USN										

CMRIT

Internal Assessment Test 3 – March 2024									
Sub:	**Network Analysis**					Sub Code:	**BEC304**	Branch:	**ECE**
Date:	**06/03/2024**	Duration:	90 Minutes	Max Marks:	50	Sem/Sec:	**3/A, B, C, D**		**OBE**
<u>Answer Any Five questions</u>							MARKS	CO	RBT

1	Determine the transmission parameters in the s- domain for the network shown in Fig 1.	[10]	CO5	L3

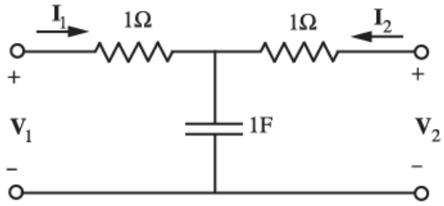
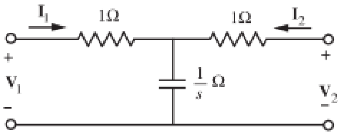


Fig 1

SOLUTION

The s domain equivalent circuit with the assumption that all the initial conditions are zero is shown in Fig. 7.47(a).



To find the parameters **A** and **C**, open-circuit the output port and connect a voltage source $\mathbf{V}_1$ at the input port. The same is shown in Fig. 7.47(b).

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{1+\frac{1}{s}} = \frac{s\mathbf{V}_1}{s+1}$$

Then $\mathbf{V}_2 = \frac{1}{s}\mathbf{I}_1$

$$\Rightarrow \quad \mathbf{V}_2 = \frac{1}{s}\frac{s\mathbf{V}_1}{s+1} = \frac{\mathbf{V}_1}{s+1}$$

$$\Rightarrow \quad \mathbf{A} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = s+1$$

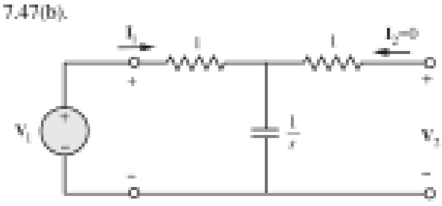


Figure 7.47(b)

Also, $\mathbf{V}_2 = \frac{1}{s}\mathbf{I}_1$

$$\Rightarrow \quad \mathbf{C} = \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = s$$

To find the parameters **B** and **D**, short-circuit the output port and connect a voltage source $\mathbf{V}_1$ to the input port as shown in Fig. 7.47(c).

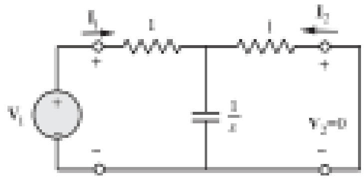


Figure 7.47(c)

The total impedance as seen by the source $\mathbf{V}_1$ is

$$\mathbf{Z} = 1 + \frac{\frac{1}{s}\times 1}{\frac{1}{s}+1}$$

$$= 1 + \frac{1}{s+1} = \frac{s+2}{s+1}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}} = \frac{\mathbf{V}_1(s+1)}{(s+2)} \qquad (7.44)$$

Using the principle of current division, we have

$$-\mathbf{I}_2 = \frac{\mathbf{I}_1\left(\frac{1}{s}\right)}{\frac{1}{s}+1} = \frac{\mathbf{I}_1}{s+1} \qquad (7.45)$$

Hence, $\mathbf{D} = \left.\frac{\mathbf{I}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = s+1$

From equation (7.44) and (7.45), we can write

$$-\mathbf{I}_2(s+1) = \frac{\mathbf{V}_1(s+1)}{(s+2)}$$

Hence, $\mathbf{B} = \left.\frac{-\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = s+2$

2	Determine the ABCD parameters for the two-port network shown in Fig 2.	[10]	CO5	L3

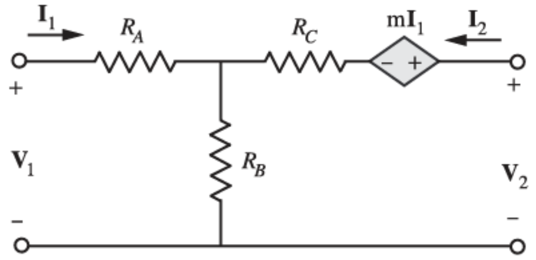


Fig 2

SOLUTION

To find the parameters **A** and **C**, open-circuit the output port as shown in Fig. 7.49(a) and connect a voltage source $\mathbf{V}_1$ to the input port.

