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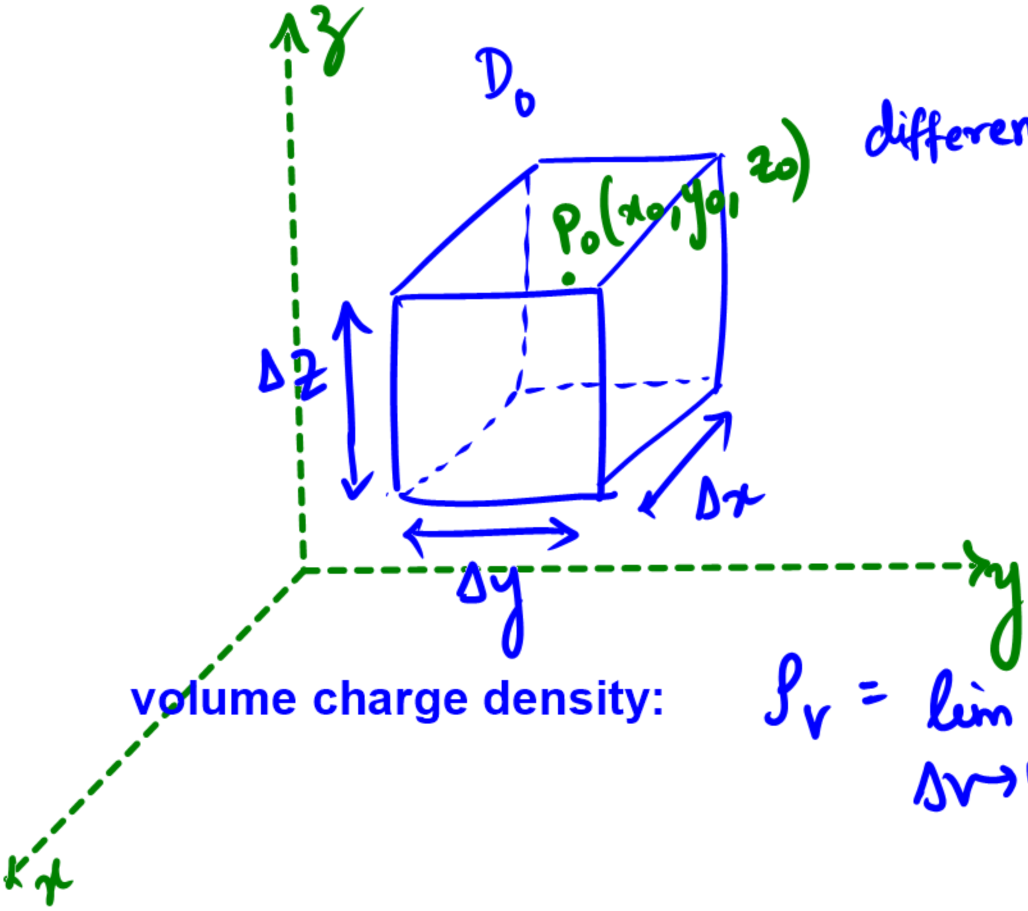
Internal Assesment Test-II									
Sub:	Electromagnetic Waves							Code:	21EC54
Date:	30/01/2024	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	ECE (A,B,C,D)
Answer any **FIVE FULL** Questions									

		Marks	OBE CO	OBE RBT
1.	Derive Maxwell's equation of electrostatics $\nabla . \boldsymbol{D} = \rho_V$.	[10]	CO2	L2
2. a)	Calculate electric field intensity because of infinite line charge using Gauss's law.	[05]	CO2	L2
2. b)	Calculate divergence of the given field. $\mathbf{D}= 1/z^2[10xyz\ \mathbf{a_x}+5x^2z\ \mathbf{a_y}+(2z^3 - 5x^2y)\mathbf{a_z}]$ C/m² at point P(2,3,5). $\mathbf{D} = 5r^2\ \mathbf{a_r}$ mC/m² at r = 0.06m,	[05]	CO2	L3
3. a)	Derive an expression for the work done in moving a point charge Q in the presence of an electric field **E**.	[05]	CO3	L2
3. b)	Given $V = \dfrac{\cos 2\phi}{\rho}$ in free space. Find the volume charge density at the point A(0.5, 60°, 1).	[05]	CO3	L3
4.	Evaluate both sides of the divergence theorem for the field $\mathbf{D} = 4xy\ \mathbf{a_x} + 2(x^2+z^2)\mathbf{a_y} + 4yz\ \mathbf{a_z}$ C/m² and the cube formed by the planes $x = 0$ and 2, $y = 0$ and 3, and $z = 0$ and 5.	[10]	CO2	L3
5. a)	Derive the expression for capacitance of coaxial cable using Laplace's equation. Consider the radius of the inner conductor 'a' and outer conductor 'b'. Potential at radius 'a' is maintained at V_0 and the outside surface is grounded.	[08]	CO3	L2
5. b)	Verify if the given field satisfies Laplace's equation: $V = 2x^2y-5z$ V.	[02]	CO3	L3
6. a)	State and prove the Uniqueness theorem.	[06]	CO3	L2
6. b)	If the potential field $V = 3x^2+2y^2+2z^3$ volts find V and **E** at point P(-4,5,4).	[04]	CO3	L3
7. a)	Derive an expression of differential force between differential current elements.	[06]	CO4	L2
7. b)	Discuss the force on a differential current element when placed in a magnetic field and also obtain the expression for force.	[04]	CO4	L2

1)

Application of Gauss's law to differential volume element: Derive Maxwell's First Equation of electrostatics from Gauss's law.



