Digital communication:
IA - 02

① In a linear block code the syndrome is given by
$s_1 = r_1 + r_2 + r_3 + r_5 \quad s_2 = r_1 + r_2 + r_4 + r_6 \quad s_3 = r_1 + r_3 + r_4 + r_7$
ⓐ Generate all the possible code vectors.
ⓑ A single bit error has occured in 1011011 detect & correct this error.

→ $n = 7$
$n - K = 3$
$K = 7 - 3 = 4$.

$$H = [P^T \; I] = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$G = [I_4 | P]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \qquad H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G = [I_4 | P]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The code vector are written using.
$C = DG$

$$C_1 = [0000] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [0000000]$$

$$C_2 = [0001] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [0001011]$$

$$C_3 = [0010]\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix} = [0010001]$$

$$C_4 = [0011]\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix} = [0011110]$$

$C_5 = [0100] \Rightarrow [0100110]$

$C_6 = [0101] \Rightarrow [0101101]$

$C_7 = [0110] \Rightarrow [0110011]$

$C_8 = [0111] \Rightarrow [0111000]$

$$C_9 = [1000]\begin{bmatrix}1&0&0&0&1&1&1\\0&1&0&0&1&1&0\\0&0&1&0&1&0&1\\0&0&0&1&0&1&1\end{bmatrix} = [1000111]$$

$C_{10} = [1001] \quad [1001100]$

$C_{11} = [1010] = [1010010]$

$C_{12} = [1011] = [1011001]$

$C_{13} = [1100] = [1100001]$

$C_{14} = [1101] = [1101010]$

$C_{15} = [1110] = [1110100]$

$C_{16} = [1111] = [1111111]$

5) $S = RH^T$

$$= [1011011]\begin{bmatrix}1&1&1\\1&1&0\\1&0&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix} = [0\ 1\ 0]$$

This is present in the 6^{th} row of H^T

$v_3(x) = 1 + x + x^3 + x^5 + x^6$

$v_1 = 1\ 1\ 1\ 1\ 1\ 1\ 0$

$v_2 = 1\ 0\ 0\ 0\ 0\ 0\ 1$

$v_3 = 1\ 1\ 0\ 1\ 0\ 1\ 1$

$C = 111\ \ 101\ \ 100\ \ 101\ \ 100\ \ 101\ \ 011$

(II) (a) Derive the expression for error probability of binary BPSK using coherent detection.

→ In binary phase shift keying, bit 1 & bit 0 are represented by the following symbols.

bit 1

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \quad 0 \le t \le T_b \quad f_c = \frac{n}{T_b}$$

$n \to$ non zero integer

$T_b \to$ bit duration.

bit 0

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi), \quad 0 \le t \le T_b \quad f_c = \frac{n}{T_b}$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

To find the basis function

$$\text{Energy of } s_1(t) = \int_0^{T_b} |s_1(t)|^2 \, dt$$

$$= \int_0^{T_b} \frac{2E_b}{T_b} \cos^2(2\pi f_c t) \, dt$$

$$= \frac{2E_b}{T_b} \int_0^{T_b} \frac{1 + \cos 4\pi f_c t}{2} \, dt$$

$$= \frac{E_b}{T_b} \int_0^{T_b} 1 \Rightarrow E_b$$

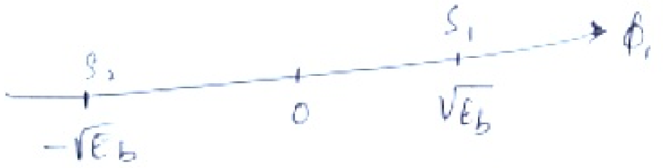
basis function $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_b}}$