Applying KVL to the output mesh, we get

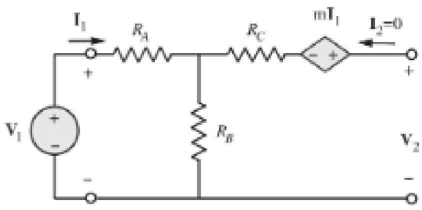


Figure 7.49(a)

$$-\mathbf{V}_2 + \mathrm{m}\mathbf{I}_1 + 0 \times R_C + \mathbf{I}_1 R_A = 0$$

$$\Rightarrow \quad \mathbf{V}_2 = \mathbf{I}_1 (\mathrm{m} + R_A)$$

Hence, $\mathbf{C} = \left.\dfrac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \dfrac{1}{\mathrm{m} + R_A}$

Applying KVL to the input mesh, we get

$$\mathbf{V}_1 = \mathbf{I}_1 (R_A + R_B)$$

Hence, $\mathbf{A} = \left.\dfrac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \dfrac{R_A + R_B}{\mathrm{m} + R_A}$

To find the parameters **B** and **D**, short-ciruit the output port and connect a voltage source $\mathbf{V}_1$ to the input port as shown in Fig. 7.49(b).

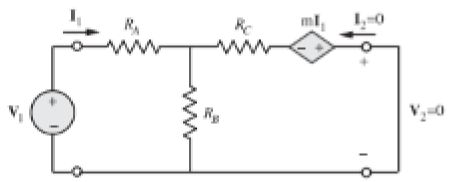


Figure 7.49(b)

Applying KVL to the right-mesh, we get

$$\mathrm{m}\mathbf{I}_1 + R_C \mathbf{I}_2 + R_B (\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$\Rightarrow \quad (\mathrm{m} + R_B)\mathbf{I}_1 = -(R_C + R_B)\mathbf{I}_2$$

$$\Rightarrow \quad \mathbf{I}_1 = \frac{-(R_C + R_B)}{(\mathrm{m} + R_B)}\mathbf{I}_2$$

Hence, $\mathbf{D} = \left.\dfrac{\mathbf{I}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = \dfrac{(R_C + R_B)}{(\mathrm{m} + R_B)}$

Applying KVL to the left-mesh, we get

$$-\mathbf{V}_1 + R_A \mathbf{I}_1 + R_B (\mathbf{I}_1 + \mathbf{I}_2) = 0$$

$$\Rightarrow \quad \mathbf{V}_1 = (R_A + R_B)\mathbf{I}_1 + R_B \mathbf{I}_2$$

$$= (R_A + R_B)\left[\frac{-(R_C + R_B)}{(\mathrm{m} + R_B)}\mathbf{I}_2\right] + R_B \mathbf{I}_2$$

$$= -\left[\frac{R_C R_A + R_C R_B + R_B R_A - \mathrm{m}R_B}{\mathrm{m} + R_B}\right]\mathbf{I}_2$$

Hence, $\mathbf{B} = \left.\dfrac{\mathbf{V}_1}{-\mathbf{I}_2}\right|_{\mathbf{V}_2=0}$

$$= \frac{R_C R_A + R_C R_B + R_B R_A - \mathrm{m}R_B}{\mathrm{m} + R_B}$$

3	Define Q factor, selectivity and band width. Prove that for a resonant circuit $f0=\sqrt{f1f2}$, **where** $f1$ and $f2$ are two half power frequencies.	[10]	CO5	L2

Q-factor (Quality Factor) (3)

- Q factor is also known as figure of merit.
- In resonance circuit there are two storage elements i.e inductor (L) and capacitor (C).

The efficiency at which these two energy storing elements store the energy is called quality factor.

