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Internal Assessment Test 2 – January 2024

Sub:	**Network Analysis**				Sub Code:	**BEC304**	Branch:	**ECE**
Date:	**19/01/2024**	Duration:	90 Minutes	Max Marks: 50	Sem/Sec:	**3/A, B, C, D**		**OBE**

	Answer Any Five Questions	MARKS	CO	RBT
1a	State and explain Superposition Theorem.	[3]	CO2	L2

Superposition Theorem :- The principle of superposition applicable only for linear systems.

It states that,

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as algebraic sum of the individual combinations of each source acting alone.

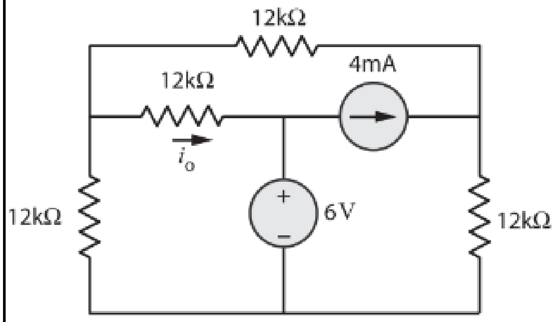
Step wise approach :-

Step 1: In a circuit comprising of many independent sources, only one source is allowed to be active, the rest are deactivated.

Step 2: To deactivate voltage source, replace it with a short circuit and to deactivate current source, replace it with an open circuit.

Step 3: The response obtained by applying each source, one at a time are then added algebraically to obtain a solution.

1 b	Find $\mathbf{i_0}$ in the network shown in Fig. using superposition.	[7]	CO2	L2



As a first step, set the current source to zero. That is, the current source appears as an open circuit as shown in Fig. 3.5.

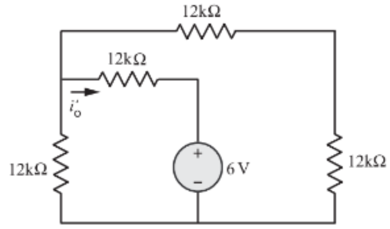
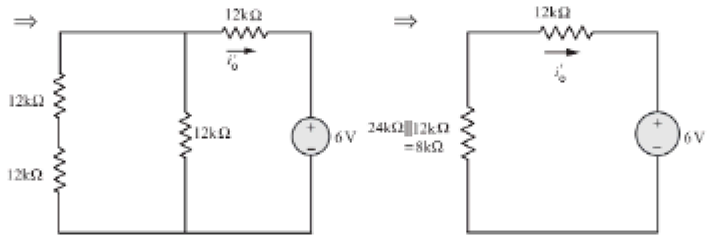


Figure 3.5



$$i_o' = \frac{-6}{(8+12)\times 10^3} = -0.3\text{ mA}$$

As a second step, set the voltage source to zero. This means the voltage source in Fig. 3.4 is replaced by a short circuit as shown in Figs. 3.6 and 3.6(a). Using current division principle,

$$i_A = \frac{iR_2}{R_1+R_2}$$

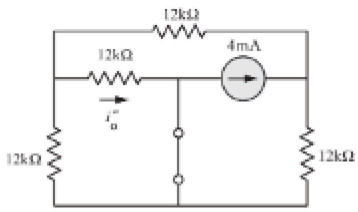


Figure 3.6

where $R_1 = (12\text{ k}\Omega || 12\text{ k}\Omega) + 12\text{ k}\Omega$

$= 6\text{ k}\Omega + 12\text{ k}\Omega$

$= 18\text{ k}\Omega$

and $R_2 = 12\text{ k}\Omega$

⇒ $i_A = \dfrac{4\times 10^{-3}\times 12\times 10^3}{(12+18)\times 10^3}$

$= 1.6\text{ mA}$

Again applying the current division principle,

$$i_o'' = \frac{i_A \times 12}{12+12} = 0.8\text{ mA}$$

Thus, $i_o = i_o' + i_o'' = -0.3 + 0.8 = \mathbf{0.5\text{ mA}}$

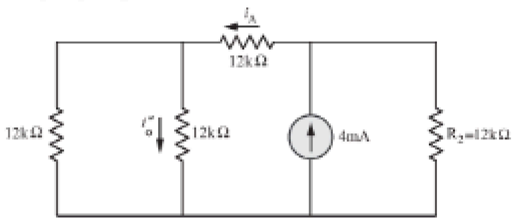
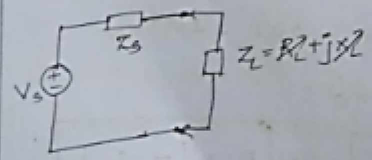


