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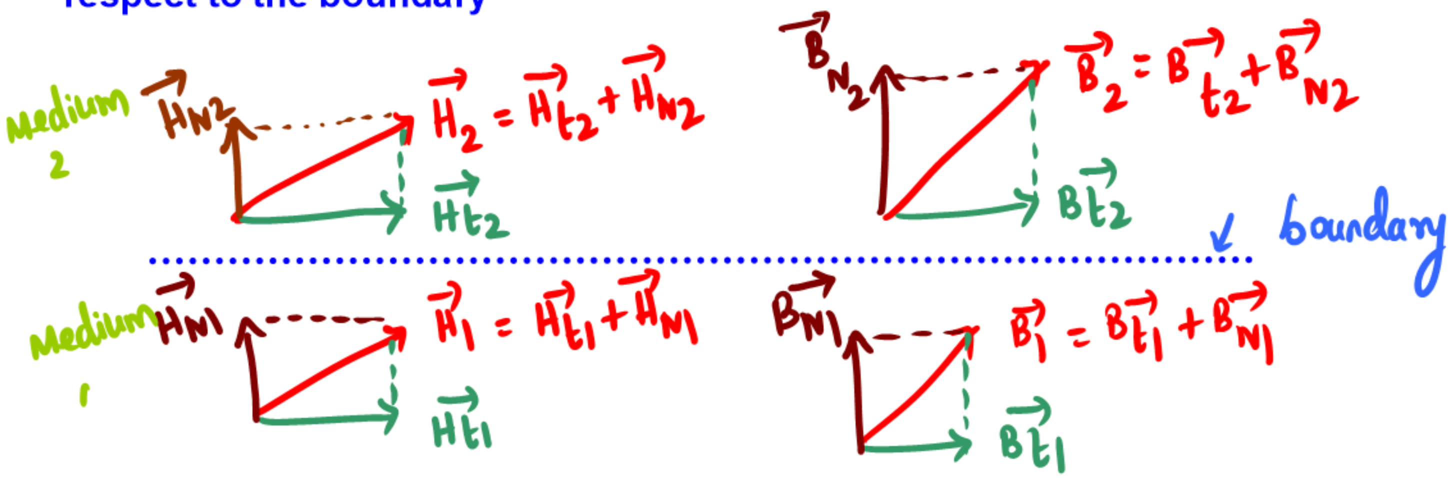
CMR
INSTITUTE OF
TECHNOLOGY

Internal Assesment Test-III								
Sub:	Electromagnetic Waves						Code:	21EC54
Date:	13/03/2024	Duration:	90 mins	Max Marks:	50	Sem: 5th	Branch:	ECE(A,B,C,D)
Answer any **FIVE FULL** Questions								

		Marks	OBE CO	OBE RBT
1.	Obtain the boundary conditions at the interface between two magnetic materials.	[10]	CO4	L2
2. (a)	Using Faraday's law derive an expression for e.m.f induced in stationary conductor placed in a time varying magnetic field.	[06]	CO5	L2
(b)	Compare the properties of electric and magnetic circuit with equations,	[04]	CO4	L2
3.	Explain displacement current. What is the inconsistency of Ampere's law with the equation of continuity? Derive a modified form of Ampere's law for time-varying fields.	[10]	CO5	L2
4.	State and prove Poynting's theorem. Give the expression for Poynting's vector.	[10]	CO5	L2
5.	Starting from Maxwell's equations, derive the wave equation for TEM wave.	[10]	CO5	L2
6.(a)	Derive the expression for Intrinsic Impedance for uniform plane wave in free space.	[08]	CO5	L2
6.(b)	List Maxwell's equations in point forms.	[02]	CO5	L2
7.	Derive the magnetic field intensity for a finite conducting line carrying direct current I placed along the z-axis.	[10]	CO4	L2

Q.No. 1) **Magnetic Bounday Conditions:**

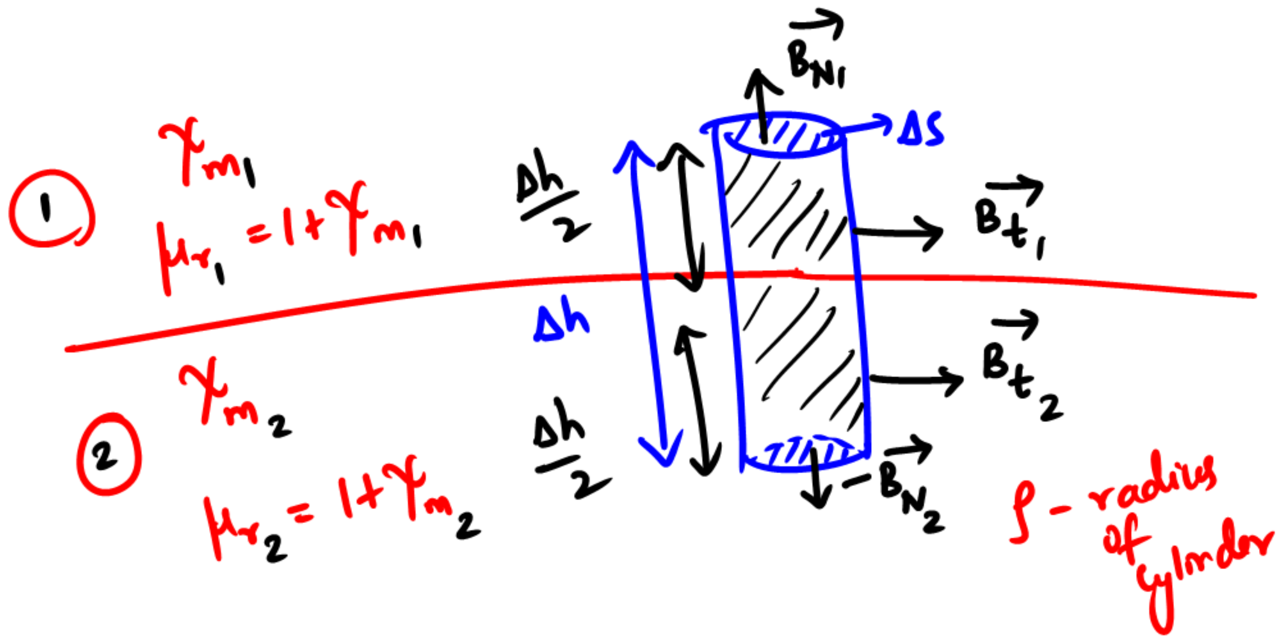
Magnetic field vectors can be split into tangential and normal components with respect to the boundary



Magnetic boundary conditions can be obtained by applying (i)Gauss's law for magnetic fields and (ii) Ampere's Circuital Law

Gauss's law:

(i) $$\oint\!\!\!\oint_S \vec{B}\cdot\vec{ds} = 0$$



$$\iint_{bottom} \vec{B}\cdot\vec{ds} + \iint_{top} \vec{B}\cdot\vec{ds} + \iint_{side\ 1} \vec{B}\cdot\vec{ds} + \iint_{side\ 2} \vec{B}\cdot\vec{ds} = 0$$

$$B_{N_1} \cdot \pi \rho^2 - B_{N_2} \cdot \pi \rho^2 + B_{t_1} \underbrace{2\pi\rho \cdot \frac{\Delta h}{2}}_{\to 0} + B_{t_2} \cdot \underbrace{2\pi\rho \cdot \frac{\Delta h}{2}}_{\to 0} = 0$$

To obtain conditions at the boundary,

$$\Delta h \to 0,$$

$$B_{N_1} \cdot \pi \rho^2 - B_{N_2} \pi \rho^2 = 0$$

$$\boxed{B_{N_1} = B_{N_2}} \Rightarrow \boxed{\vec{B_{N_1}} = \vec{B_{N_2}}} \to ①$$