Q factor is defined as

$$Q = 2\pi \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipated per cycle}}$$

Selectivity (7)

It is defined by the equation:

$$\text{selectivity} = \frac{f_0}{\text{Bandwidth}} = \frac{f_0}{f_2 - f_1}$$

At resonance

$$\text{selectivity} = \frac{f_0}{R/2\pi L} = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R}$$

$[\text{as } \omega_0 = 2\pi f_0]$

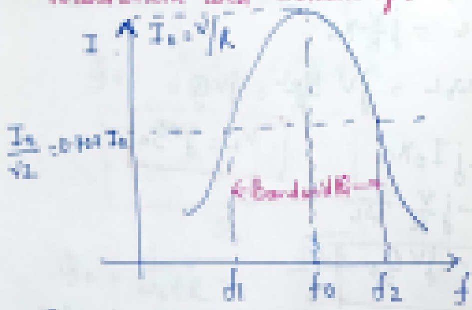
$$\therefore \text{Selectivity} = \frac{\omega_0 L}{R} = Q.$$

$$\therefore \boxed{\text{Selectivity} = \frac{f_0}{BW} = \frac{f_0}{f_2 - f_1} = \frac{\omega_0 L}{R} = Q_0}$$

Low selectivity Low Q_0
High selectivity High Q_0.

Selectivity is defined as the ability of the ~~resonating~~ resonant circuit to distinguish or discriminate between desired and undesired frequencies.

Bandwidth and Selectivity :



Power $P = I^2 R$

at resonance, $P = (I)^2 R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R}$

At series resonance condition, power is maximum

Half power $\frac{P}{2} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$

Bandwidth is defined as range or band of frequ… at which power in the circuit is half of its ma… value of current is $\frac{1}{\sqrt{2}} = 0.707$ time of maxi… current in the circuit.

i.e. $BW = (f_2 - f_1)\,Hz = \frac{R}{2\pi L}$

where f_2 is upper cut off frequency (or upper half power frequency) and f_1 is lower cut off frequency (or lower half power frequency)

Current in the series RLC circuit is given by

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \quad -(1)$$

Also at ~~resonance~~ cut off frequency,

$I = \frac{I_0}{\sqrt{2}}$, where I_0 is maximum curr…

$$I = \frac{V}{R\sqrt{2}} \quad \text{—} (2)$$

Equating (1) and (2)

$$\frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{R\sqrt{2}}$$

$$\Rightarrow 2R^2 = R^2 + (X_L - X_C)^2$$

$$\Rightarrow R^2 = \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\text{or } \omega L - \frac{1}{\omega C} = \pm R. \quad \text{—} (3)$$

At upper cut off frequency, f_2

$$R = \omega_2 L - \frac{1}{\omega_2 C} \quad \text{—} (4)$$

At lower cut off frequency

$$-R = \omega_1 L - \frac{1}{\omega_1 C} \quad \text{—} (5)$$

Adding (4) and (5)

$$\omega_2 L - \frac{1}{\omega_2 C} + \omega_1 L - \frac{1}{\omega_1 C} = R - R$$

$$\Rightarrow L(\omega_2 + \omega_1) - \frac{1}{C}\left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right) = 0$$

$$\Rightarrow L(\omega_2 + \omega_1) = \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)$$

$$L = \frac{1}{C} \times \frac{1}{\omega_1 \omega_2}$$

(9)

$$\Rightarrow \omega_1 \omega_2 = \frac{1}{LC}$$

$$\Rightarrow \omega_1 \omega_2 = \omega_0^2 \quad \left[\text{as } \omega_0^2 = \frac{1}{LC}\right].$$

$$\Rightarrow \boxed{f_1 f_2 = f_0^2}$$

$\Rightarrow$ In series resonance circui
resonant frequency f_0 is
geometrical mean of f_1 & f_2

4	Express Z parameter in terms of Y and h parameters.	[10]	CO5	L1

a) Relation between Z and Y parameters

The two defining functions of Z parameters are

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{---} (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{---} (2)$$