differential volume element

$$\Delta V = \Delta x \, \Delta y \, \Delta z$$

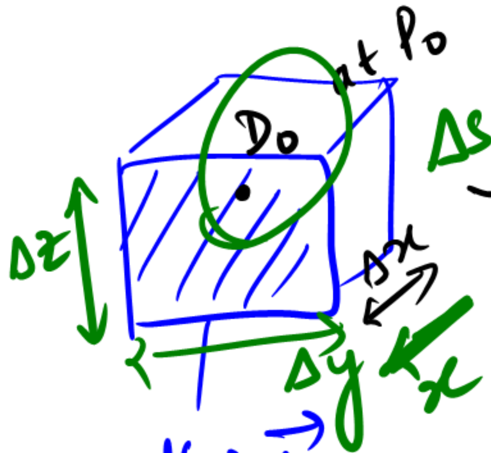
volume charge density: $\rho_v = \lim_{\Delta v \to 0} \dfrac{Q}{\Delta V}$ $\quad C/m^3$

Gauss's law: $\displaystyle \oiint_S \vec{D} \cdot \vec{ds} = Q_{enc}$

$$\iint_{front} \vec{D} \cdot \vec{ds} + \iint_{back} \vec{D} \cdot \vec{ds} + \iint_{right} \vec{D} \cdot \vec{ds} + \iint_{left} \vec{D} \cdot \vec{ds} + \iint_{top} \vec{D} \cdot \vec{ds} + \iint_{bottom} \vec{D} \cdot \vec{ds} = \rho_v \Delta V$$

Taylor Series: $f_0 = f(x_0)$

$$f(x) = f_0 + \frac{(x - x_0)}{1!} f_0' + \frac{(x - x_0)^2}{2!} f_0'' + \cdots$$



$$\Delta s = \Delta y \Delta z$$

$$\iint_{front} \vec{D}\cdot\vec{ds} = D_{front}\iint_s ds = \boxed{D_{front}}\,\Delta y \Delta z$$

$$D_{front} = D_{x_0} + \frac{\Delta x}{2}\frac{\partial}{\partial x}D_x + \text{higher order terms}$$

$$\boxed{\iint_{front} \vec{D}\cdot\vec{ds}_x = \left[D_{x0} + \boxed{\frac{\Delta x}{2}}\frac{\partial D_x}{\partial x} + \text{higher order terms}\right]\Delta y \Delta z}$$

$$\iint_{back} \vec{D}\cdot\vec{ds}_{-x} = D_{back}\iint_s ds_{-x} = -D_{back}\,\Delta y \Delta z$$

$$D_{back} = D_{x_0} - \frac{\Delta x}{2}\frac{\partial D_x}{\partial x} + \text{higher order terms}$$

$$\iint_{back} \vec{D}\cdot\vec{ds} = \left[-D_{x_0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x} - h.o.t.\right]\Delta y \Delta z$$

$$\iint_{front} + \iint_{back} \vec{D}\cdot\vec{ds} = \frac{\partial D_x}{\partial x}\,\Delta x\,\Delta y\,\Delta z$$

$$\iint_{left} + \iint_{right} \vec{D}\cdot\vec{ds} = \frac{\partial D_y}{\partial y}\,\Delta x\,\Delta y\,\Delta z$$

$$\iint_{top} + \iint_{bottom} \vec{D}\cdot\vec{ds} = \frac{\partial D_z}{\partial z}\,\Delta x\,\Delta y\,\Delta z$$

$$\oiint_S \vec{D}\cdot\vec{ds} = \left[\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right]\Delta x\,\Delta y\,\Delta z$$

$$\oiint_S \vec{D}\cdot\vec{ds} = Q_{enc} = \rho_v\,\Delta x\,\Delta y\,\Delta z$$

$$\boxed{\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)}\cancel{\Delta v} = \rho_v\,\cancel{\Delta v}$$

$$\operatorname{div}\vec{D} = \vec{\nabla}\cdot\vec{D} = \rho_v$$

Differential form of Gauss's law (or) Maxwell's first equation of electrostatics.

The divergence of the vector flux density $\vec{A}$ is the outflow of flux from a small closed surface per unit volume as the volume shrinks to zero.

$$\text{Divergence of } \mathbf{A} = \text{div } \mathbf{A} = \lim_{\Delta v \to 0} \frac{\oint_S \mathbf{A} \cdot d\mathbf{S}}{\Delta v}$$

Divergence of electric flux density $= \text{div } \vec{D} = \lim_{\Delta V \to 0} \frac{\oiint_S \vec{D} \cdot \vec{ds}}{\Delta V}$

$$\boxed{\text{div } \vec{D} = \lim_{\Delta V \to 0} \frac{Q_{enc}}{\Delta V} = \rho_V}$$

Gauss's Divergence Theorem:

Gauss's law $\Rightarrow$ $\oiint_S \vec{D} \cdot \vec{ds} = Q_{enc}$

$$\oiint_S \vec{D} \cdot \vec{ds} = \iiint_V \rho_V \, dv$$

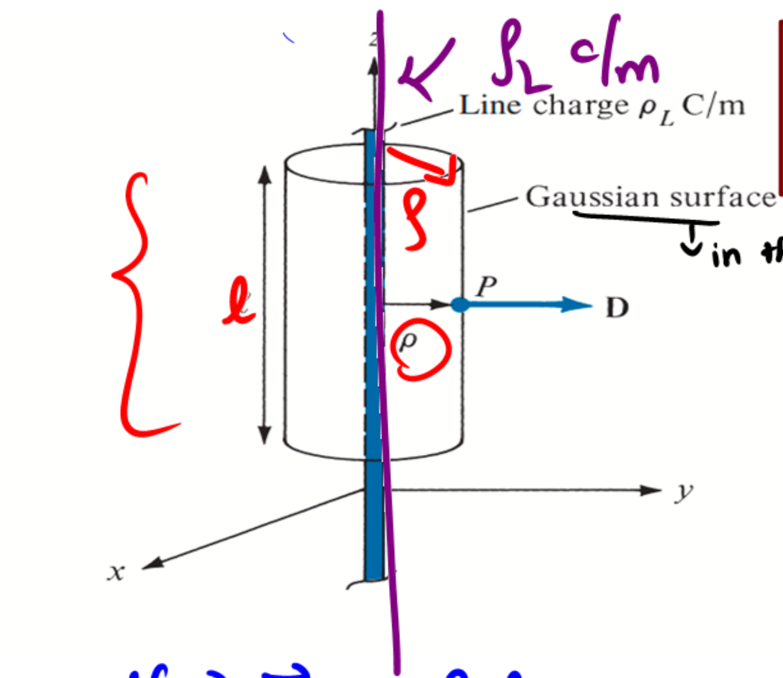
w.k.t. $\rho_V = \vec{\nabla} \cdot \vec{D}$

$$\boxed{\oiint_S \vec{D} \cdot \vec{ds} = \iiint_V \vec{\nabla} \cdot \vec{D} \, dv}$$

Gauss's Divergence Theorem:

2)a)

Infinite line charge $[\rho_L \ C/m]$



Gauss's law

$$\boxed{\oint\!\!\oint_S \vec{D} \cdot \vec{ds} = Q_{enc}}$$

Gaussian surface ↓ in the form of closed cylinder of radius 'ρ' and height 'l'.

$$RHS = Q_{enc} = \rho_L \cdot l$$

$$\oint\!\!\oint_S \vec{D} \cdot \vec{ds} = \rho_L l$$

$$\underbrace{\underbrace{\iint_{top} \vec{D} \cdot \vec{ds}}_{0} + \underbrace{\iint_{bottom} \vec{D} \cdot \vec{ds}}_{0}}_{\vec{D} \text{ is tangential to the surface}} + \underbrace{\iint_{\text{curved surface}} \vec{D} \cdot \vec{ds}}_{\vec{D} \text{ is normal to the surface}} = \rho_L l$$

$$\iint_S D \, ds = \rho_L l$$

$$D \left[\iint_S ds \right] = \rho_L l$$

surface area of cylinder $= 2\pi\rho l$

$$D \times 2\pi\rho l = \rho_L l$$

$$D = \frac{\rho_L}{2\pi\rho} \Rightarrow \boxed{\vec{D} = \frac{\rho_L}{2\pi\rho} \vec{a_\rho}} \ c/m^2$$

Electric field intensity, $\boxed{\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\rho_L}{2\pi\varepsilon\rho} \hat{a}_\rho \ V/m}$

2)b)

Find $\vec{\nabla}\cdot\vec{D}$.

i) $\vec{D} = \frac{1}{z^2}\left(10xyz\ \vec{a_x} + 5x^2z\ \vec{a_y} + (2z^3 - 5x^2y)\ \vec{a_z}\right)$ C/m² at P (2,3,5).

$$\vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}\left[\frac{10xyz}{z^2}\right] + \frac{\partial}{\partial y}\left[\frac{5x^2z}{z^2}\right] + \frac{\partial}{\partial z}\left[\frac{2z^3 - 5x^2y}{z^2}\right]$$

$$= \frac{10y}{z} + 0 + 2 - 5x^2y\left(-2z^{-3}\right)$$

$$\vec{\nabla}\cdot\vec{D} = \frac{10y}{z} + 2 + \frac{10x^2y}{z^3}$$

$$\vec{\nabla}\cdot\vec{D}\Big|_{(2,3,5)} = \frac{10\times3}{5} + 2 + \frac{\cancel{10}^{2}\times4\times3}{(5)^{\cancel{3}\,2}} = 6 + 2 + \frac{24}{25} = 8.96\ \text{C/m}^3$$

Let $\vec{D} = 5r^2\vec{a_r}$ m C/m²

2)b)

(i)

$r = 0.06$

$\vec{D} = 5r^2\vec{a_r}$

$$\rho_v = \vec{\nabla}\cdot\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_r\right) + 0 + 0$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(5r^4\right)\ \text{m}$$

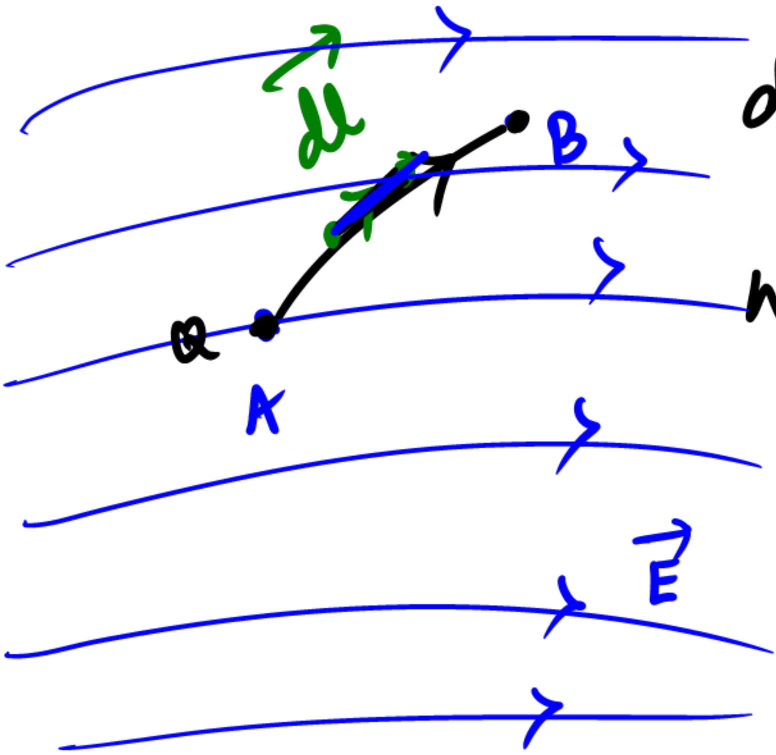
$$= \frac{1}{r^2}\cdot 20r^3\ \text{m}$$