$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \quad 0 \le t \le T_b$$

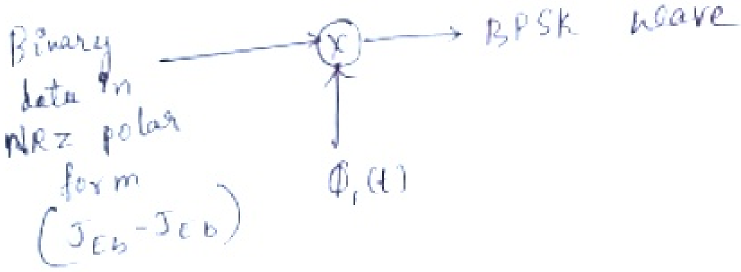
$$\therefore S_1(t) = \sqrt{E_b}\,\phi_1(t)$$

$$S_2(t) = -\sqrt{E_b}\,\phi_1(t)$$

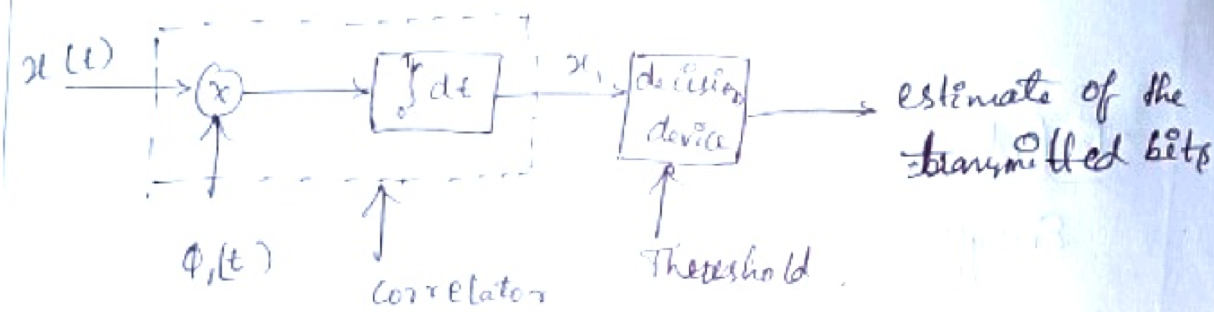
<u>Signal Space diagram</u>



<u>Block diagram of transmitter</u>



<u>Block diagram of receiver</u>



Let $x(t)$, $0 \le t \le T_b$ be the received signal.

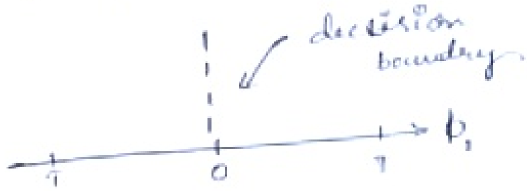
$$x(t) = S_i(t) + w(t) \longrightarrow (1) \quad , 0 \le t \le T_b$$

where $w(t)$ represents additive white Gaussian noise with zero mean & PSD $\frac{N_0}{2}$ i.e variance $\frac{N_0}{2}$

<u>Decision logic</u>

Let x_1 be the o/p of the correlator.
If $x_1 > 0$ decide in favour of bit 1

$x_1 < 0$, decide in favour of bit 0



Probability of error

Suppose that bit 0 was transmitted i.e $s_1(t)$ was transmitted

Then from the block diagram of the receiver may write o/p of the correlator.

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \left[s_2(t) + w(t)\right]\phi_1(t)\,dt$$

$$= \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt + \int_0^{T_b} w(t)\,\phi_1(t)\,dt$$

$$= -\sqrt{E_b} + w_1 \longrightarrow (2)$$

coordinate of $s_2(t)$

Mean of x_1 when 0 was transmitted.

$$\mu = E[x_1] \Rightarrow -\sqrt{E_b} \longrightarrow (3)$$

Variance of x_1 when 0 was transmitted.

$$\sigma^2 = VAR[w_1]$$

$$\sigma^2 = \frac{N_0}{2} \longrightarrow (4)$$

$\therefore$ probability density function (PDF) of output of correlator when bit 0 was transmitted.

$$f_{x_1}(x/0) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \longrightarrow$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1 + \sqrt{E_b})^2}{N_0}} \longrightarrow (5)$$

wrong decision is made where $s_2(t)$ was th... ...tted and $x_1 > 0$

$\therefore$ probability of error when bit 0 was transmitted

$$P_e(0) = P(x_1 > 0/0)$$

$$= \int_0^\infty f_{x_1}(x_1/0)\, dx_1$$

$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1 + \sqrt{E_b})^2}{N_0}} dx_1 \longrightarrow (6)$$

we know that

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{z^2}{2}} dz \longrightarrow (7)$$

Let us represent $P_e(0)$ in terms of Q function.