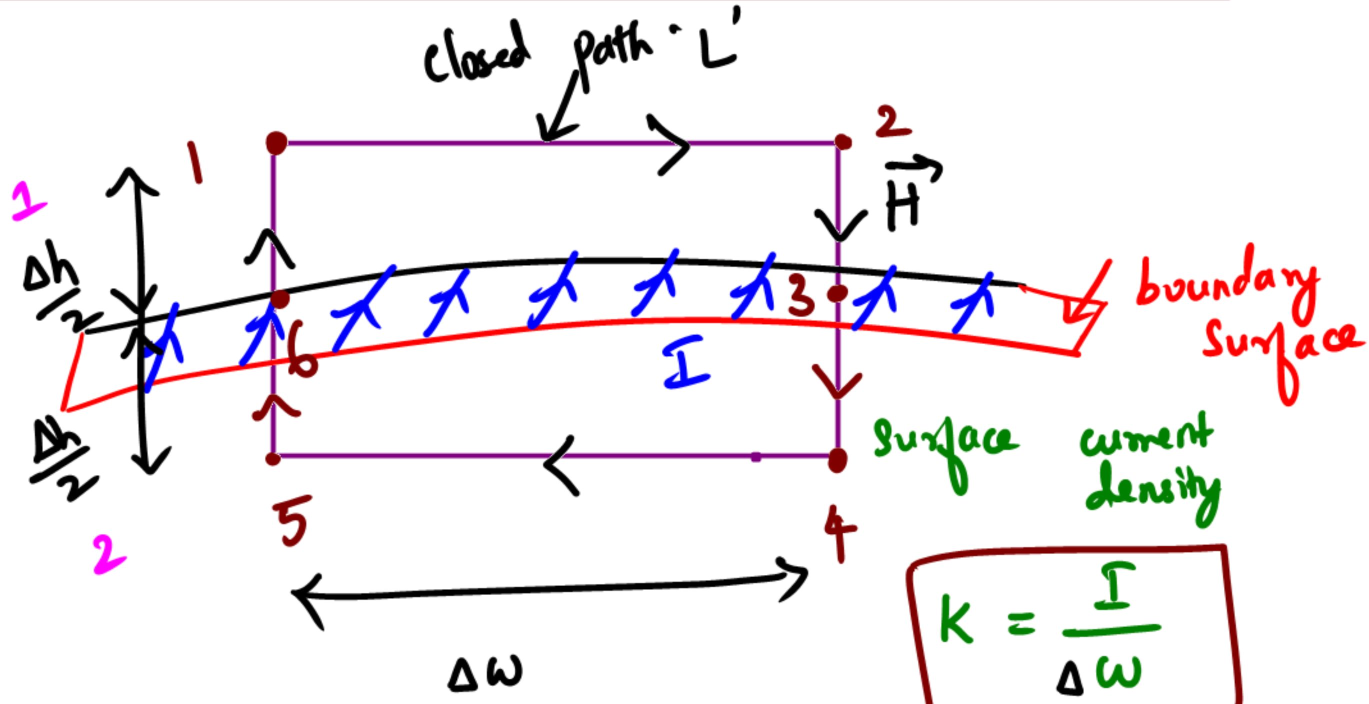
$$\vec{B} = \mu_0 \mu_r \vec{H}$$

$$\boxed{\mu_0 \mu_{r_1} H_{N_1} = \mu_0 \mu_{r_2} H_{N_2}} \Rightarrow \boxed{\mu_{r_1} \vec{H_{N_1}} = \mu_{r_2} \vec{H_{N_2}}} \to ②$$

$$\boxed{\vec{M} = \chi_m \vec{H}} \Rightarrow \boxed{\vec{H} = \frac{\vec{M}}{\chi_m}}$$

$$\boxed{\mu_{r_1} \frac{M_{N_1}}{\chi_{m_1}} = \mu_{r_2} \frac{M_{N_2}}{\chi_{m_2}}} \Rightarrow \boxed{\frac{\mu_{r_1}}{\chi_{m_1}} \vec{M_{N_1}} = \frac{\mu_{r_2}}{\chi_{m_2}} \vec{M_{N_2}}} \to ③$$

$$\boxed{\text{(ii) Ampere's circuital Law: } \oint_L \vec{H}\cdot\vec{dl} = I_{enc}}$$



$$\boxed{K = \frac{I}{\Delta w}}$$

$$\oint_L \vec{H}\cdot\vec{dl} = I_{enc} = K\Delta w$$

$$\int_1^2 + \int_2^3 + \int_3^4 + \int_4^5 + \int_5^6 + \int_6^1 \vec{H}\cdot\vec{dl} = K\Delta w$$

$$H_{t_1}\cdot\Delta w - \cancel{H_{N_1}\cdot\frac{\Delta h}{2}}^{\,0} - \cancel{H_{N_2}\frac{\Delta h}{2}}^{\,0}$$

$$- H_{t_2}\Delta w + \cancel{H_{N_2}\frac{\Delta h}{2}}^{\,0} + \cancel{H_{N_1}\cdot\frac{\Delta h}{2}}^{\,0} = K\Delta w$$

To obtain conditions at the boundary,

$\Delta h \rightarrow 0$

$$H_{t_1}\,\Delta w - H_{t_2}\,\Delta w = K\,\Delta w$$

$$\boxed{H_{t_1} - H_{t_2} = K}$$

$$\Rightarrow \boxed{\vec{H}_{t_1} - \vec{H}_{t_2} = \vec{a}_{N_{12}} \times \vec{K}} \rightarrow (4)$$

where

$\vec{a}_{N_{12}}$ – unit normal vector from region 1 to region 2.

w.k.t

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0\mu_r}} \quad \text{and} \quad \boxed{\vec{H} = \frac{\vec{M}}{\chi_m}}$$

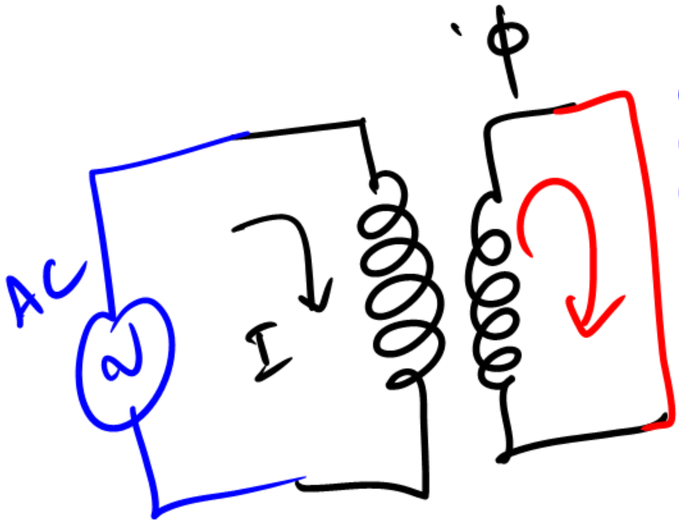
$$\boxed{\frac{\vec{B}_{t_1}}{\mu_0\mu_{r_1}} - \frac{\vec{B}_{t_2}}{\mu_0\mu_{r_2}} = \vec{a}_{N_{12}} \times \vec{K}} \rightarrow (5)$$

$$\boxed{\frac{\vec{M}_{t_1}}{\chi_{m_1}} - \frac{\vec{M}_{t_2}}{\chi_{m_2}} = \vec{a}_{N_{12}} \times \vec{K}} \rightarrow (6)$$

Equations (1), (2), (3), (4), (5) and (6) are the magnetic boundary conditions

Q. No. 2 a)

Faraday's Law and induced e.m.f.:



Faraday discovered that the induced emf, Vemf (in volts) in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit

$$\boxed{V_{emf} = -N\frac{d\phi}{dt}}$$

Lenz's law

Lenz's law states the direction of current flow in the circuit is such that the induced magnetic field produced by the induced current opposes change in the original magnetic field.

$$V = -\int_L \vec{E}\cdot\vec{dl}$$

$$\boxed{V_{induced} = +\oint_L \vec{E}\cdot\vec{dl}}$$

$$\phi = \iint_S \vec{B}\cdot\vec{ds} = \iint_S \mu\vec{H}\cdot\vec{ds}$$

$$\boxed{\frac{d\phi}{dt} = \frac{d}{dt}\left[\iint_S \vec{B}\cdot\vec{ds}\right]}$$

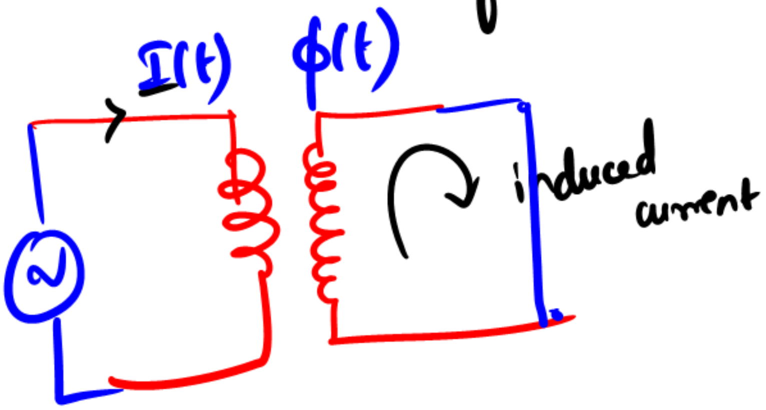
An e.m.f. will be induced under the following two conditions:

(i) when the magnetic field is static and the coil is moving - such e.m.f is called Generator e.m.f. (or) Motional e.m.f.