Figure 3.6(a)

2a	State and prove Maximum Power Transfer Theorem when both $\mathbf{R_L}$ **and** $\mathbf{X_L}$ are varying.	[5]	CO2	L2

Case (5):- AC source with $Z_S = R_S + jX_S$ connected to a $Z_L = R_L + jX_L$



$$I = \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$$|I| = \frac{V_S}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

$$P = |I|^2 R_L = \frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

For max. power transfer, $\frac{dP}{dX_L} = 0$

$$\frac{d}{dX_L}\left[\frac{V_S^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}\right] = 0$$

$$0 - R_L[0 + 2(X_S + X_L)] = 0$$

$$X_S + X_L = 0$$

$$X_L = -X_S$$

The condition for max. power transfer is the load reactance should be conjugate of source reactance. Under this condition,

$$P_{max} = \frac{V_S^2 R_L}{(R_S + R_L)^2}$$

$\therefore \frac{dP}{dR_L} = 0$

$$\frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S + R_L)^2}\right] = 0$$

$$(R_S + R_L)^2(1) - R_L[2(R_S + R_L)] = 0$$

$$R_S^2 + R_L^2 + 2R_SR_L - 2R_SR_L - 2R_L^2 = 0$$

$$R_S^2 - R_L^2 = 0$$

$$R_S^2 = R_L^2$$

WKT,

$Z_L = R_L + jX_L$

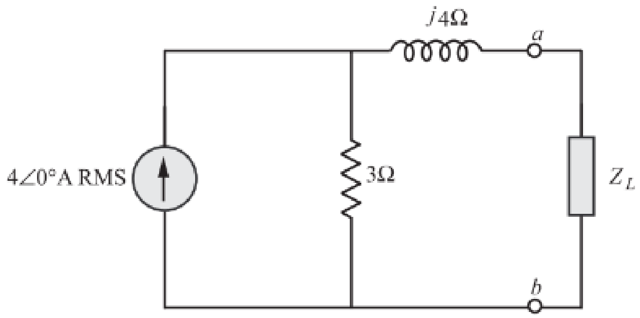
put $R_L = R_S$ & $X_L = -X_S$

$\therefore Z_L = R_S - jX_S$

$\boxed{Z_L = Z_S^*}$

$\therefore$ The condition for max. power transfer is the load impedance should be complex conjugate of source impedance.

2b	Find the load impedance that transfers the maximum average power to the load and determine the maximum average power transferred to the load Z_L shown in Fig.	[5]	CO2	L2



SOLUTION

The first step in the analysis is to find the Thevenin equivalent circuit by disconnecting the load $\mathbf{Z}_L$. This leads to a circuit diagram as shown in Fig. 3.114.

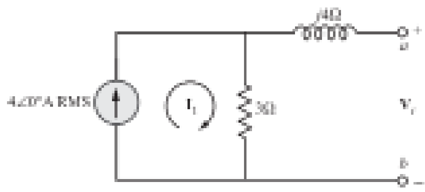


Figure 3.114

Hence
$$\mathbf{V}_t = \mathbf{V}_{oc} = 4\angle 0^\circ \times 3$$
$$= 12\angle 0^\circ \text{ Volts(RMS)}$$

To find $\mathbf{Z}_t$, let us deactivate all the independent sources of Fig. 3.114. This leads to a circuit diagram as shown in Fig 3.114 (a):

$$\mathbf{Z}_t = 3 + j4\ \Omega$$

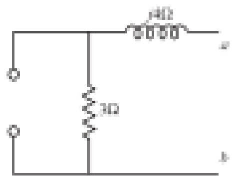


Figure 3.114 (a)

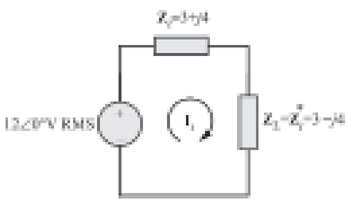


Figure 3.115

The Thevenin equivalent circuit with $\mathbf{Z}_L$ is as shown in Fig. 3.115.
For maximum average power transfer to the load, $\mathbf{Z}_L = \mathbf{Z}_t^* = 3 - j4$.

$$\mathbf{I}_1 = \frac{12\angle 0^\circ}{3 + j4 + 3 - j4} = 2\angle 0^\circ \text{ A(RMS)}$$

Hence, maximum average power delivered to the load is

$$P = |I_1|^2 R_L = 4(3) = 12 \text{ W}$$

It may be noted that the scaling factor $\frac{1}{2}$ is not taken since the phase current is already expressed by its RMS value.