Put $\frac{(x_1 + \sqrt{E_b})^2}{N_0} = \frac{z^2}{2} \longrightarrow (8)$

i.e. $\frac{x_1 + \sqrt{E_b}}{\sqrt{N_0}} = \frac{z}{\sqrt{2}}$

$$\frac{dx_1}{\sqrt{N_0}} = \frac{dz}{\sqrt{2}}$$

$$\therefore dx_1 = \sqrt{\frac{N_0}{2}}\, dz \longrightarrow (9)$$

when $x_1 = 0$, $z = \sqrt{\frac{2E_b}{N_0}} \longrightarrow (10)$

when $x_1 = \infty$, $z = \infty \longrightarrow (11)$

using (8) (9) (10) (11) we may write (6) as

$$P_e(0) = \int_{\sqrt{\frac{2E_b}{N_0}}}^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{z^2}{2}} \sqrt{\frac{N_0}{2}}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} e^{-\frac{z^2}{2}} dz$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \longrightarrow (12)$$

Similarly we may prove that probability of error when bit 1 was transmitted

$$P_e(1) = P\left(\sqrt{\frac{2E_b}{N_0}}\right) \longrightarrow (13)$$

∴ Average probability of error

$$= \frac{1}{2} P_e(0) + \frac{1}{2} P_e(1)$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \longrightarrow (14)$$

5 (a) With a neat block diagram. Explain the generation and detection of BFSK signal.

→ In binary frequency shift keying (FSK) bit 1 & bit 0 are represented by the following symbols. bit 1

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \quad 0 \le t \le T \quad f_1 = \frac{n}{T_b}$$

$n \rightarrow$ non zero integer

$T_b \rightarrow$ bit duration

Bit 0

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_0 t \quad 0 \le t \le T_b$$

To find basic function.

$$\int_0^{T_b} s_1(t)\, s_2(t)\, dt = 0$$

⟹ $s_1(t)$ & $s_2(t)$ are orthogonal to each other Basic function $\phi_1(t) = \frac{s_1(t)}{\sqrt{E_b}}$

$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t) \quad , \quad 0 \le t \le T_b$$

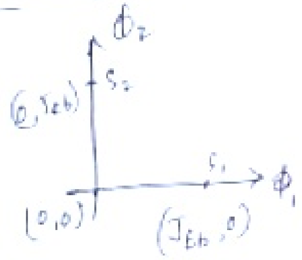
Basis function $\phi_2(t) = \frac{S_1(t)}{\sqrt{E_b}}$

$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

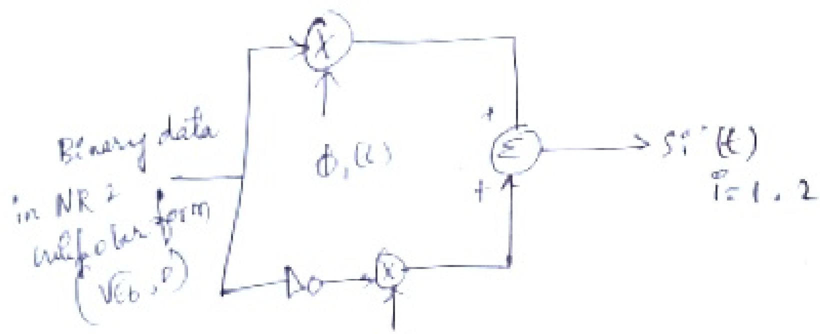
$$\therefore S_1(t) = \sqrt{E_b}\,\phi_1(t) + 0\,\phi_2(t) \ , \ 0 \le t \le T_b \ .$$