The two defining function of Y parameters

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{---} (3)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{---} (4)$$

from eqn (3) $y_{11} V_1 = I_1 - y_{12} V_2$

$$V_1 = \frac{I_1}{y_{11}} - \frac{y_{12}}{y_{11}} V_2 \quad \text{---} (5)$$

from eqn (4)

$$y_{22} V_2 = I_2 - y_{21} V_1$$

$$V_2 = \frac{I_2}{y_{22}} - \frac{y_{21}}{y_{22}} V_1 \quad \text{---} (6)$$

Substituting eqn (6) in eqn (5)

$$V_1 = \frac{I_1}{y_{11}} - \frac{y_{12}}{y_{11}} \left(\frac{I_2}{y_{22}} - \frac{y_{21}}{y_{22}} V_1 \right)$$

or $V_1 = \frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11} y_{22}} I_2 + \frac{y_{12} y_{21}}{y_{11} y_{22}} V_1$

$$\Rightarrow V_1 - \frac{y_{12} y_{21}}{y_{11} y_{22}} V_1 = \frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11} y_{22}} I_2$$

$$\frac{(y_{11} y_{22} - y_{12} y_{21}) V_1}{\cancel{y_{11}} y_{22}} = \frac{1}{\cancel{y_{11}}} \left[I_1 - \frac{y_{12}}{y_{22}} I_2 \right]$$

But $y_{11} y_{22} - y_{12} y_{21} = \Delta y$.

$$V_1 = I_1 \frac{y_{22}}{\Delta y} - \frac{y_{12}}{\Delta y} I_2 \quad \text{--- (7)}$$

Lly substituting eqn (5) in eqn (6).

$$\Rightarrow V_2 = \frac{1}{y_{22}} I_2 - \frac{y_{21}}{y_{22}} \left[\frac{I_1}{y_{11}} - \frac{y_{12}}{y_{11}} V_2 \right].$$

$$\text{Or} \quad V_2 - \frac{y_{21} y_{12}}{y_{22} y_{11}} V_2 = \frac{1}{y_{22}} I_2 - \frac{y_{21}}{y_{22} y_{11}} I_1$$

$$\text{Or} \quad V_2 \left(\frac{y_{22} y_{11} - y_{21} y_{12}}{\cancel{y_{22}} y_{11}} \right) = \frac{1}{\cancel{y_{22}}} \left[I_2 - \frac{y_{21}}{y_{11}} I_1 \right]$$

$$\text{Or} \quad V_2 = \frac{y_{11} I_2}{\Delta y} - \frac{y_{21}}{\Delta y} I_1$$

$$V_2 = -\frac{y_{21}}{\Delta y} I_1 + \frac{y_{11}}{\Delta y} I_2 \quad \text{--- (8)}$$

Comparing eqn (1) with eqn (7) and eqn (2) with eqn (8), we get

Z parameters in term of y parameters

$$Z = \begin{bmatrix} \frac{y_{22}}{\Delta y} & -\frac{y_{12}}{\Delta y} \\ -\frac{y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix} \Omega.$$

Relation between Z parameter and h parameters

The Z parameter equations are $V_1 = Z_{11} I_1 + Z_{12} I_2$ — (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — (2)

h-parameter equations are

$V_1 = h_{11} I_1 + h_{12} V_2$ — (3)

$I_2 = h_{21} I_1 + h_{22} V_2$ — (4)

from eqn (4) $h_{22} V_2 = I_2 - h_{21} I_1$

$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1$ — (5)

Substituting (5) in eqn (3)

$V_1 = h_{11} I_1 + h_{12} \left[\frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1 \right]$

$\Rightarrow V_1 = I_1 \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} \right] + \frac{h_{12}}{h_{22}} I_2$

$\Rightarrow V_1 = I_1 \left[\frac{\Delta h}{h_{22}} \right] + \frac{h_{12}}{h_{22}} I_2 \rightarrow$ (6)

from eqn (5) $V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1$

or $V_2 = -\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2$ — (7)

Comparing (1) with (6) and (2) with (7)

$$Z = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

5	Find the y parameters of the two-port network shown in Fig 3. Then determine the current in a 4Ω load, that is connected to the output port when a 2A source is applied at the input port.	[10]	CO5	L3