3a	State and explain Millman's Theorem.	[4]	CO2	L1

Millman's theorem states that if n number of generators having generated emfs $\mathbf{E}_1, \mathbf{E}_2, \cdots \mathbf{E}_n$ and internal impedances $\mathbf{Z}_1, \mathbf{Z}_2, \cdots \mathbf{Z}_n$ are connected in parallel, then the emfs and impedances can be combined to give a single equivalent emf of E with an internal impedance of equivalent value Z.

where
$$\mathbf{E} = \frac{\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2 + \ldots + \mathbf{E}_n\mathbf{Y}_n}{\mathbf{Y}_1 + \mathbf{Y}_2 + \ldots + \mathbf{Y}_n}$$

and
$$\mathbf{Z} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2 + \ldots + \mathbf{Y}_n}$$

where $\mathbf{Y}_1, \mathbf{Y}_2 \cdots \mathbf{Y}_n$ are the admittances corresponding to the internal impedances $\mathbf{Z}_1, \mathbf{Z}_2 \cdots \mathbf{Z}_n$ and are given by

$$\mathbf{Y}_1 = \frac{1}{\mathbf{Z}_1}$$
$$\mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2}$$
$$\vdots$$
$$\mathbf{Y}_n = \frac{1}{\mathbf{Z}_n}$$

Fig. 3.134 shows a number of generators having emfs $\mathbf{E}_1, \mathbf{E}_2 \cdots \mathbf{E}_n$ connected in parallel across the terminals x and y. Also, $\mathbf{Z}_1, \mathbf{Z}_2 \cdots \mathbf{Z}_n$ are the respective internal impedances of the generators.

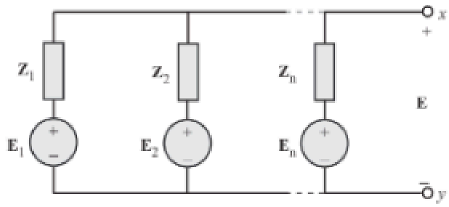


Figure 3.134

The Thevenin equivalent circuit of Fig. 3.134 using Millman's theorem is shown in Fig. 3.135. The nodal equation at x gives

$$\frac{\mathbf{E}_1 - \mathbf{E}}{\mathbf{Z}_1} + \frac{\mathbf{E}_2 - \mathbf{E}}{\mathbf{Z}_2} + \cdots + \frac{\mathbf{E}_n - \mathbf{E}}{\mathbf{Z}_n} = 0$$

$$\Rightarrow \quad \left[\frac{\mathbf{E}_1}{\mathbf{Z}_1} + \frac{\mathbf{E}_2}{\mathbf{Z}_2} + \cdots + \frac{\mathbf{E}_n}{\mathbf{Z}_n}\right] = \mathbf{E}\left[\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_n}\right]$$

$$\Rightarrow \quad \mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2 + \cdots + \mathbf{E}_n\mathbf{Y}_n = \mathbf{E}\left[\frac{1}{\mathbf{Z}}\right]$$

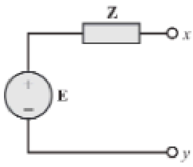


Figure 3.135

where $\mathbf{Z}$ = Equivalent internal impedance.

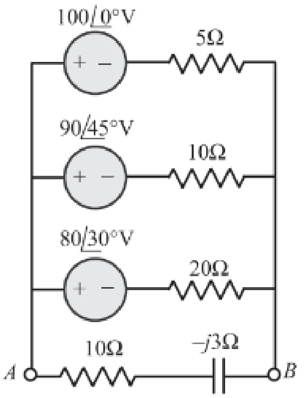
or
$$[\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2 + \cdots + \mathbf{E}_n\mathbf{Y}_n] = \mathbf{E}\mathbf{Y}$$

$$\Rightarrow \quad \mathbf{E} = \frac{\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2 + \cdots + \mathbf{E}_n\mathbf{Y}_n}{\mathbf{Y}}$$

where
$$\mathbf{Y} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_n$$

and
$$\mathbf{Z} = \frac{1}{\mathbf{Y}} = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_n}$$

3b	Find the current through **(10 -j3) Ω** using Millman's theorem. Refer Fig.	[6]	CO2	L2



SOLUTION

The circuit shown in Fig. 3.138 is replaced by its Thevenin equivalent circuit as seen from the terminals, *A* and *B* using Millman's theorem. Fig. 3.139 shows the Thevenin equivalent circuit along with $\mathbf{Z}_L = 10 - j3\ \Omega$.