$$S_1(t) = 0\,\phi_1(t) + \sqrt{E_b}\,\phi_2(t) \ , \ 0 \le t \le T_b$$

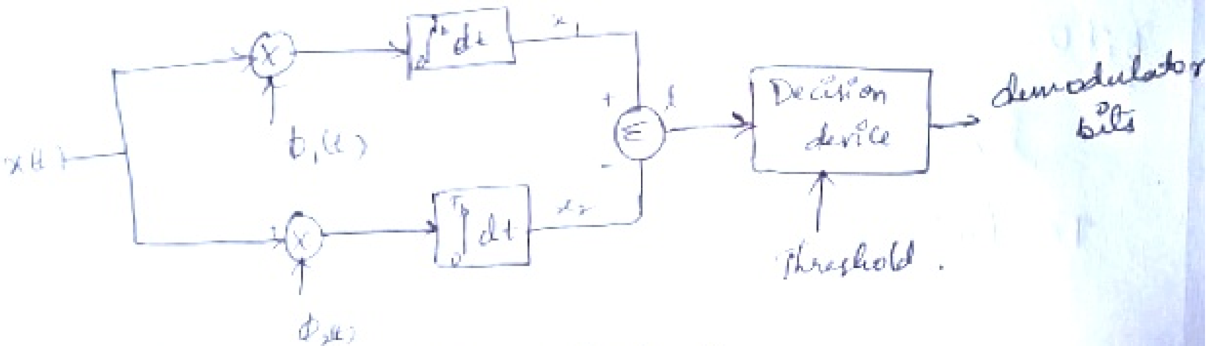
Constellation diagram



Block diagram of transmitter



Block diagram of receiver



Let $x(t)$ be the received sig

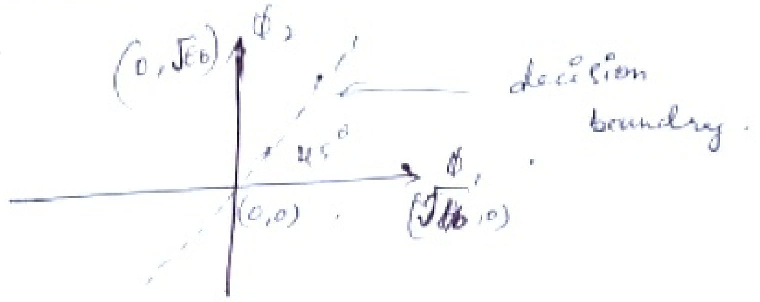
$$x(t) = S_i(t) + w(t) \qquad 0 \le t \le T_b$$
$$i = 1, 2$$

where $n(t)$ represents additive white Gaussian noise with zero mean & PSD $\frac{N_0}{2}$ i,e variance $\frac{N_0}{2}$.

Decision logic

If $x_1 > x_2$ decide in favour of bit 1

If $x_1 \not< x_2$ decide in favour of bit 0



From the block diagram of receiver

$$l = x_1 - x_2$$

Hence we may state decision rule as follows

If $l > 0$ decide in favour of 1

If $l < 0$ decide in favour of 0

(b) In an FSK system the transmitted binary data is 2.5×10^6 bps PSD of zero mean AWGN is 10^{-20} w/Hz the amplitude of the received sig is $1\mu v$. find the probability of error using coherent detection Given $(2.5) = 0.99959$.

Given $R_b = 2.5 \times 10^6$ bps

$$T_b = \frac{1}{2.5 \times 10^6}$$

$$= 4 \times 10^{-7}\ Sec = 0.4 \times 10^{-6}\ Sec.$$

$$\frac{N_0}{2} = 10^{-20}\ W/Hz$$

Amp $= 1\mu V$

$$\sqrt{\frac{2E_b}{T_b}} = 1\times10^{-6}$$

$$\frac{2E_b}{T_b} = 10^{-12}$$

$$E_b = 0.4\times10^{-6}\times10^{-12}$$

$$= 0.2\times10^{-18}\ J$$

Probability of error for coherent detection.

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$= Q\left(\sqrt{\frac{0.2\times10^{-18}}{2\times10^{-20}}}\right)$$

