VtU Solution

Ques-1 (a)

In binary phase shift keying (BPSK), bit '1' and bit '0' are represented by the following symbols.

Bit 1:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \quad 0 \le t \le T_b$$

$f_c = \frac{n}{T_b}$

n - nonzero integer

T_b - bit duration

Bit 0:

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi), \quad 0 \le t \le T_b$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t); \quad 0 \le t \le T_b$$

To find basis function

$$\text{Energy of } s_1(t) = \int_0^{T_b} |s_1(t)|^2 dt$$

$$= \int_0^{T_b} \frac{2E_b}{T_b} \cos^2(2\pi f_c t)\, dt$$

$$= \frac{2E_b}{T_b} \int_0^{T_b} \frac{1 + \cos(4\pi f_c t)}{2}\, dt$$

Probability of Error

$$= E_b$$

$\therefore$ Basis function, $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_b}}$

$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

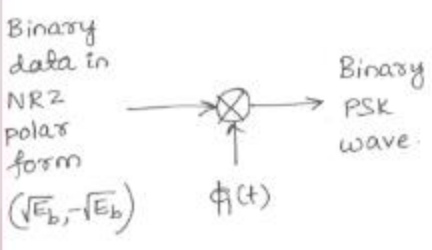
$$0 \leq t \leq T_b$$

$\therefore \quad s_1(t) = \sqrt{E_b}\, \phi_1(t), \quad 0 \leq t \leq T_b$

$\quad s_2(t) = -\sqrt{E_b}\, \phi_1(t), \quad 0 \leq t \leq T_b$

Signal-space diagram

Block diagram of transmitter.



Suppose that bit '0' was transmitted.
ie, $s_2(t)$ was transmitted.
Then, from the block diagram of the receiver, we may write,
output of the correlator,

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$$

$$= \int_0^{T_b} \left[s_2(t) + w(t)\right] \phi_1(t)\,dt$$

$$= \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt + \int_0^{T_b} w(t)\,\phi_1(t)\,dt$$

$$= -\sqrt{E_b} + w_1 \quad \cdots (2)$$

coordinate of $s_2(t)$.

Mean of x_1 when '0' was transmitted.

$$\mu = E[x_1]$$

$$= -\sqrt{E_b} \quad \cdots (3)$$

Variance of x_1 when '0' was transmitted,

$\sigma^2 = VAR[W_1]$

$= \frac{N_0}{2} \quad \cdots (4)$

($\because$ variance does not change by the addi-tion of a constant to a random varia-ble)

$\therefore$ Probability density function (PDF) of output of correlator when bit '0' was transmitted,

$$f_{X_1}(x_1/0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1+\sqrt{E_b})^2}{N_0}} \quad \cdots (5)$$

Wrong decision is made when $s_2(t)$ was transmitted and $x_1 > 0$.

$\therefore$ Probability of error when bit '0' was transmitted,

$$P_e(0) = P(X_1 > 0 / 0)$$

$$= \int_0^\infty f_{X_1}(x_1/0)\, dx_1$$

$$= \int_0^\infty \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(x_1+\sqrt{E_b})^2}{N_0}} dx_1 \quad \ldots (6)$$

We know that,

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{z^2}{2}} dz \quad \ldots (7)$$

Let us represent $P_e(0)$ in terms of Q function.

Put $$\frac{(x_1+\sqrt{E_b})^2}{N_0} = \frac{z^2}{2} \quad \ldots (8)$$

ie, $$\frac{x_1+\sqrt{E_b}}{\sqrt{N_0}} = \frac{z}{\sqrt{2}}$$

$$\therefore \frac{dx_1}{\sqrt{N_0}} = \frac{dz}{\sqrt{2}}$$

$$\therefore dx_1 = \sqrt{\frac{N_0}{2}} dz \quad \ldots (9)$$

when $x_1 = 0$, $z = \sqrt{\frac{2E_b}{N_0}}$ ... (10)

when $x_1 = \infty$, $z = \infty$ ... (11)