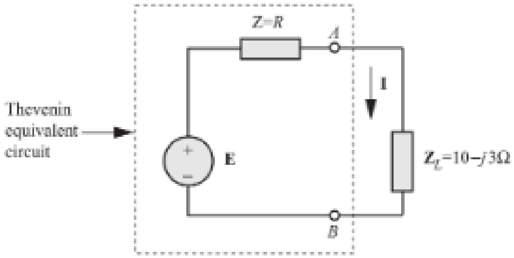


Figure 3.139

$$\mathbf{E} = \frac{\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2 - \mathbf{E}_3\mathbf{Y}_3}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3}$$

$$= \frac{100\,\underline{/0^\circ}\left(\frac{1}{5}\right) + 90\,\underline{/45^\circ}\left(\frac{1}{10}\right) + 80\,\underline{/30^\circ}\left(\frac{1}{20}\right)}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}}$$

$$= 88.49\,\underline{/15.66^\circ}\ \text{V}$$

$$\mathbf{Z} = R = \frac{1}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3} = \frac{1}{\frac{1}{5} + \frac{1}{10} + \frac{1}{20}} = 2.86\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z} + \mathbf{Z}_L} = \frac{88.49\,\underline{/15.66}}{2.86 + 10 - j3} = 6.7\,\underline{/28.79^\circ}\ \mathbf{A}$$

Alternately,

$$\mathbf{E} = \frac{\mathbf{E}_1\mathbf{Y}_1 + \mathbf{E}_2\mathbf{Y}_2 + \mathbf{E}_3\mathbf{Y}_3 + \mathbf{E}_4\mathbf{Y}_4}{\mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \mathbf{Y}_4}$$

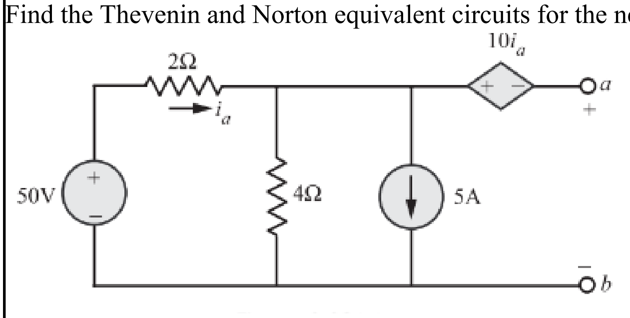
$$= \frac{100 \times 5^{-1} + 90\,\underline{/45^\circ} \times 10^{-1} + 80\,\underline{/30^\circ} \times 20^{-1}}{5^{-1} + 10^{-1} + 20^{-1} + (10 - j3)^{-1}}$$

$$= 70\,\underline{/12^\circ}\ \text{V}$$

Therefore, $$I = \frac{70\,\underline{/12^\circ}}{10 - j3}$$

$$= 6.7\,\underline{/28.8^\circ}\ \text{A}$$

4	Find the Thevenin and Norton equivalent circuits for the network shown in Fig.	[10]	CO2	L3



SOLUTION

To find V_{oc} :

Performing source transformation on 5A current source, we get the circuit shown in Fig. 3.80 (b).

Applying KVL around Left mesh :

$$-50 + 2i_a - 20 + 4i_a = 0$$

$$\Rightarrow \quad i_a = \frac{70}{6}\text{A}$$

Applying KVL around right mesh:

$$20 + 10i_a + V_{oc} - 4i_a = 0$$

$$\Rightarrow \quad V_{oc} = -90\text{ V}$$

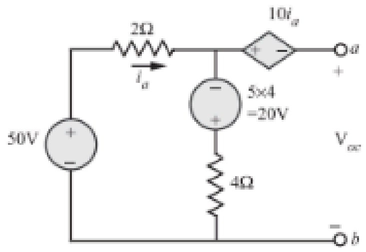


Figure 3.80(b)

