CMR INSTITUTE OF TECHNOLOGY

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| Internal Assesment Test-I Solution | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Sub: | Electromagnetic Waves | | | | | | Code: | 21EC54 |
| Date: | 19/12/2023 | Duration: | 90 mins | Max Marks: | 50 | Sem: 5th | Branch: | ECE(A,B,C,D) |
| Answer any **FIVE FULL** Questions | | | | | | | | |

| | Marks | OBE CO | OBE RBT |
|---|---|---|---|
| 1. a) Transform the vector field G = (xz/y) $\mathbf{a}_x$ into spherical components and variables. | [06] | CO1 | L3 |

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$$\boxed{G_x = \frac{r\cos\theta\cos\phi}{\sin\phi}}$$

$$\begin{bmatrix} G_r \\ G_\theta \\ G_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} G_x \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{G} = r\cos\theta\cos\phi\,(\sin\theta\cot\phi\,\mathbf{a}_r + \cos\theta\cot\phi\,\mathbf{a}_\theta - \mathbf{a}_\phi)$$

| | Marks | CO | RBT |
|---|---|---|---|
| 1. b) Given the two points, C(−3, 2, 1) and D(r = 5, θ = 20°, φ = −70°), find: (a) the spherical coordinates of C; (b) the rectangular coordinates of D. | [04] | CO1 | L3 |

a)

$r = \sqrt{x^2+y^2+z^2}$

$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

$\phi = \tan^{-1}(y/x)$

$\phi_{\text{II}} = 180 - |\phi|$

$r = 3.741$ $\quad \theta = 74.49^\circ$ $\quad \phi = 146.3^\circ$

$C(r = 3.741,\ \theta = 74.49^\circ,\ \phi = 146.3^\circ)$

b) $D(x = ?,\ y = ?,\ z = ?)$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$x = 0.58$

$y = -1.6$

$z = 4.69$

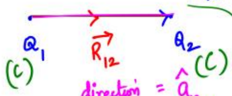
$D(0.58,\ -1.6,\ 4.69)$

2. a) State and explain Coulomb's law in vector form. [05] CO1 L1

Point charges

$\boxed{\text{The force between two very small objects separated in Vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.}}$

Force on $Q_2$ by $Q_1$, $\boxed{\vec{F}_{12} = k\dfrac{Q_1 Q_2}{|\vec{R}_{12}|^2} \cdot \hat{a}_{R_{12}}}$ N




direction = $\hat{a}_{R_{12}}$

Free space $k = 9 \times 10^{9} = \dfrac{1}{4\pi\varepsilon_0}$

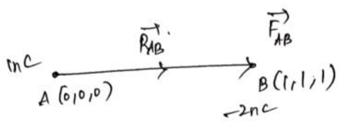
$\varepsilon_0$ - permittivity of free space

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

force on $Q_1$ by $Q_2 \Rightarrow \vec{F}_{21} = ?$

$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$

2. b) Point charges of 1 nC and -2nC are located at A (0,0,0) and B (1,1,1) respectively, in free space. Determine the vector force acting on the charge at B due to the charge at A. [05] CO1 L3




$\vec{R}_{AB} = \hat{a}_x + \hat{a}_y + \hat{a}_z$

$|\vec{R}_{AB}| = \sqrt{1+1+1} = \sqrt{3}$

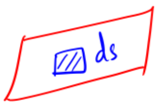
$\vec{F}_{AB} = \dfrac{Q_A Q_B}{4\pi\varepsilon_0} \cdot \dfrac{\vec{R}}{|\vec{R}|^3}$

$\vec{F}_{AB} = \dfrac{1 \times 10^{-9} \times (-2) \times 10^{-9} \times 9 \times 10^{9} (\hat{a}_x + \hat{a}_y + \hat{a}_z)}{(\sqrt{3})^3}$

$\boxed{\vec{F}_{AB} = (-3.46\,\hat{a}_x - 3.46\,\hat{a}_y - 3.46\,\hat{a}_z)\ \text{nN}}$