Using (8), (9), (10), (11), we may write (6) as

(17)

$$P_e(0) = \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{z^2}{2}} \sqrt{\frac{N_0}{2}}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} e^{-\frac{z^2}{2}}\, dz$$

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (12)$$

Similarly, we may prove that, probability of error when bit '1' was transmitted,

$$P_e(1) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (13)$$

$\therefore$ Average probability of error

$$= \frac{1}{2} P_e(0) + \frac{1}{2} P_e(1)$$

(Assuming equiprobable 0s & 1s)

$$= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \ldots (14)$$

In M-ary PSK, phase of the carrier takes one of the M-possible values.

ie. $\theta_i = \frac{2\pi}{M}(i-1)$, $i = 1, 2, \cdots M$

Accordingly M-ary PSK modulated signal can be written as follows

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \frac{2\pi}{M}(i-1)\right] , \quad 0 \le t \le T$$

$i = 1, 2, \cdots M-1, M$.

E is the symbol energy

T is the symbol duration.

$f_c = \frac{n}{T}$ where n is a non-zero integer.

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\frac{2\pi}{M}(i-1)\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[\frac{2\pi}{M}(i-1)\right] \sin(2\pi f_c t)$$

$$0 \le t \le T$$

Basis functions are.

(43)

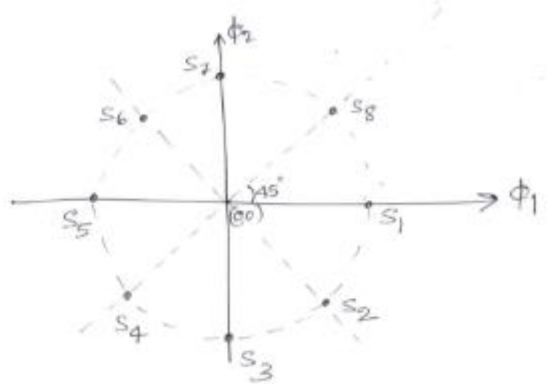
$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$, $0 \le t \le T$

$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$, $0 \le t \le T$

The coordinates of $s_i(t)$ are

$$\begin{bmatrix} \sqrt{E} \cos\left[\frac{2\pi}{M}(i-1)\right] \\ -\sqrt{E} \sin\left[\frac{2\pi}{M}(i-1)\right] \end{bmatrix}, \; i = 1, 2, \dots M.$$

Signal space diagram for $M = 8$



Note that there are 8 possible symbols

$\therefore$ We can transmit 3 bits at a time

Module-2

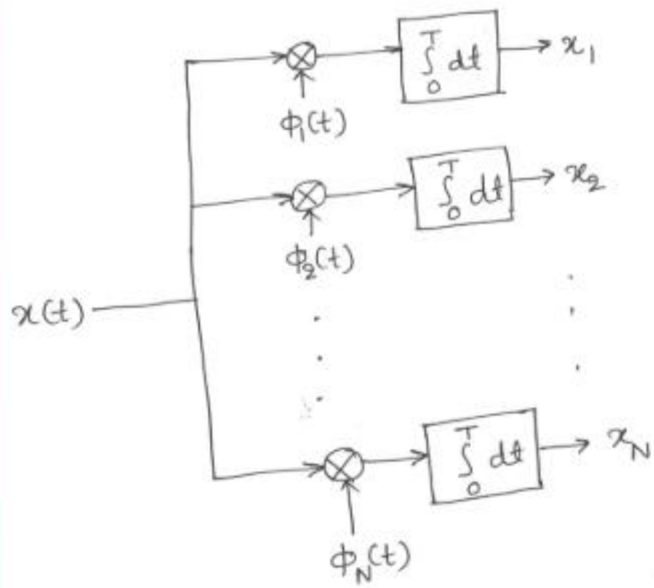
Q-3 (a)

The optimum receiver for an AWGN chan-nel and for the case when the trans-smitted signals $s_1(t), s_2(t), \ldots s_M(t)$ are equally likely is called correla-tion receiver.

It consists of 2 subsystems.