To find i_{sc}(referring Fig 3.80 (c)) :

KVL around Left mesh :

$$-50 + 2i_a - 20 + 4(i_a - i_{sc}) = 0$$

$$\Rightarrow \quad 6i_a - 4i_{sc} = 70$$

KVL around right mesh :

$$4(i_{sc} - i_a) + 20 + 10i_a = 0$$

$$\Rightarrow \quad 6i_a + 4i_{sc} = -20$$

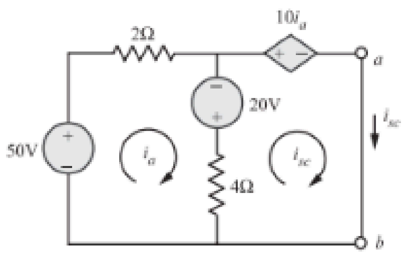
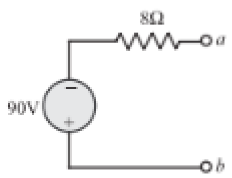


Figure 3.80(c)

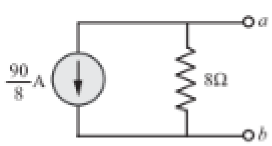
Solving the two mesh equations simultaneously, we get $i_{sc} = -11.25$ A

Hence, $R_t = R_N = \frac{v_{oc}}{i_{sc}} = \frac{-90}{-11.25} = 8\ \Omega$

Performing source transformation on Thevenin equivalent circuit, we get the norton equivalent circuit (both are shown below).

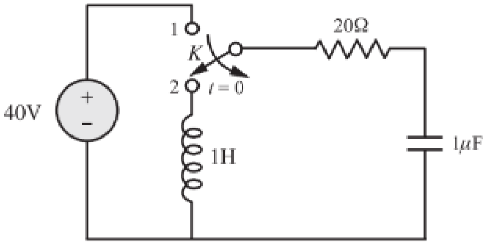


Thevenin equivalent circuit



Norton equivalent circuit

5	In the network shown in Fig. the switch is moved from position 1 to position 2 at t= 0. Calculate i, $\frac{di}{dt}$, and $\frac{d^2i}{dt^2}$ at $t = 0^+$.	[10]	CO3	L2



SOLUTION

The symbol for switch K implies that it is in position 1 at $t = 0^-$ and in position 2 at $t = 0^+$. Under steady-state condition, a capacitor acts as an open circuit. Hence at $t = 0^-$, the circuit diagram is as shown in Fig. 4.18(a).

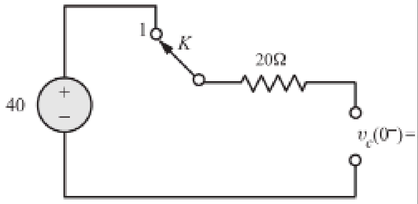


Figure 4.18(a)

We know that the voltage across a capacitor cannot change instantaneously. This means that $v_C(0^+) = v_C(0^-) = 40$ V.

At $t = 0^-$, inductor is not energized. This means that $i(0^-) = 0$. Since current in an inductor cannot change instantaneously, $i(0^+) = i(0^-) = \mathbf{0}$. Hence, the circuit diagram at $t = 0^+$ is as shown in Fig. 4.18(b).

The circuit diagram for $t \geq 0^+$ is as shown in Fig.4.18(c).

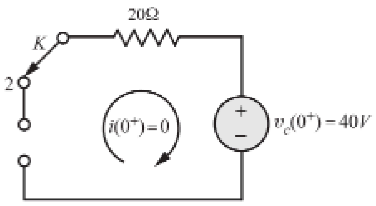


Figure 4.18(b)

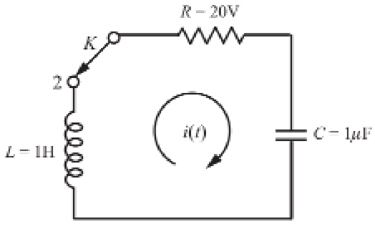


Figure 4.18(c)

