{12D\sin x - 9\sin x}{(4D)^2-(3)^2} + \frac{16D\cos x - 12\cos x}{(4D)^2-(3)^2}$$

$$PI = \frac{12\frac{d}{dx}(\sin x) - 9\sin x + 16\frac{d}{dx}(\cos x) - 12\cos x}{16D^2-9}$$

$$PI = \frac{12\cos x - 9\sin x + 16(-\sin x) - 12\cos x}{16D^2-9}$$

$$PI = \frac{-25\sin x}{16D^2-9} \quad (\because D^2=-1)$$

$$PI = \frac{-\cancel{25}\sin x}{-\cancel{25}}$$

$$\boxed{PI = \sin x}$$

∴ General solution:- $y = CF + PI$

$$\boxed{y = (c_1 + c_2 x)e^{-2x} + \sin x}$$

⑧ ⓑ Solve $(2x+3)^{2}\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$

Ⓐ

Given,

$$(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x \quad - ①$$

As WKT,

$2x+3 = ax+b$

$\boxed{a=2}, \boxed{b=3}$

$(2x+3) = e^t \qquad t = \log(2x+3)$

$2x = e^t - 3$

$\boxed{x = \frac{e^t-3}{2}}$

$$(ax+b)^2\frac{d^2y}{dx^2} = a^2D(D-1)y \quad . \text{ where } D = \frac{d}{dt}$$

$$(ax+b)\frac{dy}{dx} = aDy$$

∴ By substituting these value in eqⁿ ① we get

$$(2)^2D(D-1)y - 2Dy - 12y = \overset{3}{\cancel{6}}\left(\frac{e^t-3}{\cancel{2}}\right)$$

$$4(D^2-D)y - 2Dy - 12y = 3e^t - 9$$

$$4D^2_y - 4Dy - 2Dy - 12\cancel{D}y = 3e^t - 9$$

$$4D^2y - 6Dy - 12\cancel{D}y = 3e^t - 9$$

$$(4D^2 - 6D - 12)y = 3e^t - 9 \quad - ②$$

∴ Consider Homogeneous Part of eqⁿ ②

$$(4D^2 - 6D - 12)y = 0$$

∴ The Auxillary eqⁿ is:-

$$4m^2 - 6m - 12 = 0$$

$$2m^2 - 3m - 6 = 0$$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$m = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-6)}}{2(2)}$$

$$m = \frac{3 \pm \sqrt{9+48}}{4}$$

$$m = \frac{3 \pm \sqrt{57}}{4}$$

$$m = \frac{3}{4} \pm \frac{\sqrt{57}}{4}$$

∴ The Complementary function is:- $CF = e^{(3/4)t}\left(c_1 \cos\left(\frac{\sqrt{57}}{4}\right)t + c_2 \sin\left(\frac{\sqrt{57}}{4}\right)t\right)$

$$PI = \frac{\phi(x)}{f(D)}$$

$$PI = \frac{3e^t - 9}{f(D)}$$

$$PI = \frac{3e^t}{f(D)} - \frac{9 \cdot e^{0t}}{f(D)}$$

$$PI = \left.\frac{3e^t}{4D^2-6D-12}\right|_{D=1} - \left.\frac{9 \cdot e^{0t}}{4D^2-6D-12}\right|_{D=0}$$

$$PI = \frac{3e^t}{-14} - \frac{9}{-12}$$ (∵ Replace 'D' with 'a')

$$PI = \frac{\cancel{9}^{3}}{\cancel{12}_{4}} - \frac{3e^t}{14}$$

$$\boxed{PI = \frac{3}{4} - \frac{3e^t}{14}}$$

∴ The General solution is:- $y = CF + PI$

$$y = e^{(3/4)t}\left(c_1 \cos\left(\frac{\sqrt{57}}{4}\right)t + c_2 \sin\left(\frac{\sqrt{57}}{4}\right)t\right) + \frac{3}{4} - \frac{3e^t}{14}$$

$$y = (2x+3)^{3/4}\left(c_1 \cos\left(\frac{\sqrt{57}}{4}\right)\log(2x+3) + c_2 \sin\left(\frac{\sqrt{57}}{4}\right)\log(2x+3)\right)$$
$$+ \frac{3}{4} - \frac{3}{14}(2x+3).$$

5. a) least square straight line is given by

$y = ax + b$ —①

The normal equations of the given line are

$\Sigma y = a\Sigma x + bn$ —②

$\Sigma xy = a\Sigma x^2 + b\Sigma x$ —③

x	y	x^2	xy
1	6	1	6
2	4	4	8
3	3	9	9
4	5	16	20
5	4	25	20
6	2	36	12

$n = 6$

$\Sigma x = 21$

$\Sigma y = 24$

$\Sigma x^2 = 91$

$\Sigma xy = 75$

eqns ② and ③ becomes

$24 = 21a + 6b$

$75 = 91a + 21b$.

solving these we get

$a = -0.51$

$b = 5.8$

eq^n ① becomes.

$y = -0.51x + 5.8$ //.