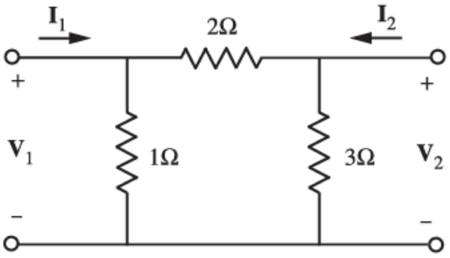


Fig 3

SOLUTION

To find y_{11} and y_{21}, short-circuit the output terminals and connect a current source I_1 to the input terminals. The resulting circuit diagram is shown in Fig. 7.8(a).

$$I_1 = \frac{V_1}{1\Omega||2\Omega} = \frac{V_1}{\frac{1\times 2}{1+2}}$$

$$\Rightarrow \quad I_1 = \frac{3}{2}V_1$$

Hence, $$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{3}{2}S$$

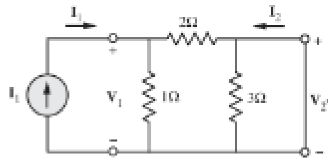


Figure 7.8(a)

Using the principle of current division,

$$-I_2 = \frac{I_1 \times 1}{1+2}$$

$$\Rightarrow \quad -I_2 = \frac{1}{3}I_1$$

$$\Rightarrow \quad -I_2 = \frac{1}{3}\left[\frac{3}{2}V_1\right]$$

Hence, $$y_{21} = \frac{I_2}{V_1} = \frac{-1}{2}S$$

To find y_{12} and y_{22}, short the input terminals and connect a current source I_2 to the output terminals. The resulting circuit diagram is shown in Fig. 7.8(b).

$$I_2 = \frac{V_2}{2\Omega||3\Omega} = \frac{V_2}{\frac{2\times 3}{2+3}} = \frac{5V_2}{6}$$

$$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0} = \frac{5}{6}S$$

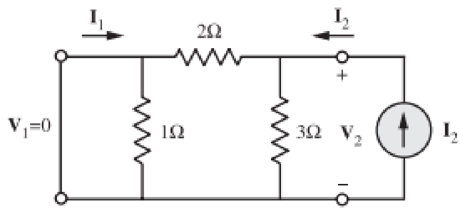


Figure 7.8(b)

Employing the current division principle,

$$-I_1 = \frac{I_2 \times 3}{2+3}$$

$$\Rightarrow \quad -I_1 = \frac{3}{5}I_2$$

$$\Rightarrow \quad -\mathbf{I}_1 = \frac{3}{5}\left[\frac{5\mathbf{V}_2}{6}\right]$$

$$\Rightarrow \quad \mathbf{I}_1 = \frac{-1}{2}\mathbf{V}_2$$

Hence,
$$\mathbf{y}_{12} = \left.\frac{-\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} = \frac{-1}{2}\text{S}$$

Therefore, the equations that describe the two-port network are

$$\mathbf{I}_1 = \frac{3}{2}\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 \qquad (7.3)$$

$$\mathbf{I}_2 = -\frac{1}{2}\mathbf{V}_1 + \frac{5}{6}\mathbf{V}_2 \qquad (7.4)$$

Putting the above equations (7.3) and (7.4) in matrix form, we get

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

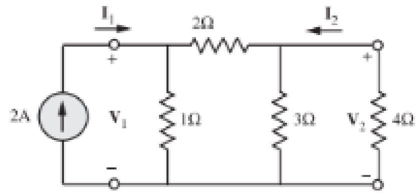


Figure 7.8(c)

Referring to Fig. 7.8(c), we find that $\mathbf{I}_1 = 2\text{A}$ and $\mathbf{V}_2 = -4\mathbf{I}_2$

Substituting $\mathbf{I}_1 = 2\text{A}$ in equation (7.3), we get

$$2 = \frac{3}{4}\mathbf{V}_1 - \frac{1}{2}\mathbf{V}_2 \qquad (7.5)$$

Multiplying equation (7.4) by -4, we get

$$-4\mathbf{I}_2 = 2\mathbf{V}_1 - \frac{20}{6}\mathbf{V}_2$$

$$\Rightarrow \quad \mathbf{V}_2 = 2\mathbf{V}_1 - \frac{20}{6}\mathbf{V}_2$$

$$\Rightarrow \quad 0 = 2\mathbf{V}_1 - \left(\frac{20}{6} + 1\right)\mathbf{V}_2$$

$$\Rightarrow \quad 0 = \frac{-1}{2}\mathbf{V}_1 + \frac{13}{12}\mathbf{V}_2 \qquad (7.6)$$