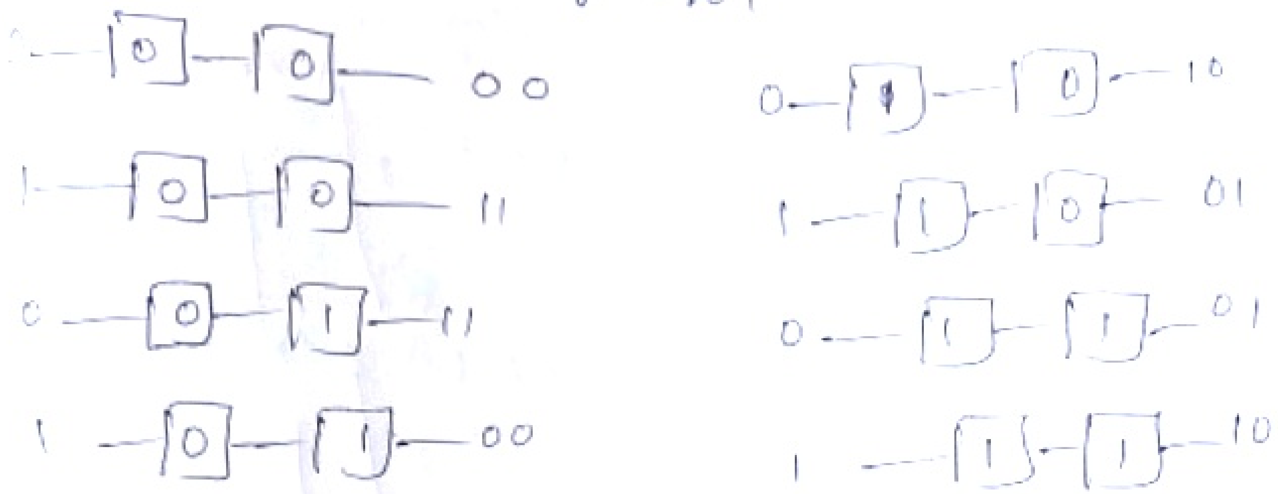
$$= Q(3.1622) \qquad \text{(from Q function table)}$$

$$= 2.077\times10^{-4} \qquad Q = 6.57\times10^{-5}$$

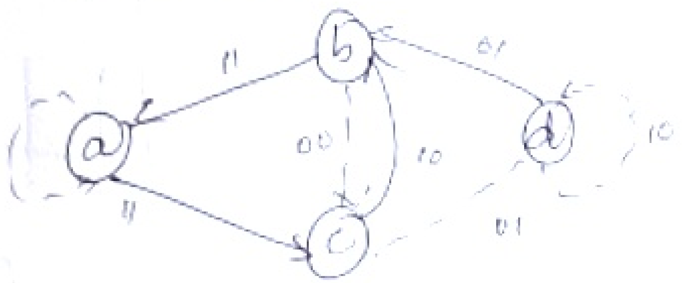
(3) For $r = 1/2$ and $K = 3$ convolutional encoder, $g^1 = 111$ & $g^2 = 101$ Encoder the i/p message sequence 110110 using a neat Trellis diagram.

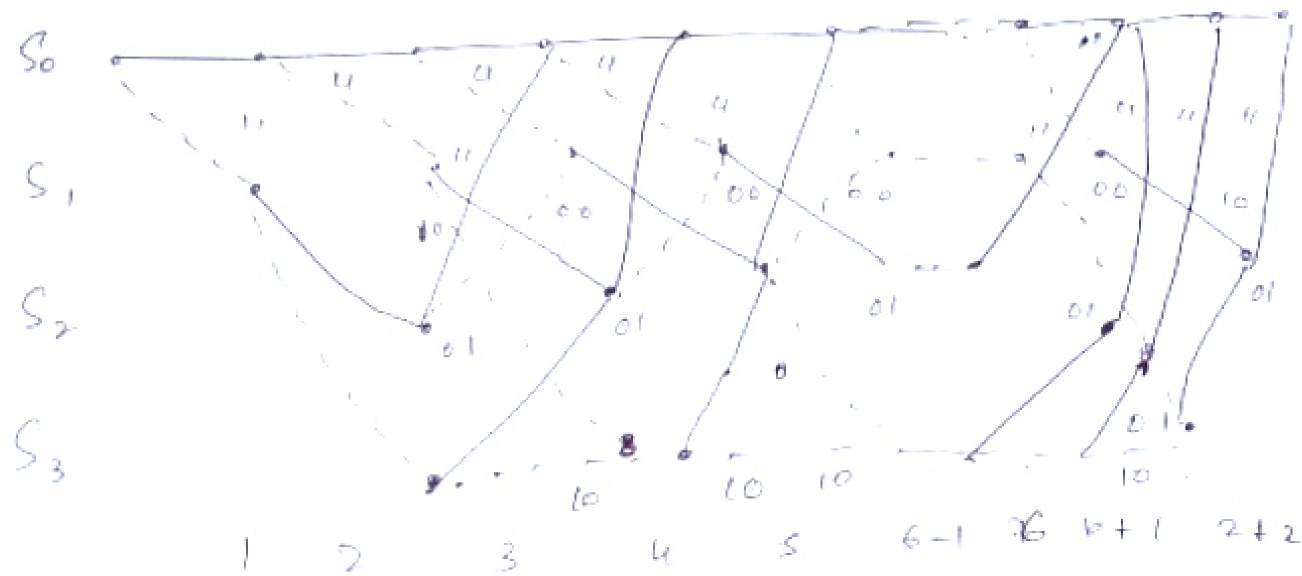
present State	Binary description	i/p	binary description next state	next State	d_i	d_{i-1}	d_{i-2}	o/p c_i c
a	00	0	00	a	0	0	0	0 0
		1	10	b	1	0	0	1 1
b	10	0	01	c	D	1	0	1 0
		1	11	d	1	1	0	0 1
c	01	0	00	a	0	0	1	1
		1	10	b	1	0	0	1 00
d	11	0	01	c	0	1	1	0 1
		1	11	d	1	1	1	1 0