3. Define surface charge density. Obtain an expression of electric field intensity due to an infinite sheet of charge with uniform surface charge distribution $\rho_s$ C/m$^2$. Assume the charge is placed over x-y plane. [10] CO1 L2

Surface charge density '$\rho_s$' C/m$^2$ (Charge per unit surface area)

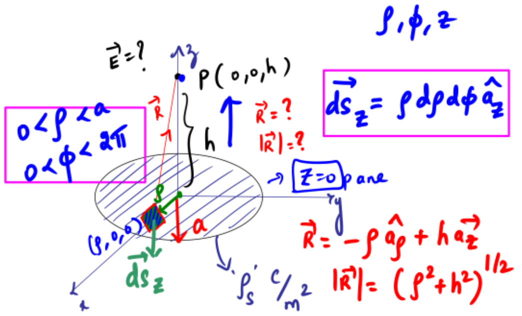



$Q_{total} = \iint_S \rho_s \, ds$

where $\boxed{\rho_s = \lim_{\Delta s \to 0} \dfrac{Q}{\Delta s}}$

## (i) Circular disc of charge on Z= 0 plane

$\rho, \phi, z$




Differential Charge in differential surface ds can be written as, $dq = \rho_s\, ds = \rho_s\, \rho\, d\rho\, d\phi$

Differential Electric field intensity due to differential charge dQ is given as, $d\vec{E} = \frac{dq}{4\pi\varepsilon_0 |\vec{R}|^3} \cdot \vec{R}$

$$d\vec{E} = \frac{\rho_s\, \rho\, d\rho\, d\phi\, (-\rho\hat{a}_\rho + h\hat{a}_z)}{4\pi\varepsilon_0\, (\rho^2 + h^2)^{3/2}}$$

Electric field intensity due to entire circular disc of charge is given as,

$$\vec{E} = \iint_S d\vec{E} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \frac{\rho_s\, \rho\, d\rho\, d\phi\, (-\rho\hat{a}_\rho + h\hat{a}_z)}{4\pi\varepsilon_0 (\rho^2+h^2)^{3/2}}$$

Due to radial symmetry of the point P about the axis of the circular disc, the resultant $\vec{E}$ has only $\vec{a_z}$ component, whereas the radial components $\vec{a_\rho}$ cancel each other at point P.

$$\vec{E} = \frac{\rho_s}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \frac{\rho\, d\rho\, d\phi\, h}{(\rho^2+h^2)^{3/2}}\, \hat{a}_z$$

Let $\rho^2 + h^2 = u^2$

Diff. $2\rho\, d\rho = 2u\, du$

Substitution

Let, $\rho^2 + h^2 = u^2$

$\rho = 0 \Rightarrow 0 + h^2 = u^2 \rightarrow u = h$

$\rho = a \Rightarrow a^2 + h^2 = u^2 \rightarrow u = \sqrt{a^2+h^2}$

Change in Limits

| | | |
|---|---|---|
| $\rho$ : | 0 | a |
| u : | h | $\sqrt{a^2+h^2}$ |

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi}\int_{h}^{\sqrt{a^2+h^2}} \frac{u\, du}{(u^2)^{3/2}}\, d\phi\, \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi}\int_{h}^{\sqrt{a^2+h^2}} \frac{1}{u^2}\, du\, d\phi\, \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi} \left[-\frac{1}{u}\right]_h^{\sqrt{a^2+h^2}} d\phi\, \hat{a}_z$$

$$\vec{E} = \frac{\rho_s h}{4\pi\varepsilon_0} \left[\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right] \int_{\phi=0}^{2\pi} d\phi\, \hat{a}_z$$

(i) $$\boxed{\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left[\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right] \hat{a}_z} \text{ V/m}$$

$\vec{E}$ due to circular disc of charge along z=0 plane

(ii) $\vec{E}$ due to infinite sheet of charge

$a \longrightarrow \infty$

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left[\frac{1}{h} - \cancel{\frac{1}{\sqrt{a^2+h^2}}}\right]\hat{a}_z$$