$$\rho_v = 20r\ \text{m}$$

$$\boxed{\rho_v\Big|_{r=0.06} = 1.2\ \text{m}}\ \text{C/m}^2$$

3) a)

Energy expended in moving a point charge in an electric field - (Work Done)



$$dW = \vec{F} \cdot \vec{dl} \quad J$$

$$W = \int_A^B \vec{F} \cdot \vec{dl} \quad J$$

Force on the point charge in the electric field, $\vec{F} = Q\vec{E}$

Force applied to move the charge against electric field,

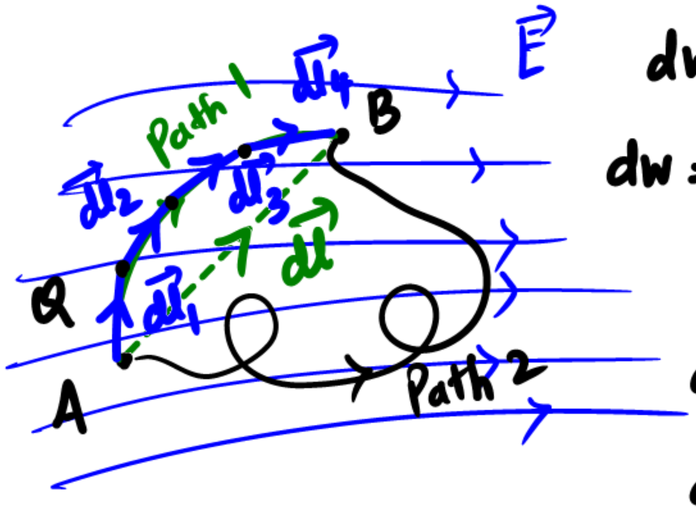
$$\vec{F}_{applied} = -Q\vec{E}$$

Work done $dW = \vec{F}_{applied} \cdot \vec{dl}$

$$dW = -Q\vec{E} \cdot \vec{dl}$$

$$\boxed{W = -Q\int_{initial}^{Final} \vec{E} \cdot \vec{dl} \quad J}$$

The line integral :



$$dW = -Q\vec{E}\cdot\vec{dl}$$

$$dW = -Q\,\vec{E}\cdot\vec{dl}_1 - Q\vec{E}\cdot\vec{dl}_2 - Q\vec{E}\cdot\vec{dl}_3 - Q\vec{E}\cdot\vec{dl}_4$$

$$dW = -Q\,\vec{E}\cdot(\vec{dl}_1+\vec{dl}_2+\vec{dl}_3+\vec{dl}_4)$$

$$dW = -Q\vec{E}\cdot\vec{dl}$$

$$\boxed{W = -Q\int_A^B \vec{E}\cdot\vec{dl}} \quad J$$

*) Work done remains same irrespective of path chosen in moving the charge from A to B

*) Work done around a closed path is zero

$$= -Q\oint_L \vec{E}\cdot\vec{dl} = 0$$

*) In general, vectors whose line integral does not depend on the path of integration are called conservative. Thus, E is conservative field

Problem:

① Let $V = \frac{\cos 2\phi}{\rho}$ in free space. find the volume charge density at point $A(\rho = 0.5, \phi = 60^\circ, z = 1)$

3)b)

Ans: $-106\ pC/m^3$

$\vec{E} = -\nabla V$

$\vec{D} = \varepsilon \vec{E}$

$\frac{1}{4\pi\varepsilon_0} = 9\times10^9$

$\nabla \cdot \vec{D} = \rho_v$

$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$

$$\rho_v = -\varepsilon_0 \left[\nabla^2 V\right]$$

$$= -\varepsilon_0 \left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}\right]$$

$$= -\frac{10^{-9}}{36\pi}\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\left(\frac{\cos 2\phi}{\rho}\right)\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2}\left(\frac{\cos 2\phi}{\rho}\right) + 0\right]$$

$$= -\frac{10^{-9}}{36\pi}\left[\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \cos 2\phi \left(\frac{-1}{\rho^2}\right)\right) + \frac{1}{\rho^3}\left(\frac{\partial}{\partial \phi}\left[-\sin 2\phi\right] 2\right)\right]$$

$$= -\frac{10^{-9}}{36\pi}\left[\frac{1}{\rho} \cdot \cos 2\phi \left(\frac{1}{\rho^2}\right) - \frac{4}{\rho^3}\cos 2\phi\right]$$

$$= -\frac{10^{-9}}{36\pi} \cdot \frac{1}{\rho^3}\left[\cos 2\phi - 4\cos 2\phi\right]$$