Given $g^1 = 111$ $g^2 = 101$



$x = 0$ — Solid line $x = 1$ → Dotted line





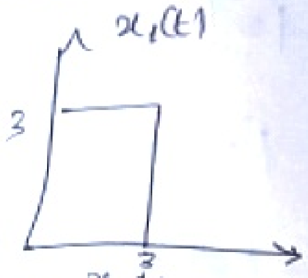
⑥ obtain a set of orthogonal basis function for the following set of signal.

$x_1(t) = \begin{cases} 3 & \text{from } 0 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$ & $x_2(t) = \begin{cases} 3 & \text{from } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

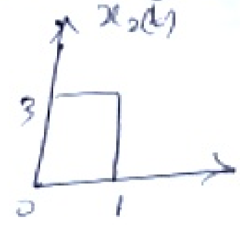
$x_3(t) = \begin{cases} 3 & \text{from } 1 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$ -7-

Express the sig. as a linear combination of basic function draw the sig space diagram.

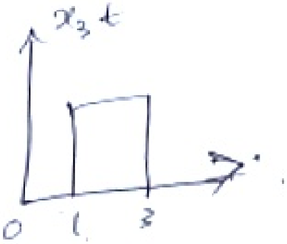
$\rightarrow x_1(t) = \begin{cases} 3 & \text{from } 0 \le t \le 3 \\ 0 \end{cases}$



$x_2(t) = \begin{cases} 3 & \text{from } 0 \le t \le 1 \\ 0 \end{cases}$



$x_3(t) = \begin{cases} 3 & \text{from } 1 \le t \le 3 \\ 0 \end{cases}$



$x_1(t) = x_2(t) + x_3(t)$

∴ Signals are linearly independent

Consider $x_2(t)$ & $x_3(t)$ are linearly independent

then, there must be 2 basis function further

$$\int_0^3 x_2(t)\, x_3(t)\, dt = 0$$

∴ $x_2(t)$ & $x_3(t)$ are orthogonal.

<u>Step 1</u> :- Energy of $x_1(t)$

$$E_2 = \int_0^1 3^2\, dt$$

$$= 9(t)_0^1$$

$$= 9$$

<u>Step 2</u> :- Basis function of $\phi_1(t) = \frac{x_2(t)}{\sqrt{9}} = \frac{x_2(t)}{3}$

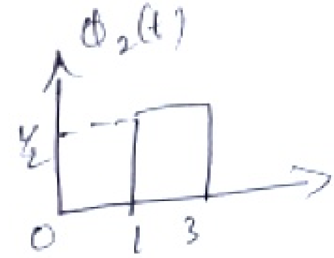
<u>Step 3</u> :- Energy of $x_3(t)$ $E_3 = \int_1^3 3^2\, dt$

$$= 9(t)_1^3$$

$$= 9 \times 2 = 18$$

∴ Basis function $\phi_2(t) = \frac{x_2(t)}{\sqrt{18}} = \frac{3(t)}{3\sqrt{2}}$

Expressing s/g as linear combination of basis function



$x_1(t) = 3\phi_1(t) + 3\sqrt{2}\,\phi_2(t)$

$x_2(t) = 3\,\phi_1(t) + 0\,\phi_2(t)$

$x_3(t) = 0\,\phi_1(t) + 3\sqrt{2}\,\phi_2(t)$