0

$$\vec{E} = \frac{\rho_s h}{2\varepsilon_0}\left[\frac{1}{h}\right]\hat{a}_z$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_z}$$

$\vec{E}$ due to infinite sheet of charge

In general, $\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_n}$ where $\hat{a}_n$ is unit normal to the surface

4. a) Define Electric flux density. [02] CO2 L1

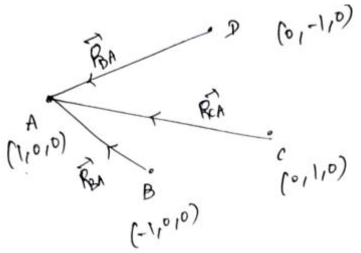
**Electric Flux density is defined as the number of electric field lines crossing per unit area.**

$$\text{Electric flux density} = \frac{\Psi}{\text{Area}} \; C/m^2$$

$$\text{Electric flux density } (\vec{D}) = \frac{\Psi}{4\pi r^2}\vec{a}_r \; C/m^2$$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\vec{a}_r} \; C/m^2$$

4. b) Point charges of 50 nC each are located at A(1, 0, 0), B(−1, 0, 0), C(0, 1, 0), and D(0,−1, 0) in free space. Find the total force on the charge at A. [08] CO1 L3




$\vec{R}_{BA} = 2\vec{a_x}$

$\vec{R}_{CA} = \hat{a_x} - \hat{a_y}$

$\vec{R}_{DA} = \hat{a_x} + \hat{a_y}$

$\vec{F}_A = \frac{Q_1 Q_2}{4\pi\epsilon_0} \frac{\vec{R}}{|\vec{R}|^3}$

$|\vec{R}_{BA}| = 2$

$|\vec{R}_{CA}| = \sqrt{2}$

$|\vec{R}_{DA}| = \sqrt{2}$

$$\vec{F}_A = 50\times10^{-9} \times 50\times10^{-9} \times 9\times10^{9} \left[ \frac{2\hat{a_x}}{2^3} + \frac{\hat{a_x} - \hat{a_y}}{(\sqrt{2})^3} + \frac{\hat{a_x}+\hat{a_y}}{(\sqrt{2})^3} \right]$$

$$\vec{F}_A = 22.5 \times 10^{-6} \left[ \frac{1}{4}\hat{a_x} + 0.3535\,(\hat{a_x} - \hat{a_y}) + 0.3535\,(\hat{a_x} + \hat{a_y}) \right]$$

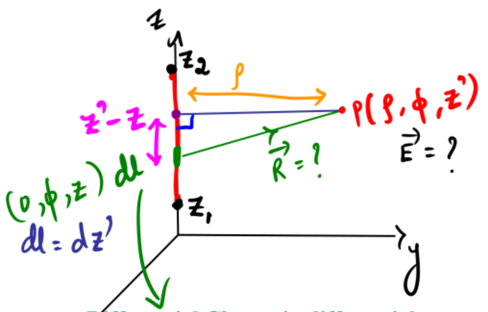
$\boxed{\vec{F}_A = 0.957\,\hat{a_x}\ \mu N} \times 22.5$

$\boxed{\vec{F}_A = 21.5\,\hat{a_x}\,\mu N}$

5. Define electric field intensity. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution [10] CO1 L2

**Electric field intensity is defined as the force per unit test charge**

$$\boxed{\vec{E} = \frac{\vec{F}}{Q_t} = \frac{Q}{4\pi\epsilon_0 |\vec{R}|^2}\,\hat{a}_R}$$ N/C (or) V/m




$\rho, \phi, z'$ are constants at point P.