Applying KVL clockwise, we get

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{0^+}^{t} i(\tau)d\tau = 0 \qquad (4.4)$$

$$\Rightarrow \qquad Ri + L\frac{di}{dt} + v_C(t) = 0$$

At $t = 0^+$, we get

$$Ri(0^+) + L\frac{di(0^+)}{dt} + v_C(0^+) = 0$$

$$\Rightarrow \qquad 20 \times 0 + 1\frac{di(0^+)}{dt} + 40 = 0$$

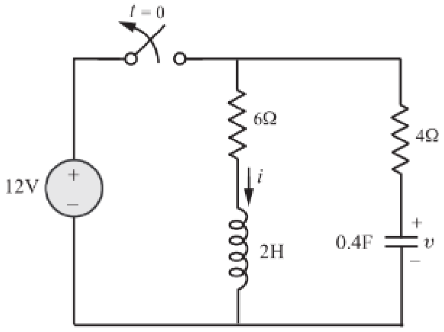
$$\Rightarrow \qquad \frac{di(0^+)}{dt} = \mathbf{-40A/sec}$$

Diferentiating equation (4.4) with respect to t, we get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

	Putting $t = 0^+$ in the above equation, we get $$R\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{C} = 0$$ $$\Rightarrow \quad R \times (-40) + L\frac{d^2i(0^+)}{dt^2} + \frac{0}{C} = 0$$ Hence $$\frac{d^2i(0^+)}{dt^2} = \mathbf{800A/sec^2}$$			

6	For the circuit shown in Fig. find:	[10]	CO3	L3



(a) $i(0^+)$ and $v(0^+)$

(b) $\dfrac{di(0^+)}{dt}$ and $\dfrac{dv(0^+)}{dt}$

(c) $i(\infty)$ and $v(\infty)$

Figure 4.28

SOLUTION

(a) From the symbol of switch, we find that at $t = 0^-$, the switch is closed and $t = 0^+$, it is open. At $t = 0^-$, the circuit has reached steady state so that the equivalent circuit is as shown in Fig.4.29(a).

$$i(0^-) = \frac{12}{6} = 2\text{A}$$

$$v(0^-) = 12 \text{ V}$$

Therefore, we have

$$i(0^+) = i(0^-) = \mathbf{2}\text{A}$$

$$v(0^+) = v(0^-) = \mathbf{12V}$$

(b) For $t \geq 0^+$, we have the equivalent circuit as shown in Fig.4.29(b).

(b) For $t \geq 0^+$, we have the equivalent circuit as shown in Fig.4.29(b).

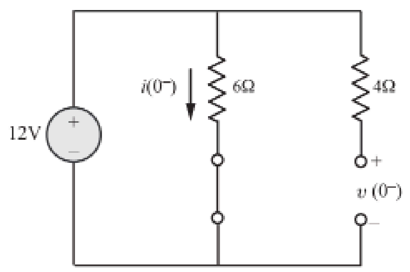


Figure 4.29(a)

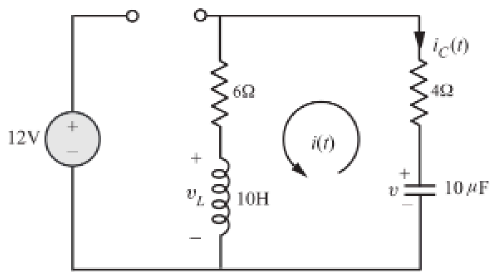


Figure 4.29(b)

Applying KVL anticlockwise to the mesh on the right, we get

$$v_L(t) - v(t) + 10i(t) = 0$$

Putting $t = 0^+$, we get

$$v_L(0^+) - v(0^+) + 10i(0^+) = 0$$

$$\Rightarrow \quad v_L(0^+) - 12 + 10 \times 2 = 0$$

$$\Rightarrow \quad v_L(0^+) = -8\text{V}$$

The voltage across the inductor is given by

$$v_L = L\frac{di}{dt}$$

$$\Rightarrow \quad v_L(0^+) = L\frac{di(0^+)}{dt}$$

$$\Rightarrow \quad \frac{di(0^+)}{dt} = \frac{1}{L}v_L(0^+)$$

$$= \frac{1}{10}(-8) = \mathbf{-0.8A/\,sec}$$

Similarly, the current through the capacitor is

$$i_C = C\frac{dv}{dt}$$

or $$\frac{dv(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-i(0^+)}{C}$$

$$= \frac{-2}{10 \times 10^{-6}} = \mathbf{-0.2 \times 10^6 V/\,sec}$$

(c) As t approaches infinity, the switch is open and the circuit has attained steady state. The equivalent circuit at $t = \infty$ is shown in Fig.4.29(c).