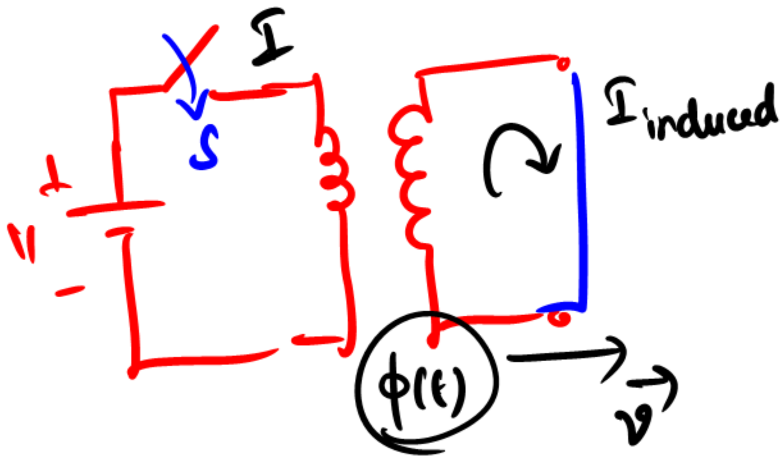
(ii) when the coil is stationary and the magnetic field is time varying - such e.m.f is called Transformer e.m.f.

Faraday's Law of electromagnetic induction:

(i) ~~Transformer~~ emf



(ii) motional emf (or) Generator emf.



$$\boxed{\text{Induced e.m.f } V_{induced} = -N\frac{d\phi}{dt}}$$

↑ Lenz's law

(i) Transformer e.m.f.

when the coil is stationary & Field is time varying

$I(t)$

$\vec{H}(x,y,z,t)$

$\vec{B}(x,y,z,t)$

$\phi(x,y,z,t) \Rightarrow \phi = \iint_S \vec{B}\cdot\vec{ds}$

$$V_{induced} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\iint_S \vec{B}\cdot\vec{ds}\right] = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}$$

$$V_{ind} = +\oint_L \vec{E}_{ind}\cdot\vec{dl}$$

where,
$\vec{E}_{induced} \leftarrow$ induced Electric field intensity

$$\boxed{\oint_L \vec{E}\cdot\vec{dl} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}}$$

Integral form of Faraday's law.

w.k.t. Stokes's Theorem: $\oint_L \vec{A}\cdot\vec{dl} = \iint_S \vec{\nabla}\times\vec{A}\cdot\vec{ds}$

$$\iint_S \vec{\nabla}\times\vec{E}\cdot\vec{ds} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{ds}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

Differential form / Point form of Faraday's law

(ii) Generator e.m.f. / Motional e.m.f.

($\vec{B}$) Field is static (not changing with respect to time)

&

Coil is moving. ($\vec{v}$ - velocity of moving coil)

Force on a moving coil

$$\vec{F}_m = Q(\vec{v} \times \vec{B})$$

$$\vec{E}_{induced} = \frac{\vec{F}}{Q} = \vec{v} \times \vec{B}$$

$$V_{induced} \text{ (induced e.m.f.)} = \oint_L \vec{E}_{induced} \cdot \vec{dl}$$

$$\boxed{V_{induced} \text{ (induced e.m.f.)} = \oint_L \vec{v} \times \vec{B} \cdot \vec{dl}}$$

(iii) Field is time varying & coil is moving

$$\text{e.m.f} = \text{transformer e.m.f} + \text{motional e.m.f.}$$

$$\boxed{V_{induced} \text{ (induced e.m.f.)} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{ds} + \oint_L (\vec{v} \times \vec{B}) \cdot \vec{dl}}$$

Q. No. 2 b)

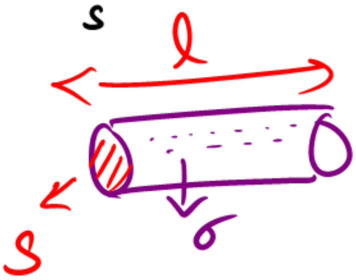
Electric Circuit Parameters:

$\vec{E} = -\vec{\nabla} V$ (electrostatics) (V/m)

$V_{ab} = -\int_b^a \vec{E} \cdot \vec{dl}$ (V)

$\vec{D} = \varepsilon \vec{E}$ (C/m^2)

$I = \iint_S \vec{J} \cdot \vec{ds}$ (A)



$R = \frac{\rho l}{S} = \frac{l}{\sigma S}$ (Ω)

Conductance

$G = \frac{1}{R}$ $(\mho)$

$\sigma = \frac{1}{\rho}$ (S/m)

$\boxed{\vec{J} = \sigma \vec{E}}$ → microscopic form of ohm's law

Macroscopic form of ohm's law: $\boxed{V = IR}$

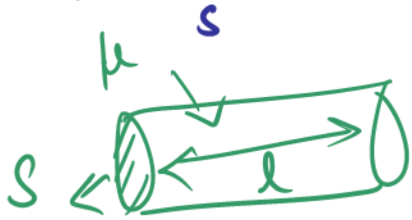
Magnetic Circuit Parameters:

$\vec{H} = -\vec{\nabla} V_m$ (A/m)

$V_m = -\int_L \vec{H} \cdot dl$ (A)

$\vec{B} = \mu \vec{H}$ (Wb/m^2)

$\phi = \iint_S \vec{B} \cdot \vec{ds}$ (Wb)



$\mathcal{R}$ - Reluctance

$\boxed{\mathcal{R} = \frac{l}{\mu S}}$ (At/Wb)

Permeance $\mathcal{P} = \frac{1}{\mathcal{R}}$ (Wb/At)

$\boxed{\mathcal{F} = m.m.f. = \phi_m \mathcal{R}}$ (At)

Electric Circuit Parameters:

$$V_{ab} = -\int_b^a \vec{E}\cdot\vec{dl} \quad (V)$$

Work done around a closed path is zero

$$\boxed{W = -Q\oint_L \vec{E}\cdot\vec{dl} = 0}$$

KVL:

$$\oint_L \vec{E}\cdot\vec{dl} = 0$$

Capacitance

$$C = \frac{Q}{V} \quad (F)$$

Magnetic Circuit Parameters:

ACL

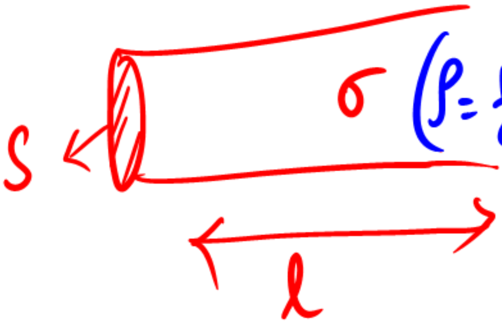
$$\oint_L \vec{H}\cdot\vec{dl} = I_{enc}$$

For N turns,

$$\oint_L \vec{H}\cdot\vec{dl} = NI$$

Inductance

$$L = \frac{N\phi}{I} \quad (H)$$



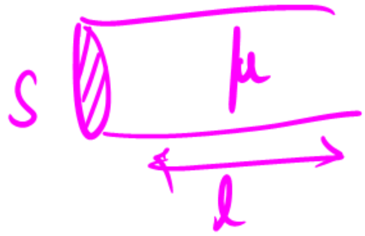
$(\rho = \frac{1}{\sigma})$ $R = \frac{\rho l}{S}$ (Ω) $G = \frac{1}{R}$ $(\mho)$

$V = IR$

m.m.f $\boxed{\mathcal{F} = \Phi \mathcal{R}}$ (At)

Reluctance

Reluctance



$\boxed{\mathcal{R} = \frac{l}{\mu S}}$ (At/wb)

Permeance $\rightarrow$ $\boxed{\mathcal{P} = \frac{1}{\mathcal{R}}}$ (wb/At)

m.m.f. (magneto motive force)