$$= \frac{3\cos 2\phi}{\rho^3} \times \frac{10^{-9}}{36\pi} \quad \left(\begin{array}{l}\rho = 0.5, \\ \phi = 60^\circ\end{array}\right)$$

$$= \frac{3 \times (-0.5)}{(0.5)^3 \cdot 2} \times \frac{10^{-9}}{36\pi}$$

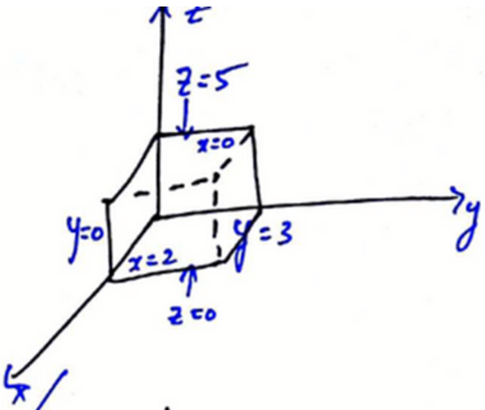
$$\boxed{\rho_v = -106.157\ p\ C/m^3}$$

4)

$$\vec{D} = 4xy\,\vec{a_x} + 2(x^2+z^2)\,\vec{a_y} + 4yz\,\vec{a_z} \quad C/m^2$$

Divergence theorem: $\oint_S \vec{D}\cdot\vec{ds} = \iiint_V \vec{\nabla}\cdot\vec{D}\,dv$

LHS $= Q_{enc} = \oint_S \vec{D}\cdot\vec{ds}$



$$Q_{enc} = \int_{z=0}^{5}\int_{y=0}^{3} \cancel{-4xy}^{\,0}\,dy\,dz\Big|_{x=0} + \int_{z=0}^{5}\int_{y=0}^{3} 4xy\,dy\,dz\Big|_{x=2}$$

$$+ \int_{z=0}^{5}\int_{x=0}^{2} \cancel{-\left(2(x^2+z^2)\right)}\,dx\,dz\Big|_{y=0} + \int_{z=0}^{5}\int_{x=0}^{2} \cancel{2(x^2+z^2)}\,dx\,dz\Big|_{y=3}$$

$$+ \int_{y=0}^{3}\int_{x=0}^{2} \cancel{-4yz}^{\,0}\,dx\,dy\Big|_{z=0} + \int_{y=0}^{3}\int_{x=0}^{2} 4yz\,dx\,dy\Big|_{z=5}$$

$$Q_{enc} = \int_{z=0}^{5}\int_{y=0}^{3} 4\times 2y\,dy\,dz + \int_{y=0}^{3}\int_{x=0}^{2} 4\times 5y\,dx\,dy$$

$$= 8\left[\frac{y^2}{2}\right]_0^3 [z]_0^5 + 20\left[\frac{y^2}{2}\right]_0^3 [x]_0^2$$

$$= 4\times 9\times 5 + 10\times 9\times 2$$

$$\boxed{Q_{enc} = 360} \quad C$$

$$RHS = \iiint_V \vec{\nabla}\cdot\vec{D}\, dV$$

$$\vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}(4xy) + \frac{\partial}{\partial y}(2(x^2+z^2)) + \frac{\partial}{\partial z}(4yz)$$

$$= 4y + 0 + 4y$$

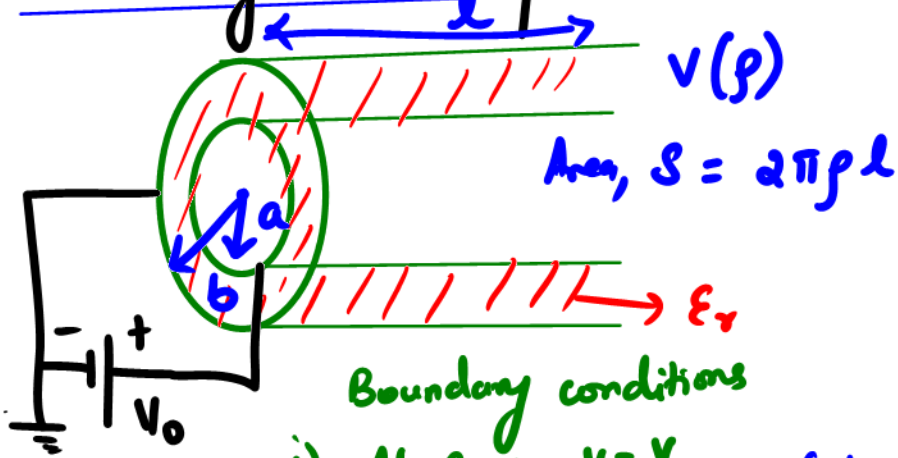
$$\boxed{\vec{\nabla}\cdot\vec{D} = 8y \ C/m^3}$$

$$RHS = \int_{z=0}^{5}\int_{y=0}^{3}\int_{x=0}^{2} 8y \, dx\,dy\,dz = 8\left[\frac{y^2}{2}\right]_0^3 [x]_0^2 [z]_0^5$$

$$= 8 \times \frac{9}{2} \times 2 \times 5$$

$$\boxed{\iiint_V \vec{\nabla}\cdot\vec{D}\, dV = 360\,C}$$

5)a) Coaxial cylindrical capacitor:



Boundary conditions

i) At $\rho = a$, $V = V_0$ $\quad (b > a)$

ii) At $\rho = b$, $V = 0$

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \cancelto{0}{\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2}} + \cancelto{0}{\frac{\partial^2 V}{\partial z^2}} = 0}$$

Laplace's eqn.