1. Detector which consists of N correl-ators supplied with orthonormal basis functions $\phi_1(t), \phi_2(t), \ldots \phi_N(t)$ that are generated locally.
   This bank of correlators operates on the received signal $x(t), 0 \le t \le T$ to produce the observation vector X.

(52)



2. Maximum likelihood decoder which operates on the observation vector x to produce an estimate $\hat{m}$ of the transmitted symbol m_i, $i = 1, 2, \ldots M$ in such a way that average probability of symbol error is minimized.

3 (c)

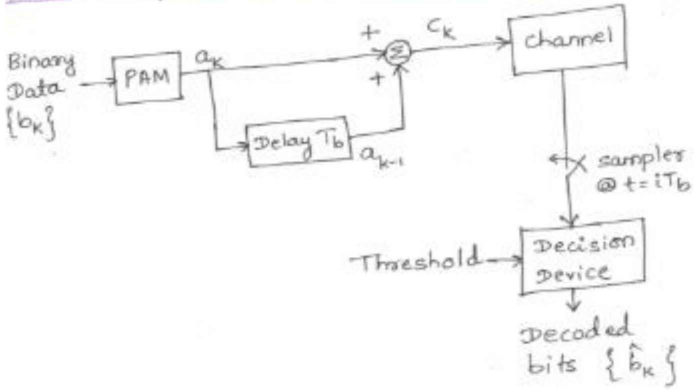
Signals with controlled

By adding inter symbol interference to the transmitted signal in a controlled manner, it is possible to achieve a bit rate of $2B_0$ bits per second in a channel of bandwidth B_0 Hz.

Such schemes are called correlative coding schemes.

Duobinary coding is one such method.

Block diagram of a duobinary system



$$a_k = PAM(b_k)$$

$$= \begin{cases} +1V & \text{if } b_k = 1 \\ -1V & \text{if } b_k = 0 \end{cases} \quad \dots (1)$$

The output of duobinary coder,

$$c_k = a_k + a_{k-1} \quad \dots (2)$$

We assume an ideal channel with frequency response.

$$H_c(f) = T_b \ , \quad -\frac{R_b}{2} \leq f \leq \frac{R_b}{2} \quad \dots (3)$$

The overall frequency response of duobinary system is given by,

$$H(f) = \left[1 + e^{-j2\pi f T_b}\right] H_c(f)$$

$$= \left[1 + e^{-j2\pi f T_b}\right] T_b \ , \quad -\frac{R_b}{2} \leq f \leq \frac{R_b}{2} \quad \dots (4)$$

$$\therefore \ H(f) = e^{-j\pi f T_b} \left[e^{j\pi f T_b} + e^{-j\pi f T_b}\right] T_b$$