Series Electric Circuit

$$I = I_1 = I_2 = \cdots = I_N$$

$$V = V_1 + V_2 + \cdots + V_N$$

$$R = \frac{V}{I} = R_1 + R_2 + \cdots + R_N$$

$$\frac{1}{G} = \frac{1}{G_1} + \frac{1}{G_2} + \cdots + \frac{1}{G_N}$$

Parallel Electric Circuit:

$$I = I_1 + I_2 + \cdots + I_N$$

$$V = V_1 = V_2 = \cdots = V_N$$

$$\frac{1}{R} = \frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

$$G = G_1 + G_2 + \cdots + G_N$$

Series Magnetic Circuit:

$$\phi_m = \phi_{m_1} = \phi_{m_2} = \cdots = \phi_{mN}$$

$$\mathcal{F} = \mathcal{F}_1 + \mathcal{F}_2 + \cdots + \mathcal{F}_N$$

$$\mathcal{F} = \phi_m \mathcal{R}$$

$$\mathcal{F}_T = \phi_{m_T} \mathcal{R}_T$$

$$\mathcal{F}_T = \phi_{m_1}\mathcal{R}_1 + \phi_{m_2}\mathcal{R}_2 + \cdots + \phi_{m_N}\mathcal{R}_N$$

$$\mathcal{R}_T = \mathcal{R}_1 + \mathcal{R}_2 + \cdots + \mathcal{R}_N$$

Parallel Magnetic Circuit:

$$\phi = \phi_1 + \phi_2 + \cdots + \phi_N$$

$$\mathcal{F} = \mathcal{F}_1 = \mathcal{F}_2 = \cdots = \mathcal{F}_N$$

$$\frac{1}{\mathcal{R}} = \frac{\phi}{\mathcal{F}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \cdots + \frac{1}{\mathcal{R}_N}$$

$$\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2 + \cdots + \mathcal{P}_N$$

Q. no. 3)

Explain the inconsistency of Ampere's law for time varying fields. Derive the consistent equation (Modified form) of Ampere's law for time varying fields.

$\vec{E}(x, y, z, t)$

$\vec{H}(x, y, z, t)$

Continuity Equation of current

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ ✓

Ampere's Circuital Law

Integral form

$$\boxed{\oint_L \vec{H} \cdot \vec{dl} = I_{enc}}$$

Differential form:

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}}$$

Take divergence on both sides

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \vec{J}$$

$$0 = -\frac{\partial \rho_v}{\partial t}$$

But $\frac{\partial \rho_v}{\partial t}$ cannot be zero

This shows that A.C.L. for static fields is inconsistent for time varying fields

Modified A.C.L. for time varying fields,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{G}$$

$\vec{G}$ → unknown vector ??

Take divergence on both sides

$$\vec{\nabla} \cdot \vec{\nabla} \times \vec{H} = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{G}$$

$$0 = -\frac{\partial \rho_v}{\partial t} + \vec{\nabla} \cdot \vec{G}$$

$$\vec{\nabla} \cdot \vec{G} = \frac{\partial \rho_v}{\partial t}$$

w.k.t. $\rho_v = \vec{\nabla} \cdot \vec{D}$

$$\vec{\nabla} \cdot \vec{G} = \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{D})$$

$$\vec{\nabla} \cdot \vec{G} = \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$$

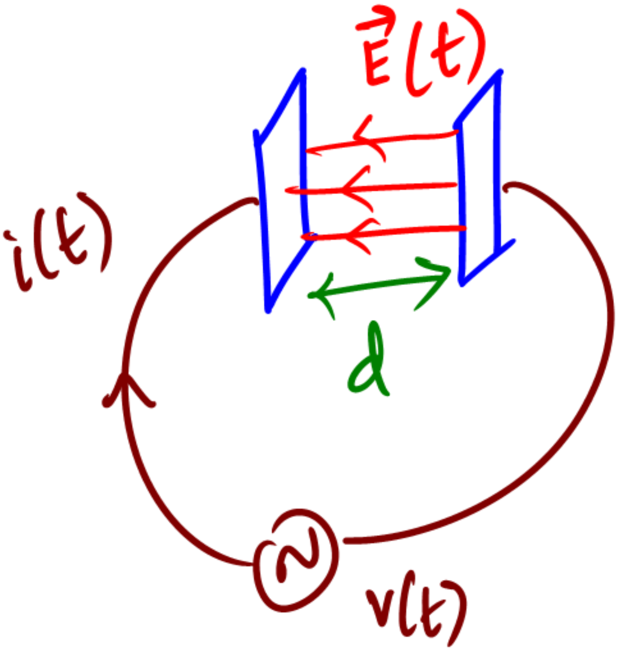
$$\boxed{\vec{G} = \frac{\partial \vec{D}}{\partial t}}$$

Modified form of Ampere's Circuital Law for Time varying fields:(Differential or Point Form)

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

where $\vec{J} = \sigma \vec{E}$ (conduction current density)

and $\frac{\partial \vec{D}}{\partial t} = \vec{J}_D$ (Displacement current density)



$$C = \frac{\varepsilon_0 \varepsilon_r S}{d}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{D}(t) = \varepsilon_0 \varepsilon_r \vec{E}(t)$$

$$I = I_C + I_D$$

$$I_C = \iint_S \vec{J_C} \cdot \vec{ds} = \iint_S \sigma \vec{E} \cdot \vec{ds}$$

$$I_D = \iint_S \vec{J_D} \cdot \vec{ds} = \iint_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$

$$\oint_L \vec{H} \cdot \vec{dl} = I_C + I_D$$

$$\boxed{\oint_L \vec{H} \cdot \vec{dl} = \iint_S \sigma \vec{E} \cdot \vec{ds} + \iint_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}}$$

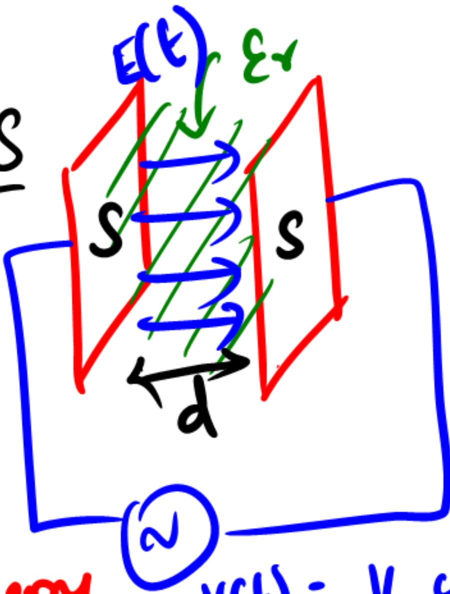
Modified form of A.C.L. for time varying fields in Integral form

Modified form of A.C.L. for time varying fields in differential (Point) form

$$\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

The displacement current is a result of time-varying electric field. A typical example of such current is the current through a capacitor when an alternating voltage source is applied to its plate.

$$C = \frac{\varepsilon_0 \varepsilon_r S}{d}$$



$V(t) = V_0 \cos\omega t$

Ckt theory:

$$i_{cap} = C\frac{dv}{dt}$$

$$i_{cap} = \frac{\varepsilon_0 \varepsilon_r S}{d} V_0(-\omega \sin\omega t)$$

$$\boxed{i_{cap} = -\frac{\varepsilon_0 \varepsilon_r V_0 S \omega}{d} \sin\omega t}$$

A

Field theory:

$$\boxed{I_D = \iint_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}} = ?$$

$$|\vec{D}(t)| = \varepsilon |\vec{E}(t)|$$

$$|\vec{E}(t)| = \frac{V(t)}{d}$$

$$E(t) = \frac{V_0 \cos\omega t}{d} \quad V/m$$

$$D(t) = \frac{\varepsilon_0 \varepsilon_r V_0 \cos\omega t}{d} \quad C/m^2$$

$$I_D = \frac{\partial D}{\partial t} \cdot S$$

$$\boxed{I_D = -\frac{\varepsilon_0 \varepsilon_r V_0 S}{d} \omega \sin\omega t}$$

A

Q. No. 4) **State and Explain Poynting's theorem with derivation.**

(or)

State and Prove Poynting's theorem.

(or)

State Poynting's theorem. Prove that P = E × H.