$V(\rho)$

$$\nabla^2 V = \frac{1}{\rho}\frac{d}{d\rho}\left(\rho \frac{dV}{d\rho}\right) = 0$$

$$\frac{d}{d\rho}\left(\rho \frac{dV}{d\rho}\right) = 0$$

Integrating w.r.t. ρ $\Rightarrow$ $\displaystyle\int \frac{d}{d\rho}\left(\rho \frac{dV}{d\rho}\right) d\rho = \int 0 \, d\rho$

$$\rho \frac{dV}{d\rho} = 0 + C_1$$

$$\frac{dV}{d\rho} = \frac{C_1}{\rho}$$

Integrating w.r.t. ρ $\Rightarrow$ $\displaystyle\int \frac{dV}{d\rho} d\rho = \int \frac{C_1}{\rho} d\rho$

$$\boxed{V = C_1 \ln \rho + C_2}$$

i) At $\rho = a$, $V = V_0$ $\Rightarrow$ $\boxed{V_0 = c_1 \ln a + c_2}$ $(b > a)$

ii) At $\rho = b$, $V = 0$ $\Rightarrow$ $\boxed{0 = c_1 \ln b + c_2}$

$$V_0 - 0 = c_1 \ln a - c_1 \ln b$$

$$-V_0 = c_1 \ln(b/a)$$

$$c_2 = -c_1 \ln b$$

$$\boxed{c_2 = \frac{V_0 \ln b}{\ln(b/a)}}$$

$$\boxed{c_1 = \frac{-V_0}{\ln(b/a)}}$$

1) $$\boxed{V(\rho) = \frac{-V_0}{\ln(b/a)} \ln\rho + \frac{V_0}{\ln(b/a)} \ln b} \quad V$$

2) $$\vec{E} = -\vec{\nabla} V = -\left[\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\cancelto{0}{\frac{\partial V}{\partial \phi}}\hat{a}_\phi + \cancelto{0}{\frac{\partial V}{\partial z}}\hat{a}_z\right]$$

$$= -\left[\frac{-V_0}{\ln(b/a)}\frac{1}{\rho}\hat{a}_\rho\right]$$

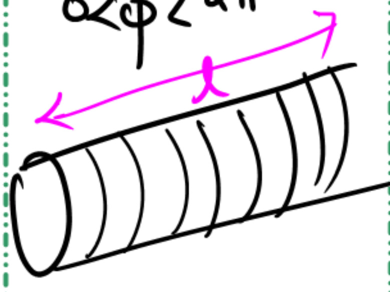
$$\boxed{\vec{E} = \frac{V_0}{\rho \ln(b/a)}\hat{a}_\rho} \quad V/m$$

3) $$\boxed{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \frac{\varepsilon_0 \varepsilon_r V_0}{\rho \ln(b/a)}\hat{a}_\rho} \quad C/m^2$$

4) $$Q = \oiint_s \vec{D}\cdot\vec{ds} = \iint \frac{\varepsilon_0 \varepsilon_r V_0}{\rho \ln(b/a)}\hat{a}_\rho \cdot \vec{ds}$$

$0 < z < \ell$

$0 < \phi < 2\pi$



$$\iint_s ds = \iint \rho\, d\phi\, dz = 2\pi\rho\ell$$

$$Q = \frac{\varepsilon_0 \varepsilon_r V_0}{\rho \ln(b/a)} \cdot 2\pi\rho l$$

$$\boxed{Q = \frac{2\pi \varepsilon_0 \varepsilon_r l V_0}{\ln(b/a)}} \quad C$$

$V_d = V_0 - 0 = V_0$

5) $C = \frac{|Q|}{V_d} = \frac{2\pi \varepsilon_0 \varepsilon_r l V_0}{\ln(b/a)} \Big/ V_0 = \frac{2\pi \varepsilon_0 \varepsilon_r l}{\ln(b/a)}$

$$\boxed{C = \frac{2\pi \varepsilon_0 \varepsilon_r l}{\ln(b/a)}} \quad F$$

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5) b) $V = 2x^2 y - 5z$

Laplace's equation : $\nabla^2 V = 0$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{\partial}{\partial x}(4xy) + \frac{\partial}{\partial y}(2x^2) + \frac{\partial}{\partial z}(-5)$$

$$= 4y + 0 + 0$$

$\nabla^2 V = 4y \neq 0$

Therefore, the given potential function V doesn't satisfy Laplace's equation.

6)a) State and Prove Uniqueness theorem:

UNIQUENESS THEOREM

This is the **uniqueness theorem:** If a solution to Laplace's equation can be found that satisfies the boundary conditions, then the solution is unique.

The theorem applies to any solution of Poisson's or Laplace's equation in a given region or closed surface.

Proof :

The theorem is proved by contradiction.
We assume that there are two solutions V1 and V2 of Laplace's equation both of which satisfy the prescribed boundary conditions.