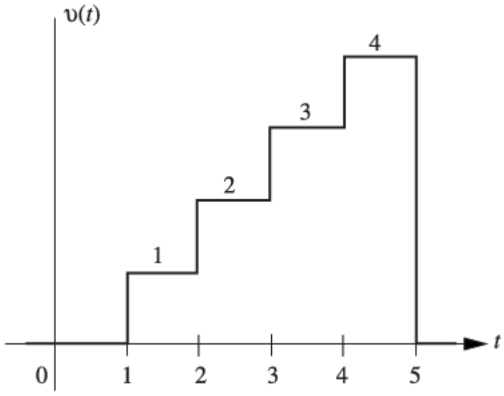
Putting equations (7.5) and (7.6) in matrix form, we get

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{13}{12} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

It may be noted that the above equations are simply the nodal equations for the circuit shown in Fig. 7.8(c). Solving these equations, we get

$$\mathbf{V}_2 = \frac{3}{2}\text{V}$$

and hence,
$$\mathbf{I}_2 = \frac{-1}{4}\mathbf{V}_2 = \frac{-3}{8}\mathbf{A}$$

6	A. Find the Laplace transform of the staircase waveform shown in Fig 4. B. If this voltage were applied to an RL series circuit with R=1Ω and L=1H, find the current *i(t)*.  **Fig 4**	[10]	CO4	L3

SOLUTION

(a) We can express mathematically, the voltage waveform shown in Fig. 5.54 as,

$$v(t) = \begin{cases} 1, & 1 < t < 2 \\ 2, & 2 < t < 3 \\ 3, & 3 < t < 4 \\ 4, & 4 < t < 5 \\ 0, & \text{elsewhere} \end{cases}$$

or

$$\begin{aligned} v(t) &= [u(t-1) - u(t-2)] + 2[u(t-2) - u(t-3)] \\ &\quad + 3[u(t-3) - u(t-4)] + 4[u(t-4) - u(t-5)] \\ &= u(t-1) + u(t-2) + u(t-3) + u(t-4) - 4u(t-5) \end{aligned}$$

Taking the Laplace transform, we get

$$\boldsymbol{V(s) = \frac{1}{s}\left[e^{-s} + e^{-2s} + e^{-3s} + e^{-4s} - 4e^{-5s}\right]}$$

(b) Assuming all initial conditions to be zero, the time domian circuit shown in Fig. 5.55 gets transformed to a circuit as shown in Fig. 5.56.

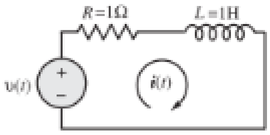


Figure 5.55 Time Domain Circuit

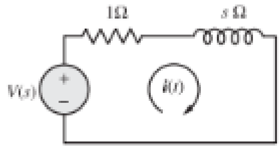


Figure 5.56 Frequency Domain Circuit

From Fig. 5.56, we can write

$$I(s) = \frac{V(s)}{s+1}$$

$$\Rightarrow I(s) = \frac{1}{s(s+1)}e^{-s} + \frac{1}{s(s+1)}e^{-2s} + \frac{1}{s(s+1)}e^{-3s} + \frac{1}{s(s+1)}e^{-4s} - \frac{4}{s(s+1)}e^{-5s}$$

$$\begin{aligned} \Rightarrow I(s) = \Bigg[&\left(\frac{1}{s} - \frac{1}{s+1}\right)e^{-s} + \left(\frac{1}{s} - \frac{1}{s+1}\right)e^{-2s} + \left(\frac{1}{s} - \frac{1}{s+1}\right)e^{-3s} \\ &+ \left(\frac{1}{s} - \frac{1}{s+1}\right)e^{-4s} - 4\left(\frac{1}{s} - \frac{1}{s+1}\right)e^{-5s}\Bigg] \end{aligned}$$