Differential Charge in differential length dl can be written as,

$dQ = \rho_L\, dl$

$dQ = \rho_L\, dz$

$\vec{R} = \rho\,\hat{a}_\rho + (z' - z)\,\hat{a}_z$

$|\vec{R}| = \left[\rho^2 + (z'-z)\right]^{1/2}$

Differential Electric field intensity due to differential charge dQ is given as,

$$\boxed{\vec{dE} = \frac{dQ}{4\pi\varepsilon\,|\vec{R}|^3}\cdot\vec{R}}$$

where $\varepsilon = \varepsilon_0\varepsilon_r$

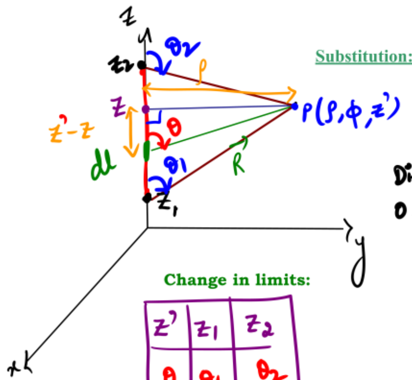
Electric field intensity due to entire length of charge is given as,

$$\boxed{\vec{E} = \int_L \vec{dE}}$$

$$\vec{dE} = \frac{\rho_L\, dz\,[\rho\,\hat{a}_\rho + (z'-z)\,\hat{a}_z]}{4\pi\varepsilon\,[\rho^2+(z'-z)^2]^{3/2}}$$

$z_1 < z < z_2$

$$\vec{E} = \int_{z'=z_1}^{z_2} \frac{\rho_L\, dz\,[\rho\,\hat{a}_\rho + (z'-z)\,\hat{a}_z]}{4\pi\varepsilon\,[\rho^2+(z'-z)^2]^{3/2}}$$




**Substitution:**

$$\int_{z_1}^{z_2} dz \rightarrow \int_{\theta_1}^{\theta_2} d\theta$$

$$\boxed{z'-z = \rho\cot\theta}$$

Differentiation

$$0 - dz = -\rho\,\mathrm{cosec}^2\theta\, d\theta$$

$$dz = \rho\,\mathrm{cosec}^2\theta\, d\theta$$

**Change in limits:**

| $z'$ | $z_1$ | $z_2$ |
|---|---|---|
| $\theta$ | $\theta_1$ | $\theta_2$ |

$$\vec{E} = \int_{\theta=\theta_1}^{\theta_2} \rho_L\,\frac{\rho\,\mathrm{cosec}^2\theta\, d\theta\,[\rho\,\hat{a}_\rho + (\rho\cot\theta)\,\hat{a}_z]}{4\pi\varepsilon\,[\rho^2+\rho^2\cot^2\theta]^{3/2}}$$

$$= [\rho^2(1+\cot^2\theta)]^{3/2}$$

$$= (\rho^2\,\mathrm{cosec}^2\theta)^{3/2}$$

$$= \rho^3\,\mathrm{cosec}^3\theta$$

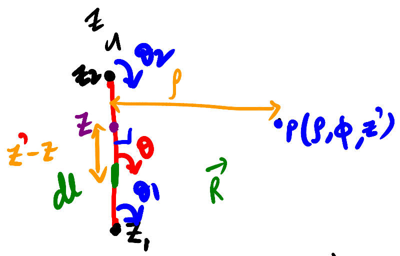
$$\vec{E} = \int_{\theta_1}^{\theta_2} \frac{\rho_L}{4\pi\varepsilon}\,\frac{d\theta}{\rho\,\mathrm{cosec}\theta}\,[\hat{a}_\rho + \cot\theta\,\hat{a}_z]$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[\int_{\theta_1}^{\theta_2} \frac{d\theta}{\operatorname{cosec}\theta}\,\hat{a}_\rho + \int_{\theta_1}^{\theta_2} \frac{\cot\theta\, d\theta}{\operatorname{cosec}\theta}\,\hat{a}_z\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta\,\hat{a}_\rho + \int_{\theta_1}^{\theta_2} \cos\theta\, d\theta\,\hat{a}_z\right]$$