Poynting's theorem and Wave Power:

Poynting's theorem states that the net power flowing out of a given volume is equal to the time rate of decrease in stored energy within the volume minus conduction losses (Ohmic losses)

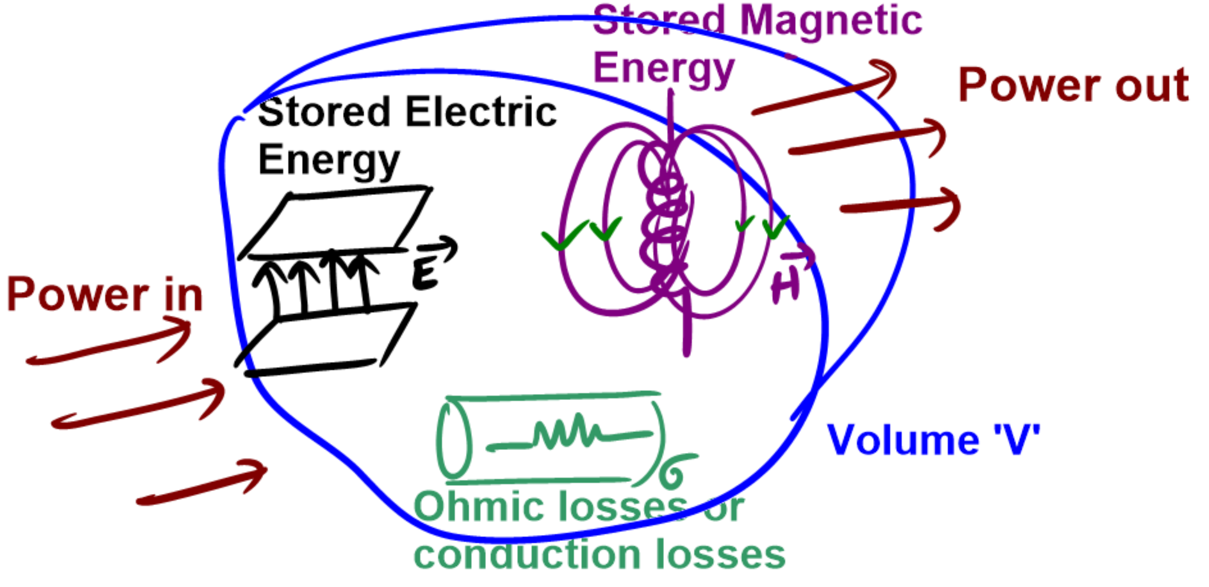


Fig. Illustration of Power Balance for EM fields

Proof:

Maxwell's Equations:

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t} \rightarrow (1)$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial\vec{E}}{\partial t} \rightarrow (2)$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_v}{\varepsilon} \rightarrow (3)$$

$$\vec{\nabla}\cdot\vec{H} = 0 \rightarrow (4)$$

Vector Identity:

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H})$$

From (1) From (2)

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\left(-\mu\frac{\partial\vec{H}}{\partial t}\right) - \vec{E}\cdot\left(\sigma\vec{E} + \varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\mu\,\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} - \sigma\vec{E}\cdot\vec{E} - \varepsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

We can also write, $\frac{\partial|\vec{H}|^2}{\partial t} = 2\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}$; $\frac{\partial|\vec{E}|^2}{\partial t} = 2\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$

Using these expressions, the above equation can be written as follows.

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\mu\frac{1}{2}\frac{\partial H^2}{\partial t} - \sigma E^2 - \varepsilon\frac{1}{2}\frac{\partial E^2}{\partial t}$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] - \sigma E^2$$

Differential form or Point form of Poynting's theorem

Integrating the above expression over the given volume "V" as depicted in the figure, provides the Integral form of Poynting's theorem.

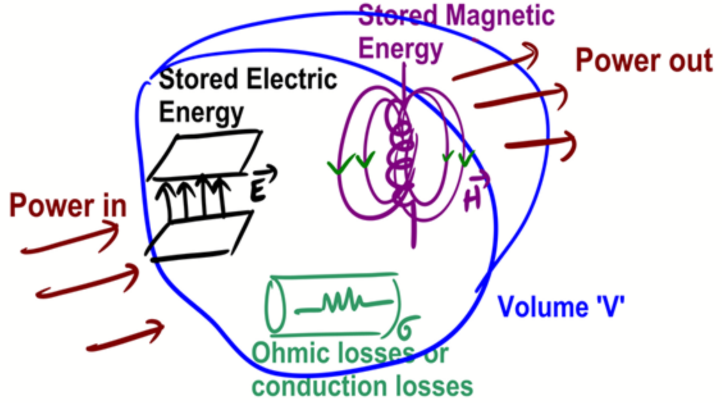


Fig. Illustration of Power Balance for EM fields

$$\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dv = -\frac{d}{dt}\left[\iiint_V \left(\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right)dv\right] - \iiint_V \sigma E^2\,dv$$

Using Divergence Theorem for the L.H.S.,

$$\iiint_V \vec{\nabla}\cdot\vec{D}\,dv = \oiint_S \vec{D}\cdot\vec{ds}$$

L.H.S. can be wrtitten as follows:

$$\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dv = \oiint_S (\vec{E}\times\vec{H})\cdot\vec{ds}$$

$$\oint\!\!\!\oint_S (\vec{E}\times\vec{H})\cdot \vec{ds} = -\frac{d}{dt}\left[\frac{1}{2}\iiint_V \mu H^2\, dv + \frac{1}{2}\iiint_V \varepsilon E^2\, dv\right] - \iiint_V \sigma E^2\, dv$$

Integral form of Poynting's theorem

Stored Magnetic Energy within the volume 'V'

Stored Electric Energy within the volume 'V'

Net power flowing out of the given Volme 'V'

Rate of decrease in stored energy within the Volume 'V'

Conduction losses or Ohmic Losses

Poynting's theorem and Wave Power:

Poynting's theorem states that the net power flowing out of a given volume is equal to the time rate of decrease in stored energy within the volume minus conduction losses (Ohmic losses)

Power of an EM wave, $P = \oiint_S (\vec{E} \times \vec{H}) \cdot \vec{ds}$ W

Power Density vector = Poynting's Vector (Power per unit Area)

$\vec{S} = \vec{P} = \vec{E} \times \vec{H}$ (W/m^2)

Instantaneous Power Density Vector, $\vec{P} = \vec{E} \times \vec{H}$ W/m^2

Active Power Density Vector or Real Power Density Vector, $\vec{P}_{avg} = \vec{P}_{real} = \frac{1}{2} \mathrm{Re}\{\vec{E} \times \vec{H}^*\}$ W/m^2

Reactive Power Density Vector, $\vec{P}_{reactive} = \frac{1}{2} \mathrm{Im}\{\vec{E} \times \vec{H}^*\}$ W/m^2

Instantaneous Power Density = Real Power Density + Reactive Power Density

Q. no. 5) Using Maxwell's equation derive an expression for uniform plane wave in free space.
(or)
Derive the expression for Wave equation for a Uniform Plane Wave for free space from Maxwell's equation.
(or)
Starting form Maxwell's equations, derive the general wave equations for electric and magnetic fields.

$$\boxed{\begin{aligned}\vec{B} &= \mu\vec{H}\\ \vec{D} &= \varepsilon\vec{E}\end{aligned}}$$

Maxwell's Equations:

1. $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

$$\boxed{\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}} \rightarrow (1)$$

2. $\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \dfrac{\partial\vec{D}}{\partial t}$

$$\boxed{\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial\vec{E}}{\partial t}} \rightarrow (2)$$

3. $\vec{\nabla}\cdot\vec{D} = \rho_v$

$\varepsilon\,\vec{\nabla}\cdot\vec{E} = \rho_v \Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \rho_v/\varepsilon}$

$\because \rho_v = 0$ for the media considered

$$\boxed{\vec{\nabla}\cdot\vec{E} = 0} \rightarrow (3)$$

4. $\vec{\nabla}\cdot\vec{B} = 0$

$\mu\,\vec{\nabla}\cdot\vec{H} = 0 \Rightarrow \boxed{\vec{\nabla}\cdot\vec{H} = 0} \rightarrow (4)$

Uniform Plane Wave(UPW)/Transverse Electromagnetic Wave(TEM):

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$\vec{E}(z,t) = E(z,t)\,\vec{a_x} = E(z)\,e^{j\omega t}\,\vec{a_x}$$

$$\vec{H}(z,t) = H(z,t)\,\vec{a_y} = H(z)\,e^{j\omega t}\,\vec{a_y}$$

Maxwell's Equations:

$$\boxed{\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}} \rightarrow (1) \qquad \boxed{\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial\vec{E}}{\partial t}} \rightarrow (2) \qquad \boxed{\vec{\nabla}\cdot\vec{E} = 0} \rightarrow (3) \qquad \boxed{\vec{\nabla}\cdot\vec{H} = 0} \rightarrow (4)$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E(z)e^{j\omega t} & 0 & 0 \end{vmatrix} = -\vec{a_y}\left[0 - \frac{\partial E}{\partial z}\right] \Rightarrow \vec{\nabla}\times\vec{E} = \frac{\partial E}{\partial z}\vec{a_y}$$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H(z)e^{j\omega t} & 0 \end{vmatrix} = \vec{a_x}\left[0 - \frac{\partial H}{\partial z}\right] \Rightarrow \vec{\nabla}\times\vec{H} = -\frac{\partial H}{\partial z}\vec{a_x}$$