case (i) Poisson's Eqn

$$\nabla^2 V_1 = -\frac{\rho_v}{\varepsilon} \rightarrow (1)$$

$$\nabla^2 V_2 = -\frac{\rho_v}{\varepsilon} \rightarrow (2)$$

$$(1)-(2) \Rightarrow \boxed{\nabla^2 V_1 - \nabla^2 V_2 = 0}$$

case (ii) Laplace's Equation

$$\nabla^2 V_1 = 0 \rightarrow (3)$$

$$\nabla^2 V_2 = 0 \rightarrow (4)$$

$$(3)-(4) \Rightarrow \boxed{\nabla^2 V_1 - \nabla^2 V_2 = 0}$$

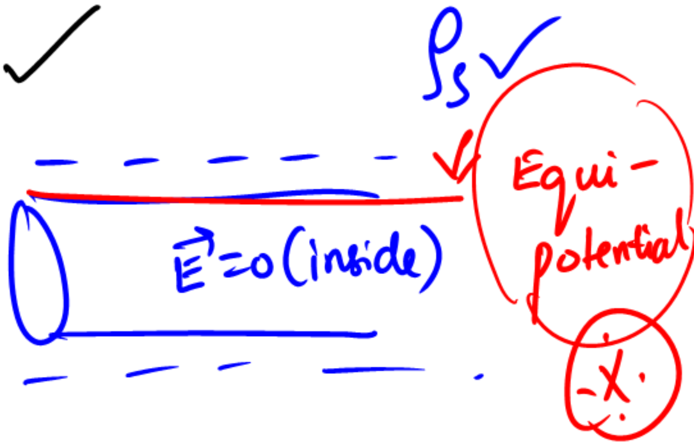
$V_1(x,y,z)$ $V_2(x,y,z)$

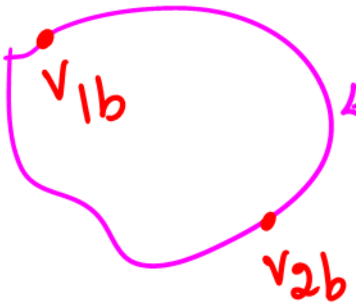
$$V_1 - V_2 = V_d(x,y,z)$$

$$\nabla^2 (V_1 - V_2) = 0$$

$$\boxed{\nabla^2 V_d = 0} \rightarrow (5)$$

Boundary condition :





closed surface ↓ Equipotential → $V_{1b} = V_{2b}$ → (6)

$$\boxed{V_1 - V_2 = V_{db} = 0}$$

Vector Identity:

$\alpha^{(x,y,z)} \rightarrow$ scalar

$\vec{A}^{(x,y,z)} \rightarrow$ vector

$$\boxed{\vec{\nabla}\cdot(\alpha\vec{A}) = \alpha\,\vec{\nabla}\cdot\vec{A} + \vec{A}\cdot\vec{\nabla}\alpha}$$

In our case,

let $\alpha = V_d$

$\vec{A} = \vec{\nabla} V_d$

$$\vec{\nabla}\cdot(V_d\,\vec{\nabla} V_d) = V_d\,\vec{\nabla}\cdot\vec{\nabla} V_d + \vec{\nabla} V_d\cdot\vec{\nabla} V_d$$

$\vec{A}\cdot\vec{A} = |\vec{A}|^2$

$$\boxed{\vec{\nabla}\cdot(V_d\,\vec{\nabla} V_d) = V_d\,\nabla^2 V_d + |\vec{\nabla} V_d|^2}$$

Integrating over volume V enclosed by closed surface

$$\boxed{\iiint_V \vec{\nabla}\cdot(V_d\,\vec{\nabla} V_d)\,dv = \iiint_V V_d\,\nabla^2 V_d\,dv + \iiint_V |\vec{\nabla} V_d|^2\,dv}$$

Boundary ≡ surface

surface Integral = ?

Using Divergence theorem

$$\oint_S \vec{D}\cdot d\vec{s} = \iiint_V \vec{\nabla}\cdot\vec{D}\,dv$$

$$\boxed{\oint_S \cancel{V_d}^{\,0}\,\vec{\nabla} V_d\cdot d\vec{s} = \iiint_V V_d\,\cancel{\nabla^2 V_d}^{\,0}\,dv + \iiint_V |\vec{\nabla} V_d|^2\,dv}$$

→ (7)

Apply ⑤ & ⑥ in ⑦

$\vec{\nabla} V_d = 0$ $\quad$ $V_{db} = 0$

$$0 = \iiint_V |\vec{\nabla} V_d|^2 \, dV \Rightarrow |\vec{\nabla} V_d|^2 = 0$$

$$\boxed{\vec{\nabla} V_d = 0}$$

If V_d = constant

$\downarrow$

Equipotential surface

$V_d = 0$

$V_1 - V_2 = 0$ $\quad$ $\boxed{V_1 = V_2}$

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6 b) $V = 3x^2 + 2y^2 + 2z^3$

$P(-4, 5, 4)$

$$V \text{ at } P = 3(-4)^2 + 2(5)^2 + 2(4)^3$$

$$\boxed{V \text{ at } P = 226 \text{ V}}$$

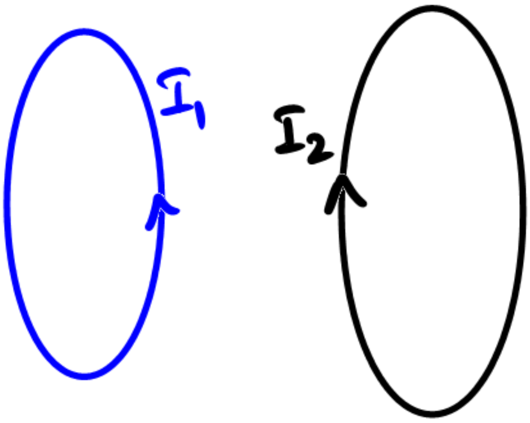
$$\vec{E} = -\vec{\nabla} V = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$\vec{E} = -\left[6x\,\hat{a}_x + 4y\,\hat{a}_y + 6z^2\,\hat{a}_z\right] \text{ V/m}$$

$$\boxed{\vec{E} \text{ at } P = 24\hat{a}_x - 20\hat{a}_y - 96\hat{a}_z \text{ V/m}}$$

7)a)

Force between differential current elements:



Magnetic Flux density due to differential current element:

(Biot - Savart law)

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \hat{a}_R}{|\vec{R}|^3}$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|}$$

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \vec{R}}{|\vec{R}|^3}$$

Differential amount of force due to differential current element:

$$d(\vec{dF}) = I\vec{dl} \times \vec{dB}$$

Magnetic flux density due to current element 1

$$\vec{dB_1} = \frac{\mu_0}{4\pi} \frac{I_1\vec{dl_1} \times \vec{R_1}}{|\vec{R_1}|^3}$$

Magnetic flux density due to current element 2

$$\vec{dB_2} = \frac{\mu_0}{4\pi} \frac{I_2\vec{dl_2} \times \vec{R_2}}{|\vec{R_2}|^3}$$

Differential amount of differential force on current element 1,

$$d(\vec{dF_1}) = I_1\vec{dl_1} \times \vec{dB_2}$$

$$d(d\vec{F_1}) = I_1\vec{dl_1} \times \frac{\mu_0}{4\pi} \frac{I_2\vec{dl_2} \times \vec{R_2}}{|\vec{R_2}|^3}$$

Total force on current element 1, $\vec{F_1} = \oint_{L_1} I_1\vec{dl_1} \times \frac{\mu_0}{4\pi} \oint_{L_2} \frac{I_2\vec{dl_2} \times \vec{R_2}}{|\vec{R_2}|^3}$

Differential amount of differential force on current element 2,

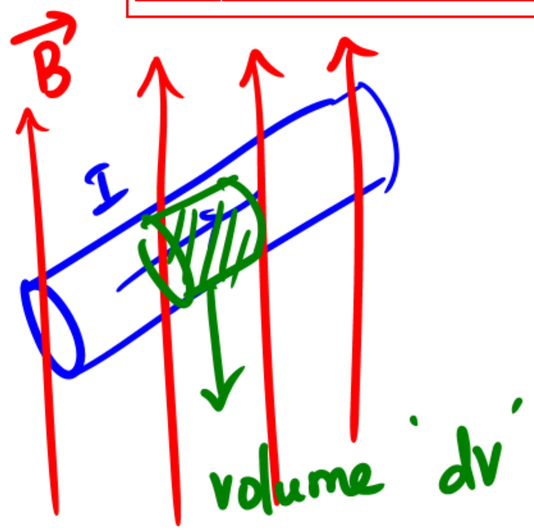
$$d(d\vec{F_2}) = I_2\vec{dl_2} \times d\vec{B_1}$$

$$d(d\vec{F_2}) = I_2\vec{dl_2} \times \frac{\mu_0}{4\pi} \frac{I_1\vec{dl_1} \times \vec{R_1}}{|\vec{R_1}|^3}$$

Total force on current element 2, $\vec{F_2} = \oint_{L_2} I_2\vec{dl_2} \times \frac{\mu_0}{4\pi} \oint_{L_1} \frac{I_1\vec{dl_1} \times \vec{R_1}}{|\vec{R_1}|^3}$

7)b)

Force on a differential current element



w.k.t. $\vec{F_m} = Q(\vec{v} \times \vec{B})$

$dQ = \rho_v \, dv$

Differential force, $d\vec{F} = dQ(\vec{v} \times \vec{B})$

$d\vec{F} = \rho_v \, dv \, (\vec{v} \times \vec{B})$

w.k.t. $\vec{J} = \rho_v \vec{v}$

Differential force on volume current $\boxed{d\vec{F} = \vec{J} \, dv \times \vec{B}}$

Total force, $\boxed{\vec{F} = \iiint_V \vec{J} \, dv \times \vec{B}}$

$\vec{J} \, dv \equiv \vec{K} \, ds \equiv I \, d\vec{l}$

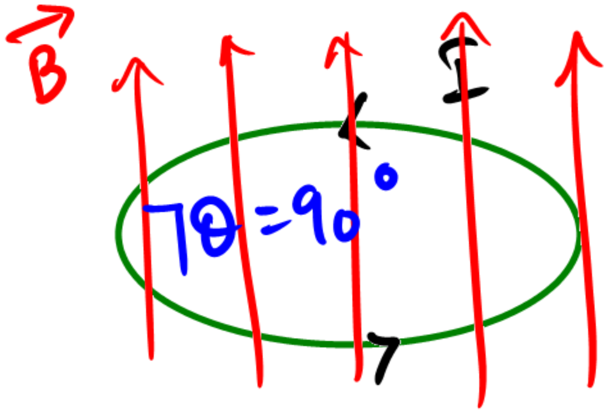
Differential force on sheet current, $\boxed{d\vec{F} = \vec{K} \, ds \times \vec{B}}$

Total force, $\boxed{\vec{F} = \iint_S \vec{K} \, ds \times \vec{B}}$

Differential force on line current, $\boxed{d\vec{F} = I \, d\vec{l} \times \vec{B}}$

Total force, $\boxed{\vec{F} = \oint_L I \, d\vec{l} \times \vec{B}}$

Force on a conductor;



l - length of the conductor

$$\vec{F} = \oint_L I\, \underbrace{\vec{dl} \times \vec{B}}_{??}$$

$$|\vec{dl} \times \vec{B}| = |\vec{dl}|\,|\vec{B}| \sin\theta$$
$$= dl\, B \sin\theta$$

Magnitude of force

$$F = |\vec{F}| = \oint_L I\, dl\, B \sin\theta$$

$$= I B \sin\theta \oint_L dl$$

Magnitude of force on a conductor

$$\boxed{F = I B l \sin\theta}$$