$\vec{E}$ due to finite length of line charge

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\cos\theta_1 - \cos\theta_2)\,\hat{a}_\rho + (\sin\theta_2 - \sin\theta_1)\,\hat{a}_z\right]$$




W.K.T.:

$$z' - z = \rho\cot\theta$$

$$\theta = \cot^{-1}\left(\frac{z'-z}{\rho}\right)$$

For infinite length,

$z_1 \longrightarrow -\infty$ and

$z_2 \longrightarrow \infty$

$$\therefore\ \theta_1 = \cot^{-1}\left(\frac{z' - (-\infty)}{\rho}\right) \qquad \theta_2 = \cot^{-1}\left(\frac{z'-\infty}{\rho}\right)$$

$$\theta_1 = \cot^{-1}(\infty) = 0 \qquad \theta_2 = \cot^{-1}(-\infty) = \pi$$

For infinite line charge, $\theta_1 = 0$ and $\theta_2 = \pi$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(\cos\underbrace{\theta_1}_{0} - \cos\underbrace{\theta_2}_{\pi})\,\hat{a}_\rho + (\sin\underbrace{\theta_2}_{\pi} - \sin\underbrace{\theta_1}_{0})\,\hat{a}_z\right]$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon\rho}\left[(1+1)\,\hat{a}_\rho\right]$$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon\rho}\,\hat{a}_\rho$$ $\vec{E}$ due to infinitely long line charge

6. a) Calculate **E** and **D** in rectangular coordinates at point P(2,−3, 6) produced by a point charge $Q_A$ = 55 mC at A(−2, 3,−6). [06] CO1 L3

$\vec{R} = 4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z$

$Q_A = 55\ mC$ $(-2, 3, -6)$ → $P\ (2, -3, 6)$

$|\vec{R}| = \sqrt{4^2+6^2+12^2}$

$|\vec{R}| = 14$

i) $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{|\vec{R}|^3} \cdot \vec{R}$

$$\vec{E} = \frac{9\times10^9 \times 55\times10^{-3} \times (4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)}{(14)^3}$$

$$\boxed{\vec{E} = (0.721\ \hat{a}_x - 1.082\ \hat{a}_y + 2.164\ \hat{a}_z)\times10^6\ V/m}$$

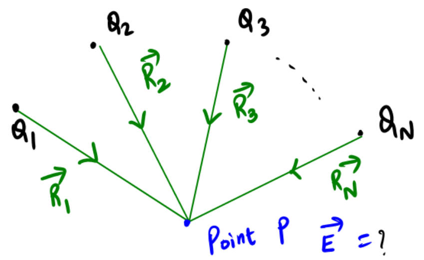
(ii) $\vec{D} = \frac{Q}{4\pi} \cdot \frac{\vec{R}}{|\vec{R}|^3}$

$$\vec{D} = \frac{55\times10^{-3}}{4\pi\times(14)^3} \times (4\hat{a}_x - 6\hat{a}_y + 12\hat{a}_z)$$

$$\boxed{\vec{D} = (6.38\ \hat{a}_x - 9.57\ \hat{a}_y + 19.14\ \hat{a}_z)\ \mu\ C/m^2}$$

6. b) Derive the expression for the electric field intensity at a point due to n number of point charges. [04] CO1 L2

**Electric Field Intensity due to a system of N number of point charges:**




$\vec{E}$ at point P, $\vec{E}_T = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_N$

$$\vec{E}_T = \frac{Q_1}{4\pi\varepsilon_0|\vec{R}_1|^3}\cdot\vec{R}_1 + \frac{Q_2}{4\pi\varepsilon_0|\vec{R}_2|^3}\cdot\vec{R}_2 + \cdots + \frac{Q_N}{4\pi\varepsilon_0|\vec{R}_N|^3}\cdot\vec{R}_N$$

$$\boxed{\vec{E}_T = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{Q_i}{|\vec{R}_i|^3}\cdot\vec{R}_i}\ V/m$$