Maxwell's Equations:

$$\boxed{\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}} \rightarrow (1) \qquad \boxed{\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}} \rightarrow (2) \qquad \boxed{\vec{\nabla} \cdot \vec{E} = 0} \rightarrow (3) \qquad \boxed{\vec{\nabla} \cdot \vec{H} = 0} \rightarrow (4)$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial E}{\partial z} \vec{a_y}$$

$$(1) \Rightarrow \quad \frac{\partial E}{\partial z} \vec{a_y} = -\mu \frac{\partial H}{\partial t} \vec{a_y}$$

$$\boxed{\frac{\partial E}{\partial z} = -\mu \frac{\partial H}{\partial t}} \longrightarrow (5)$$

$$\vec{\nabla} \times \vec{H} = -\frac{\partial H}{\partial z} \vec{a_x}$$

$$(2) \Rightarrow \quad -\frac{\partial H}{\partial z} \vec{a_x} = \sigma E \vec{a_x} + \varepsilon \frac{\partial E}{\partial t} \vec{a_x}$$

$$\boxed{-\frac{\partial H}{\partial z} = \sigma E + \varepsilon \frac{\partial E}{\partial t}} \rightarrow (6)$$

$$\boxed{\frac{\partial E}{\partial z} = -\mu\frac{\partial H}{\partial t}} \rightarrow (5)$$

$$\boxed{-\frac{\partial H}{\partial z} = \sigma E + \varepsilon\frac{\partial E}{\partial t}} \rightarrow (6)$$

Differentiating (5) w.r.t 't'

$$\boxed{\frac{\partial^2 E}{\partial t\partial z} = -\mu\frac{\partial^2 H}{\partial t^2}}$$

Diff. (5) w.r.t. 'z'

$$\boxed{\frac{\partial^2 E}{\partial z^2} = -\mu\frac{\partial^2 H}{\partial z\partial t}}$$

Differenting (6) w.r.t. 't'

$$\boxed{-\frac{\partial^2 H}{\partial t\partial z} = \sigma\frac{\partial E}{\partial t} + \varepsilon\frac{\partial^2 E}{\partial t^2}}$$

Diff. (6) w.r.t. 'z'

$$\boxed{-\frac{\partial^2 H}{\partial z^2} = \sigma\frac{\partial E}{\partial z} + \varepsilon\frac{\partial^2 E}{\partial z\partial t}}$$

$$\boxed{\frac{\partial E}{\partial z} = -\mu \frac{\partial H}{\partial t}} \rightarrow (5)$$

$$\boxed{-\frac{\partial H}{\partial z} = \sigma E + \varepsilon \frac{\partial E}{\partial t}} \rightarrow (6)$$

Differentiating (5) w.r.t 't'

$$\boxed{\frac{\partial^2 E}{\partial t \partial z} = -\mu \frac{\partial^2 H}{\partial t^2}}$$

Differenting (6) w.r.t. 't'

$$\boxed{-\frac{\partial^2 H}{\partial t \partial z} = \sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2}}$$

Diff. (5) w.r.t. 'z'

$$\boxed{\frac{\partial^2 E}{\partial z^2} = -\mu \frac{\partial^2 H}{\partial z \partial t}}$$

Diff. (6) w.r.t. 'z'

$$\boxed{-\frac{\partial^2 H}{\partial z^2} = \sigma \frac{\partial E}{\partial z} + \varepsilon \frac{\partial^2 E}{\partial z \partial t}}$$

$$\frac{\partial^2 E}{\partial z^2} = \mu \left[\sigma \frac{\partial E}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2} \right]$$

$$\boxed{\frac{\partial^2 E}{\partial z^2} = \sigma \mu \frac{\partial E}{\partial t} + \mu \varepsilon \frac{\partial^2 E}{\partial t^2}} \rightarrow \text{General wave equation for E-field.}$$

$$\boxed{\frac{\partial E}{\partial z} = -\mu\frac{\partial H}{\partial t}} \rightarrow 5$$

$$\boxed{-\frac{\partial H}{\partial z} = \sigma E + \varepsilon\frac{\partial E}{\partial t}} \rightarrow 6$$

Differentiating 5 w.r.t 't'

$$\boxed{\frac{\partial^2 E}{\partial t\partial z} = -\mu\frac{\partial^2 H}{\partial t^2}}$$

Diff. 5 w.r.t. 'z'

$$\boxed{\frac{\partial^2 E}{\partial z^2} = -\mu\frac{\partial^2 H}{\partial z\partial t}}$$

Differenting 6 w.r.t. 't'

$$\boxed{-\frac{\partial^2 H}{\partial t\partial z} = \sigma\frac{\partial E}{\partial t} + \varepsilon\frac{\partial^2 E}{\partial t^2}}$$

Diff. 6 w.r.t. 'z'

$$\boxed{-\frac{\partial^2 H}{\partial z^2} = \sigma\frac{\partial E}{\partial z} + \varepsilon\frac{\partial^2 E}{\partial z\partial t}}$$

$$-\frac{\partial^2 H}{\partial z^2} = -\sigma\mu\frac{\partial H}{\partial t} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

$$\boxed{\frac{\partial^2 H}{\partial z^2} = \sigma\mu\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}} \rightarrow$$

General wave equation for H-field.

Wave Equations for Free Space:

$$\boxed{\frac{\partial^2 E}{\partial z^2} = \sigma\mu\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}}$$ → General wave equation for E-field.

$$\boxed{\frac{\partial^2 H}{\partial z^2} = \sigma\mu\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}}$$ → General wave equation for H-field

1) Free space:

$\sigma = 0$

$\rho_v = 0$

$\varepsilon = \varepsilon_0 = \frac{1}{36\pi \times 10^9} = 8.854 \times 10^{-12}$

$\mu = \mu_0 = 4\pi \times 10^{-7}$

Loss tangent $= \frac{\sigma}{\omega\varepsilon} = 0$

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{\sqrt{4\pi\times10^{-7}\times\frac{1}{36\pi\times10^9}}}$$

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \sqrt{9\times10^{16}}$$

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^{8}\ m/s = c$$

velocity of light in free space

In general,

$$\frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = v_p$$

Velocity of the wave in any medium

Wave Equations for Free Space:

$$\boxed{\frac{\partial^2 E}{\partial z^2} = \sigma\mu\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}}$$

→ General wave equation for E-field.

$$\boxed{\frac{\partial^2 H}{\partial z^2} = \sigma\mu\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}}$$

→ General wave equation for H-field

1) Free space:

$\sigma = 0$

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^{8}\ m/s = c$$

velocity of light in free space

$$\frac{\partial^2 E}{\partial z^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 E}{\partial z^2}$$

$$\boxed{\frac{\partial^2 E}{\partial t^2} = c^2\frac{\partial^2 E}{\partial z^2}}$$

Wave Equations for Free Space:

$$\boxed{\frac{\partial^2 E}{\partial z^2} = \sigma\mu\frac{\partial E}{\partial t} + \mu\varepsilon\frac{\partial^2 E}{\partial t^2}}$$

→ General wave equation for E-field.

$$\boxed{\frac{\partial^2 H}{\partial z^2} = \sigma\mu\frac{\partial H}{\partial t} + \mu\varepsilon\frac{\partial^2 H}{\partial t^2}}$$

→ General wave equation for H-field

1) Free space:

$\sigma = 0$

$$\frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times10^8 \text{ m/s} = c$$

velocity of light in free space

$$\boxed{\frac{\partial^2 E}{\partial z^2} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}}$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 E}{\partial z^2}$$

$$\boxed{\frac{\partial^2 E}{\partial t^2} = c^2\frac{\partial^2 E}{\partial z^2}}$$

Free Space Wave Equation for E-field

$$\boxed{\frac{\partial^2 H}{\partial z^2} = \mu_0\varepsilon_0\frac{\partial^2 H}{\partial t^2}}$$

$$\frac{\partial^2 H}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2 H}{\partial z^2}$$

$$\boxed{\frac{\partial^2 H}{\partial t^2} = c^2\frac{\partial^2 H}{\partial z^2}}$$

Free Space Wave Equation for H-field

Q. No. 6 a)

Derive the relationship between E and H for a UPW for any medium.
(or)
Derive the expression for Intrinsic Impedance for any medium.
(or)
Derive the expression for Helmholtz Wave Equation and obtain the relationship between E and H for a UPW for any medium.
(or)
Derive the expression for Wave Equation and solution for a sinusoidal excitation and obtain the relationship between E and H for a UPW for any medium.

For a UPW / TEM,
Consider the sinusoidal excitation.