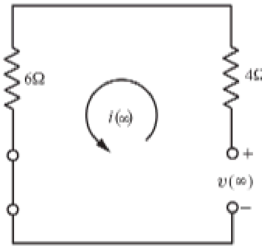
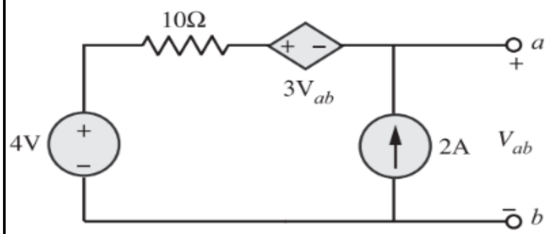
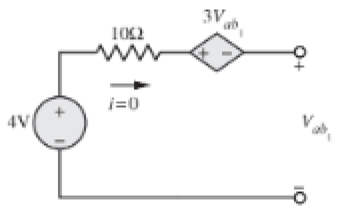
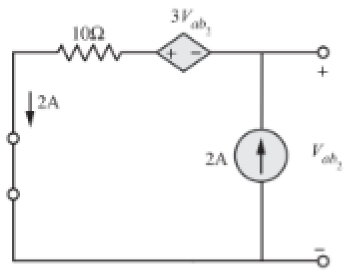
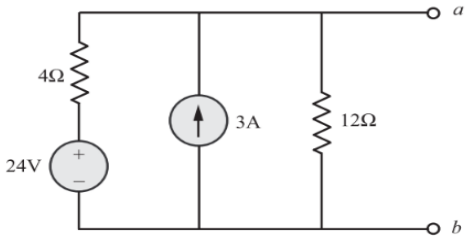


Figure 4.29(c)

$$i(\infty) = \mathbf{0}$$
$$v(\infty) = \mathbf{0}$$

7	For the circuit shown in Fig. Find the terminal voltage using superposition principle.  As a first step in the analysis, deactivate the independent current source. This results in a circuit diagram as shown in Fig. 3.15. *Applying KVL clockwise gives* $-4 + 10 \times 0 + 3V_{ab_1} + V_{ab_1} = 0$ $\Rightarrow \quad 4V_{ab_1} = 4$ $\Rightarrow \quad V_{ab_1} = 1\text{V}$  Figure 3.15 Next step in the analysis is to deactivate the independent voltage source, resulting in a circuit diagram as shown in Fig. 3.16. *Applying KVL gives* $-10 \times 2 + 3V_{ab_2} + V_{ab_2} = 0$ $\Rightarrow \quad 4V_{ab_2} = 20$ $\Rightarrow \quad V_{ab_2} = 5\text{V}$  Figure 3.16 According to superposition principle, $V_{ab} = V_{ab_1} + V_{ab_2} = 1 + 5 = \mathbf{6V}$	[10]	CO2	L2

8	Find the Norton equivalent for the circuit of Fig.	[10]	CO2	L2



SOLUTION

As a first step, short the terminals $a - b$. This results in a circuit diagram as shown in Fig. 3.67. *Applying KCL at node a*, we get

$$\frac{0-24}{4} - 3 + i_{sc} = 0$$

$$\Rightarrow i_{sc} = 9\text{A}$$

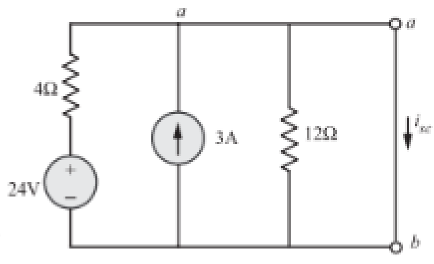


Figure 3.67

To find R_N, deactivate all the independent sources, resulting in a circuit diagram as shown in Fig. 3.68 (a). We find R_N in the same way as R_t in the Thevenin equivalent circuit.

$$R_N = \frac{4 \times 12}{4 + 12} = 3\ \Omega$$

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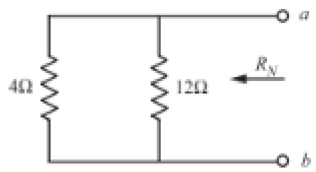

Figure 3.68(a)

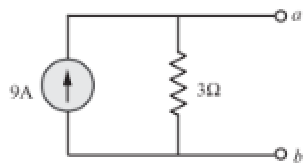

Figure 3.68(b)

Thus, we obtain Nortion equivalent circuit as shown in Fig. 3.68(b).