Taking the inverse Laplace transform, we get

$$\begin{aligned} i(t) &= \left[u(t) - e^{-t}u(t)\right]_{t\to t-1} + \left[u(t) - e^{-t}u(t)\right]_{t\to t-2} + \left[u(t) - e^{-t}u(t)\right]_{t\to t-3} \\ &\quad + \left[u(t) - e^{-t}u(t)\right]_{t\to t-4} - 4\left[u(t) - e^{-t}u(t)\right]_{t\to t-5} \end{aligned}$$

$$\boldsymbol{\begin{aligned} \Rightarrow i(t) &= \left[1 - e^{-(t-1)}\right]u(t-1) + \left[1 - e^{-(t-2)}\right]u(t-2) + \left[1 - e^{-(t-3)}\right]u(t-3) \\ &\quad + \left[1 - e^{-(t-4)}\right]u(t-4) - 4\left[1 - e^{-(t-5)}\right]u(t-5) \end{aligned}}$$

7	Express the voltage pulse shown in Fig 5 in terms of unit step function and then find V(s). Also find $\mathscr{L}\left\{\frac{dv(t)}{dt}\right\}$.	[10]	CO4	L2

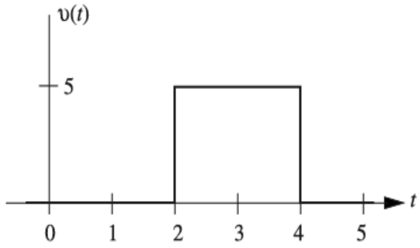


Fig 5

SOLUTION

The pulse shown in Fig. 5.37 is the gate function. This function may be regarded as a step function that switches on at $t = 2$ secs and switches off at $t = 4$ secs.

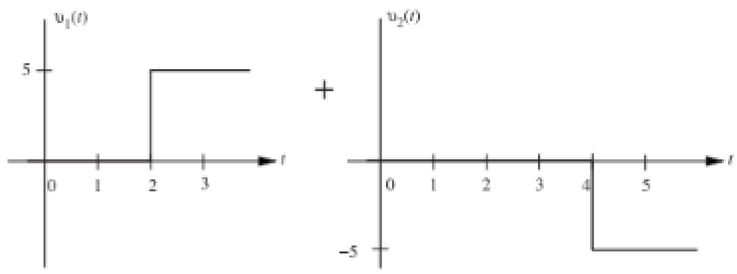


Figure 5.37(a)

Referring to Figs. 5.37 and 5.37 (a), we can write

$$v(t) = v_1(t) + v_2(t)$$

$$\Rightarrow \quad v(t) = 5u(t-2) - 5u(t-4)$$

Hence, $$V(s) = \frac{5}{s}e^{-2s} - \frac{5}{s}e^{-4s}$$

$$= \frac{5}{s}\left[e^{-2s} - e^{-4s}\right]$$

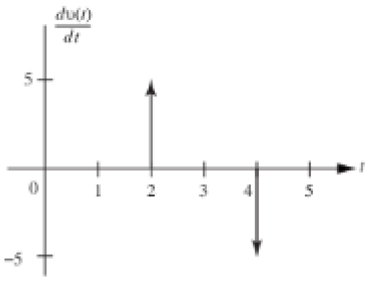


Figure 5.37(b)

Taking the derivative of $v(t)$, we get

$$\frac{dv(t)}{dt} = 5\left[\delta(t-2) - \delta(t-4)\right]$$

Fig. 5.37(b) shows the graph of $\frac{dv(t)}{dt}$.

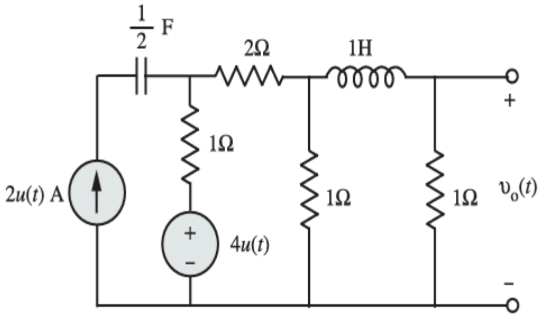
We can obtain Fig. 5.37(b) directly from Fig. 5.36 by observing that at $t = 2$ seconds, there is a sudden rise of 5V leading to $5\delta(t-2)$. Similarly, at $t = 4$ seconds, a sudden fall of 5V leading to $-5\delta(t-4)$.