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

The phasor form of Equations can be written as follows:

$$\vec{E}(z,t) = E(z,t)\,\vec{a_x} = E(z)e^{j\omega t}\,\vec{a_x}$$

$$\vec{H}(z,t) = H(z,t)\,\vec{a_y} = H(z)e^{j\omega t}\,\vec{a_y}$$

$$\boxed{\frac{\partial \vec{E}}{\partial t} = E(z)e^{j\omega t}\cdot j\omega\,\vec{a_x} = j\omega\vec{E}}$$

$$\boxed{\frac{\partial \vec{H}}{\partial t} = H(z)e^{j\omega t}\cdot j\omega\,\vec{a_y} = j\omega\vec{H}}$$

Maxwell's equations :

1. $\vec{\nabla} \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} \Rightarrow$

2. $\vec{\nabla} \times \vec{H} = \sigma \vec{E} + \varepsilon \dfrac{\partial \vec{E}}{\partial t} \Rightarrow$

3. $\vec{\nabla} \cdot \vec{E} = \rho_v / \varepsilon = 0 \Rightarrow$

   $(\because \rho_v = 0)$

4. $\vec{\nabla} \cdot \vec{H} = 0 \Rightarrow$

$$\boxed{\begin{array}{ll} \vec{\nabla} \times \vec{E} = -j\omega\mu\vec{H} & \text{(1)} \\ \vec{\nabla} \times \vec{H} = \sigma\vec{E} + j\omega\varepsilon\vec{E} & \text{(2)} \\ \vec{\nabla} \cdot \vec{E} = 0 & \text{(3)} \\ \vec{\nabla} \cdot \vec{H} = 0 & \text{(4)} \end{array}}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -j\omega\mu\vec{H}} \rightarrow (1) \qquad \boxed{\vec{\nabla}\times\vec{H} = \sigma\vec{E} + j\omega\varepsilon\vec{E}} \rightarrow (2) \qquad \boxed{\vec{\nabla}\cdot\vec{E} = 0} \rightarrow (3) \qquad \boxed{\vec{\nabla}\cdot\vec{H} = 0} \rightarrow (4)$$

Take curl on both sides of (1)

$$\underbrace{\vec{\nabla}\times\vec{\nabla}\times\vec{E}}_{} = \vec{\nabla}\times(-j\omega\mu\vec{H})$$

Vector Identity:

$$\boxed{\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E}}$$

From the vector identity

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -j\omega\mu(\vec{\nabla}\times\vec{H})$$

From (3) $\vec{\nabla}\cdot\vec{E} = 0$ From (2)

$$-\nabla^2\vec{E} = -j\omega\mu(\sigma\vec{E} + j\omega\varepsilon\vec{E})$$

$$\nabla^2\vec{E} = j\omega\mu(\sigma + j\omega\varepsilon)\vec{E}$$

where

$$k = \gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Propagation constant $\boxed{k = \gamma = \alpha + j\beta}$

α → Attenuation constant

β → Phase Constant

$$\nabla^2\vec{E} = k^2\vec{E}$$

$$\boxed{\nabla^2\vec{E} - k^2\vec{E} = 0}$$

Helmholtz Wave Equation for E - field

w.r.t. $\vec{E}(z,t)$

$$\boxed{\frac{\partial^2\vec{E}}{\partial z^2} - k^2\vec{E} = 0}$$

Equation (5)

$$\boxed{\vec{\nabla}\times\vec{E} = -j\omega\mu\vec{H}} \to (1) \qquad \boxed{\vec{\nabla}\times\vec{H} = \sigma\vec{E} + j\omega\varepsilon\vec{E}} \to (2) \qquad \boxed{\vec{\nabla}\cdot\vec{E} = 0} \to (3) \qquad \boxed{\vec{\nabla}\cdot\vec{H} = 0} \to (4)$$

Similarly for H-field:

Take curl on both sides of (2)

Vector Identity:

$$\vec{\nabla}\times\vec{\nabla}\times\vec{H} = \vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H}$$

$$\underbrace{\vec{\nabla}\times\vec{\nabla}\times\vec{H}}_{} = \vec{\nabla}\times(\sigma\vec{E} + j\omega\varepsilon\vec{E})$$

From the vector identity

$$\vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{H}}_{0}) - \nabla^2\vec{H} = \sigma(\underbrace{\vec{\nabla}\times\vec{E}}_{}) + j\omega\varepsilon(\underbrace{\vec{\nabla}\times\vec{E}}_{})$$

From (4) 0 From (1)

$$-\nabla^2\vec{H} = \sigma(-j\omega\mu\vec{H}) + j\omega\varepsilon(-j\omega\mu\vec{H})$$

$$\nabla^2\vec{H} = j\omega\mu(\sigma + j\omega\varepsilon)\vec{H}$$

where $k = \gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

Propagation constant $\boxed{k = \gamma = \alpha + j\beta}$

α: Attenuation constant

β: Phase Constant

$$\nabla^2\vec{H} = k^2\vec{H}$$

$$\boxed{\nabla^2\vec{H} - k^2\vec{H} = 0}$$

Helmholtz Wave Equation for H - field

w.r.t. $\vec{H}(z,t)$

$$\boxed{\frac{\partial^2\vec{H}}{\partial z^2} - k^2\vec{H} = 0}$$

Equation (6)

Solution for Helmholtz Wave Equations for E-field :

$$\boxed{\vec{E}(z,t) = E(z)\, e^{j\omega t}\, \vec{a_x}}$$

$$\frac{\partial^2 E}{\partial z^2} - k^2 E = 0$$

$$\sqrt{k^2} = \pm jk$$

$$E(z) = \underbrace{E_0\, e^{-jkz}}_{\text{Fwd wave}} + \underbrace{E_0'\, e^{+jkz}}_{\text{Backward wave}}$$

$$\boxed{\vec{E}(z,t) = \left[E_0\, e^{-jkz} e^{j\omega t} + E_0'\, e^{jkz} e^{j\omega t}\right]\vec{a_x}}$$

Similarly, Solution for Helmholtz Wave Equations for H-field can be given as :

$$\boxed{\vec{H}(z,t) = H(z)\, e^{j\omega t}\, \vec{a_y}}$$

$$\frac{\partial^2 H}{\partial z^2} - k^2 H = 0$$

$$\sqrt{k^2} = \pm jk$$

$$H(z) = \underbrace{H_0\, e^{-jkz}}_{\text{Fwd wave}} + \underbrace{H_0'\, e^{+jkz}}_{\text{Backward wave}}$$

$$\boxed{\vec{H}(z,t) = \left[H_0\, e^{-jkz} e^{j\omega t} + H_0'\, e^{jkz} e^{j\omega t}\right]\vec{a_y}}$$

$$\boxed{\vec{\nabla}\times\vec{E} = -j\omega\mu\vec{H}} \quad \boxed{\vec{\nabla}\times\vec{H} = \sigma\vec{E} + j\omega\epsilon\vec{E}} \quad \boxed{\vec{\nabla}\cdot\vec{E} = 0} \quad \boxed{\vec{\nabla}\cdot\vec{H} = 0}$$

↳ ① ↳ ② ↳ ③ ↳ ④

Relation between E and H:

Use ① to get relation b/w $\vec{E}$ & $\vec{H}$

$$\underbrace{\vec{\nabla}\times\vec{E}} = -j\omega\mu\vec{H}$$

$$LHS = \vec{\nabla}\times\vec{E} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E(z,t) & 0 & 0 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{E} = \vec{a_x}(0-0) - \vec{a_y}\left(0 - \frac{\partial}{\partial z}E\right) + \vec{a_z}(0-0)$$

$$\vec{\nabla}\times\vec{E} = \frac{\partial E}{\partial z}\hat{a_y}$$

$$\vec{E}(z,t) = \left[E_0\, e^{-jkz} e^{j\omega t} + E_0'\, e^{jkz} e^{j\omega t}\right]\vec{a_x}$$

$$\boxed{LHS = \vec{\nabla}\times\vec{E} = \left[E_0\, e^{j\omega t}\cdot e^{-jkz}(-jk) + E_0' e^{j\omega t} e^{jkz}(jk)\right]\vec{a_y}}$$

$$RHS = -j\omega\mu\vec{H}$$

$$\boxed{RHS = \left(-j\omega\mu H_0 e^{j\omega t} e^{-jkz} - j\omega\mu H_0' e^{j\omega t} \cdot e^{jkz}\right)\vec{a_y}}$$