9b) Given $r = 0.5$ and $\tan^{-1}\left(\frac{3}{5}\right)$ is the angle between regression lines

$$\theta = \tan^{-1}\left(\frac{3}{5}\right)$$

$$\tan\theta = \frac{3}{5}.$$

To prove:- $\sigma_x = 2\sigma_y$ or $\sigma_y = 2\sigma_x$

we know,

$$\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$$

substituting tanθ and r we get

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-(0.5)^2}{0.5}\right)$$

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}(1.5)$$

$$\frac{2}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

$$2\sigma_x^2 + 2\sigma_y^2 = 5\sigma_x \sigma_y$$

$$2\sigma_x^2 + 2\sigma_y^2 - 5\sigma_x \sigma_y = 0$$

Factorizing we get,

$$2\sigma_x^2 + 2\sigma_y^2 - 4\sigma_x \sigma_y - \sigma_x \sigma_y = 0$$

$$\sigma_x(2\sigma_x - \sigma_y) \overset{-}{\cancel{+}} 2\sigma_y(2\sigma_x - \sigma_y)$$

96)

x	y	r_1	r_2	d	d^2
6	8	3.5	7	-3.5	12.25
5	7	2	4.5	-2.5	6.25
8	7	7.5	4.5	3	9
8	10	7.5	9.5	-2	4
7	5	5.5	1	4.5	20.25
6	8	3.5	7	-3.5	12.25
10	10	10	9.5	0.5	0.25
4	6	1	2.5	-1.5	2.25
9	8	9	7	2	4
7	6	5.5	2.5	3.8	9

$\Sigma d^2 = 79.5$

$m_1 = 2$, $m_2 = 2$, $m_3 = 2$, $m_4 = 2$, $m_5 = 2$

$m_6 = 3$, $m_7 = 2$

$$R = 1 - \frac{6\left[\Sigma d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \frac{m_3(m_3^2-1)}{12} + \frac{m_4(m_4^2-1)}{12} + \frac{m_5(m_5^2-1)}{12} + \frac{m_6(m_6^2-1)}{12} + \frac{m_7(m_7^2-1)}{12}\right]}{n(n^2-1)}.$$

$$\cancel{y - \bar{y} = \frac{\Sigma xy}{\Sigma x}(x - \bar{x})}$$

$$\cancel{y - 15 = \frac{361}{}}$$

$$R = 1 - \frac{6\left[79.5 + \left[\frac{1}{2} \times 6\right] + 2\right]}{990}$$

$$= 1 - 6\left[\frac{84.5}{990}\right]$$

$$= 1 - 0.512$$

$$= 0.488 \,//.$$

10 a) The least square quadratic curve is given by $y = ax^2 + bx + c$ — ①

The normal equations of eq^n ① are

$$\Sigma y = a\Sigma x^2 + b\Sigma x + cn$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 .$$

x	y	x^2	x^3	x^4	xy	x^2y
0	12	0	0	0	0	0
1	10.5	1	1	1	10.5	10.5
2	10	4	8	16	20	40
3	8	9	27	81	24	72
4	7	16	64	256	28	112
6	2	36	216	1296	12	72
7	8.5	49	343	2401	59.5	416.5
8	9	64	512	4096	72	576
31	67	179	1171	8147	226	1299

$\therefore$ The normal equations are

$$67 = 179a + 31b + 8c$$
$$226 = 1171a + 179b + 31c$$
$$1299 = 8147a + 1171b + 179c$$

Solving the three eqns we get

$a = 0.27$
$b = -2.7$
$c = 13$

$$\therefore \quad y = 0.27x^2 - 2.7x + 13 \; //$$

At $x = 6.5$

$$y = 0.27(6.5)^2 - 2.7(6.5) + 13$$

$$y = 6.86 \; //$$

10b) The correlation coefficient r is given by

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x \sigma_y}$$

x	y	$z = x - y$	x^2	y^2	z^2
6	8	-2	36	64	4
5	7	-2	25	49	4
8	7	1	64	49	1
8	10	-2	64	100	4
7	5	2	49	25	4
6	8	-2	36	64	4
40	45	-5	274	351	21

$$\bar{x} = \frac{\Sigma x}{n} = \frac{40}{6} = 6.67$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{45}{6} = 7.5$$

$$\bar{z} = \frac{\Sigma z}{n} = \frac{-5}{6} = -0.83$$

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$$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2$$

$$= \frac{274}{6} - (6.67)^2$$

$$= 1.178.$$

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2$$

$$= \frac{351}{6} - (7.5)^2$$

$$= 2.25$$

$$\sigma_{x-y}^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2$$

$$= \frac{21}{6} - (-0.83)^2$$

$$= 2.811$$

Hence $r = \frac{1.178 + 2.25 - 2.811}{2\sqrt{1.178}\sqrt{2.25}}$

$$= 0.189$$

Hence the correlation coefficient b/w x and y is 0.189

10 c) For the given data, $\Sigma x = 21$ and

$\Sigma y = 79$

$$\therefore \ \bar{x} = \frac{\Sigma x}{n} = \frac{21}{6} = 3.5$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{79}{6} = 13.17$$

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
1	14	-2.5	0.83	6.25	0.6889	-2.075
2	12	-1.5	-1.17	2.25	1.3689	1.755
3	16	-0.5	2.83	0.25	8	-1.415
4	11	0.5	-2.17	0.25	4.7089	-1.085
5	14	1.5	0.83	2.25	0.6889	1.245
6	12	2.5	-1.17	6.25	1.3689	-2.925
21	79			17.5	16.82	-4.5

The regression lines are given by

$$(y-\bar{y}) = \frac{\Sigma XY}{\Sigma X^2}(x-\bar{x})$$

$$(y-13.17) = \frac{-4.5}{17.5}(x-3.5)$$

$$y-13.17 = -0.26x + 0.91$$

$$y = -0.26x + 14.08 \ //$$

and

$$(x-\bar{x}) = \frac{\Sigma XY}{\Sigma Y^2}(y-\bar{y})$$

$$(x-3.5) = \frac{-4.5}{16.82}(y-13.17)$$

$$x-3.5 = -0.27y + 3.55$$

$$x = -0.27y + 7.05 \ //$$

The two regression lines are

$y = -0.26x + 14.08$ and

$x = -0.27y + 7.05$