We know the Laplace trasnform pair

$$\mathscr{L}\{\delta(t-a)\} = e^{-as}\,\mathscr{L}\{\delta(t)\}$$

$$= e^{-as}$$

Hence, $$\mathscr{L}\left\{\frac{dv(t)}{dt}\right\} = 5\left[e^{-2s} - e^{-4s}\right]$$

8	For the network shown in Fig 6., find Vo(t), t >0 ,using mesh analysis.  **Fig 6**	[10]	CO4	L3

The step function $u(t)$ is defined as follows.

$$u(t) = \begin{cases} 1, & t \geq 0^+ \\ 0, & t \leq 0^- \end{cases}$$

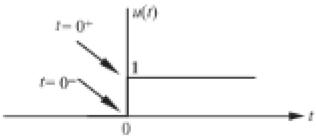


Figure 5.29(a)

Since the circuit is not energized for $t \leq 0^-$, there are no initial conditions in the circuit. For $t \geq 0^+$, the frequency domain equivalent circuit is shown in Fig. 5.29(b).

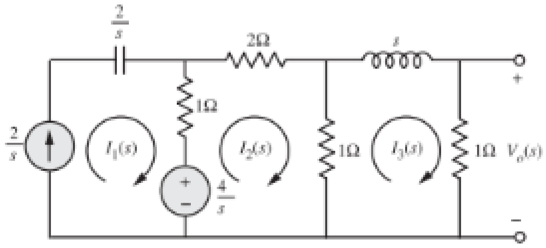


Figure 5.29(b)

By inspection, we find that $I_1(s) = \dfrac{2}{s}$

KVL clockwise for mesh 2:

$$\frac{-4}{s} + 1\left[I_2(s) - I_1(s)\right] + 2I_2(s) + 1\left[I_2(s) - I_3(s)\right] = 0$$

$$\Rightarrow \qquad \frac{-4}{s} - I_1(s) + I_2(s)\left[1+2+1\right] - I_3(s) = 0$$

Substituting the value of $I_1(s)$, we get

$$\frac{-4}{s} + 4I_2(s) - I_3(s) = \frac{2}{s}$$

$$\Rightarrow \qquad 4I_2(s) - I_3(s) = \frac{6}{s}$$

KVL clockwise for mesh 3:

$$1\left[I_3(s) - I_2(s)\right] + sI_3(s) + 1I_3(s) = 0$$

$$\Rightarrow \qquad -I_2(s) + I_3(s)\left[s+2\right] = 0$$

Putting the *KVL equations* for mesh 2 and mesh 3 in matrix form, we get

$$\begin{bmatrix} 4 & -1 \\ -1 & s+2 \end{bmatrix} \begin{bmatrix} I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} \frac{6}{s} \\ 0 \end{bmatrix}$$

Solving for $I_3(s)$, using Cramer's rule, we get

$$I_3(s) = \frac{1.5}{s\left(s+\frac{7}{4}\right)}$$

$$\Rightarrow \qquad V_o(s) = I_3(s) \times 1 = \frac{1.5}{s\left(s+\frac{7}{4}\right)}$$

Using partial fractions, we can write

$$V_o(s) = \frac{K_1}{s} + \frac{K_2}{s+\frac{7}{4}}$$

We find that, $\qquad K_1 = \dfrac{6}{7}$, and $K_2 = \dfrac{-6}{7}$

Hence, $\qquad V_o(s) = \dfrac{6}{7}\left[\dfrac{1}{s} - \dfrac{1}{s+\frac{7}{4}}\right]$

$$\Rightarrow \qquad \boldsymbol{v_o(t) = \frac{6}{7}\left[1 - e^{-\frac{7}{4}t}\right]u(t)}$$

9	A series RLC circuit has a resistance of 10Ω, an inductance of 0.3H and a capacitance of 100µF. The applied voltage is 230 V. Find i) Resonant Frequency, ii) Quality Factor, iii) Lower and upper cut off frequencies, iv) Band width, v) current at resonance, vi) currents at $f1$ and $f2$, vii) voltage across inductance at resonance.	[10]	CO5	L3