Equating LHS and RHS, gives the following relations:

$$E_0(-jk) = -j\omega\mu H_0 \quad \& \quad E_0'(jk) = -j\omega\mu H_0'$$

$$\boxed{\eta = \frac{E_0}{H_0} = \frac{j\omega\mu}{jk}} \qquad \boxed{\eta' = \frac{E_0'}{H_0'} = -\frac{j\omega\mu}{jk}}$$

$$\eta = \frac{E_0}{H_0} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}}$$

$$\boxed{\eta = \frac{E_0}{H_0} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}} \quad \Omega$$

Intrinsic Impedance of the medium

For free space, $\varepsilon = \varepsilon_0, \ \mu = \mu_0, \ \sigma = 0$

$$\eta_0 = \sqrt{\frac{\cancel{j\omega}\mu_0}{\cancel{j\omega}\varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{1}{36\pi \times 10^{9}}}} = \sqrt{4\pi \times 36\pi \times 10^{2}}$$

$$\boxed{\begin{aligned} \eta_0 &= 120\pi \ \Omega \\ \eta_0 &\simeq 377 \ \Omega \end{aligned}}$$

Intrinsic Impedance for Free Space

Q. No. 6 b)

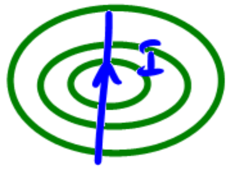
Four Equations

List the Maxwell's Equations in Integral form and differential (point) form:

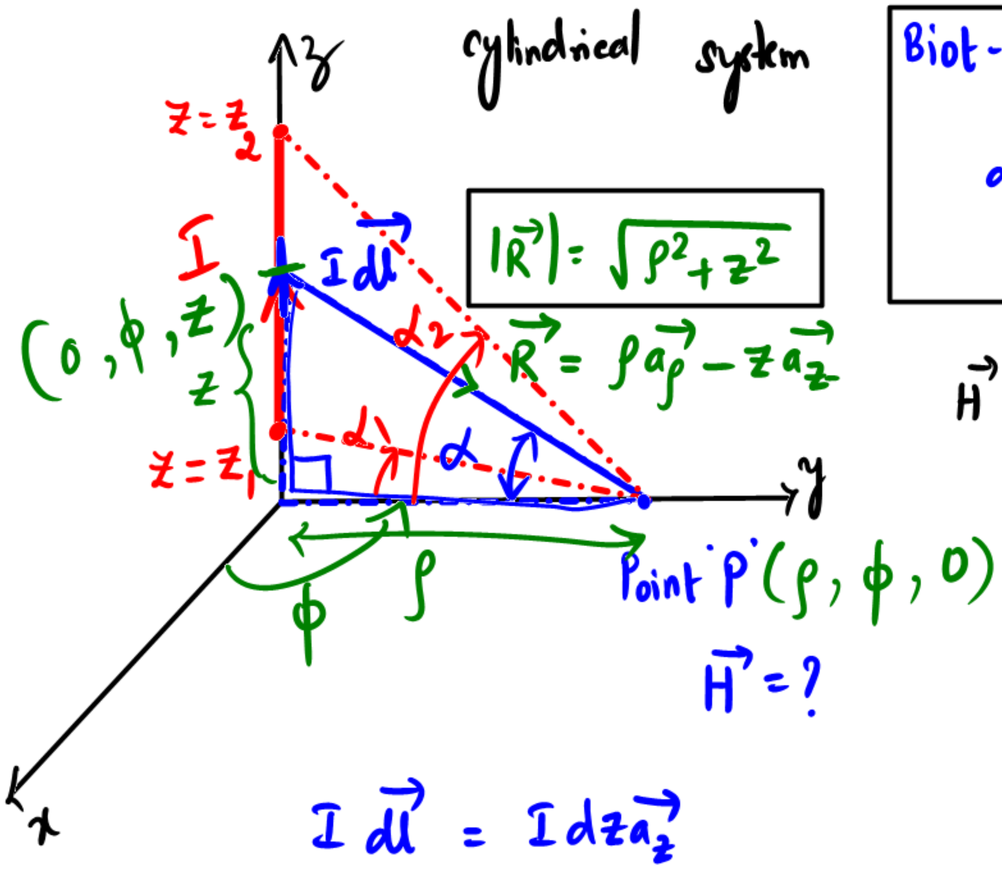
	Integral form	Point form
Faraday's law	$\oint_L \vec{E}\cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$	$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere's circuital Law	$\oint_L \vec{H}\cdot d\vec{l} = \iint_S \sigma\vec{E}\cdot d\vec{s} + \iint_S \frac{\partial \vec{D}}{\partial t}\cdot d\vec{s}$	$\vec{\nabla}\times\vec{H} = \vec{J_c} + \vec{J_D}$ $\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \frac{\partial \vec{D}}{\partial t}$
Gauss's law ($\vec{E}$- field)	$\oiint_S \vec{D}\cdot d\vec{s} = Q_{enc} = \iiint_V \rho_v \, dv$	$\vec{\nabla}\cdot\vec{D} = \rho_v$
Gauss's law ($\vec{H}$- field)	$\oiint_S \vec{B}\cdot d\vec{s} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$

Q. No. 7)

Magnetic Field Intensity due to a conductor carrying current of 'I' amperes
i) Finitely Long Conductor
ii) Infinitely Long Conductor



cylindrical system



$$|\vec{R}| = \sqrt{\rho^2 + z^2}$$

$$\vec{R} = \rho \vec{a_\rho} - z\vec{a_z}$$

Biot-Savart law:

$$d\vec{H} = \frac{1}{4\pi}\frac{I\vec{dl} \times \vec{R}}{|\vec{R}|^3} \ \text{A/m}$$

$$\vec{H} = \int_L d\vec{H}$$

$$I\vec{dl} = I\,dz\,\vec{a_z}$$

$$I\vec{dl} \times \vec{R} = \begin{vmatrix} \vec{a_\rho} & \vec{a_\phi} & \vec{a_z} \\ 0 & 0 & Idz \\ \rho & 0 & -z \end{vmatrix} = \vec{a_\rho}(0-0) - \vec{a_\phi}(0 - \rho I dz) + \vec{a_z}(0-0)$$

$$\boxed{I\vec{dl} \times \vec{R} = I\rho\,dz\,\vec{a_\phi}}$$

$$d\vec{H} = \frac{I\rho\,dz\,\vec{a_\phi}}{4\pi(\rho^2 + z^2)^{3/2}}$$

$$\vec{H} = \int_L d\vec{H} = \int_{z=z_1}^{z_2} \frac{I\rho\,dz\,\vec{a_\phi}}{4\pi(\rho^2+z^2)^{3/2}}$$

Substitution :

$$z = \rho \tan\alpha$$

$$dz = \rho \sec^2\alpha \, d\alpha$$

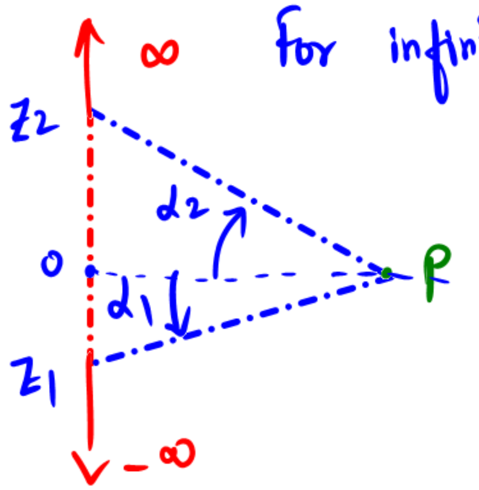
z:	z_1	z_2
α:	α_1	α_2

$$\vec{H} = \int_{\alpha=\alpha_1}^{\alpha_2} \frac{I \rho^2 \sec^2\alpha \, d\alpha \, \vec{a_\phi}}{4\pi \left(\rho^2 + \rho^2 \tan^2\alpha\right)^{3/2}}$$

$$\vec{H} = \frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{\sec\alpha} \vec{a_\phi} = \frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha \, \vec{a_\phi}$$

$$\boxed{\vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right] \vec{a_\phi}}$$

$\vec{H}$ due to finitely long conductor



For infinitely long conductor,

$$z = \rho \tan\alpha$$

$$\alpha = \tan^{-1}(z/\rho)$$

if $z_1 \to -\infty \Rightarrow \alpha_1 = -\pi/2$

if $z_2 \to \infty \Rightarrow \alpha_2 = \pi/2$

$$\vec{H} = \frac{I}{4\pi\rho}\left[1 - (-1)\right] \vec{a_\phi} \Rightarrow \boxed{\vec{H} = \frac{I}{2\pi\rho} \vec{a_\phi}}$$

$\vec{H}$ due to infinitely long conductor