$$= e^{-j\pi f T_b} \, 2\cos(\pi f T_b) T_b$$

$$-\frac{R_b}{2} \leq f \leq \frac{R_b}{2} \quad \dots (5)$$

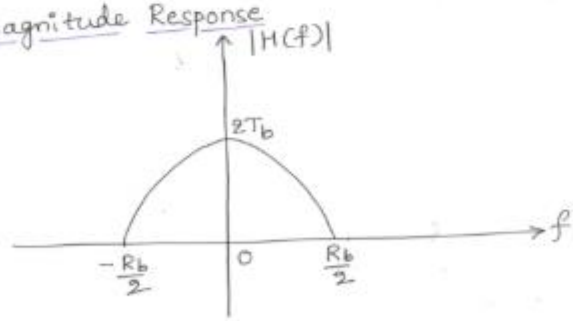
∴ Magnitude response,

$$|H(f)| = 2T_b \cos(\pi f T_b), \quad -\frac{R_b}{2} \le f \le \frac{R_b}{2}$$

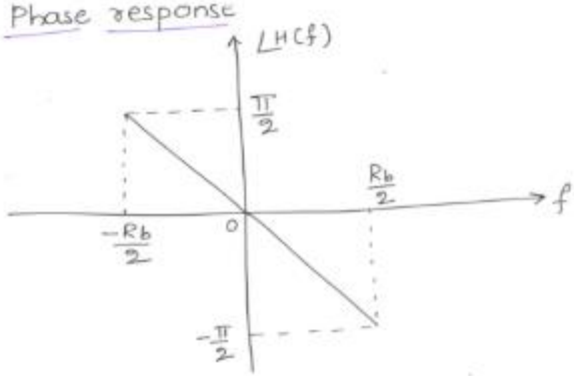
Phase response,

$$\angle H(f) = -\pi f T_b, \quad -\frac{R_b}{2} \le f \le \frac{R_b}{2}$$

Magnitude Response



Phase response



Note that $|H(f)| \neq 0$ @ $f = 0$

⇒ The system produces DC component.

The impulse response of the duobinary system can be found by computing IFT of frequency response given by (4). (19)

$$\therefore h(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi f t}\, df$$

$$= \int_{-\frac{R_b}{2}}^{\frac{R_b}{2}} \left[1 + e^{-j2\pi f T_b}\right] T_b\, e^{j2\pi f t}\, df$$

$$= T_b \int_{-\frac{R_b}{2}}^{\frac{R_b}{2}} e^{j2\pi f t}\, df + T_b \int_{-\frac{R_b}{2}}^{\frac{R_b}{2}} e^{j2\pi f (t-T_b)}\, df$$

$$= T_b \left. \frac{e^{j2\pi f t}}{j2\pi t} \right|_{-\frac{R_b}{2}}^{\frac{R_b}{2}} + T_b \left. \frac{e^{j2\pi f (t-T_b)}}{j2\pi (t-T_b)} \right|_{-\frac{R_b}{2}}^{\frac{R_b}{2}}.$$

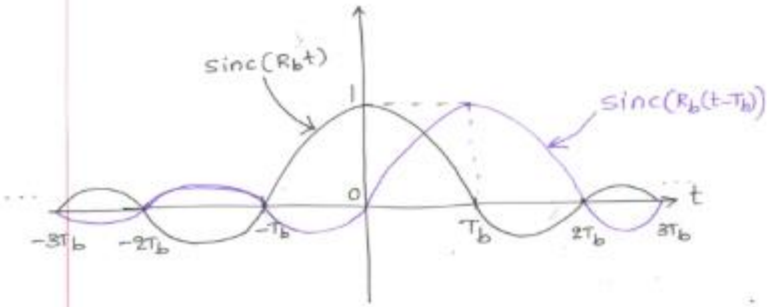
$$= \frac{T_b}{j2\pi t} \left[e^{j\pi R_b t} - e^{-j\pi R_b t} \right] + \frac{T_b}{j2\pi (t-T_b)} \left[e^{j\pi R_b (t-T_b)} - e^{-j\pi R_b (t-T_b)} \right]$$

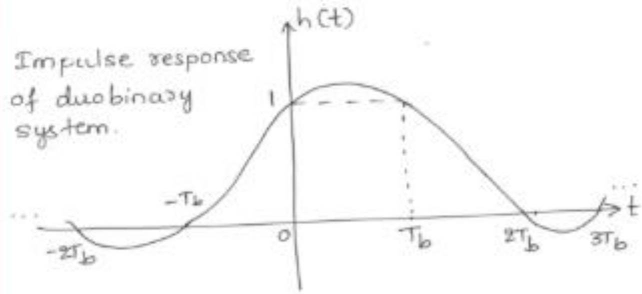
$$= \frac{T_b}{j2\pi t}\, 2j \sin(\pi R_b t) + \frac{T_b}{j2\pi (t-T_b)}\, 2j \sin\left(\pi R_b (t-T_b)\right)$$

$$= \text{sinc}(R_b t) + \text{sinc}(R_b(t-T_b)) \quad \ldots (6)$$

This is the impulse response of the duobinary system.

To plot $h(t)$ v/s t.





From (2),

$$C_K = a_K + a_{K-1}$$

@ Receiver,

we evaluate,

$$\hat{a}_k = \hat{c}_k - \hat{a}_{k-1} \quad \dots (7)$$

Decision rule:

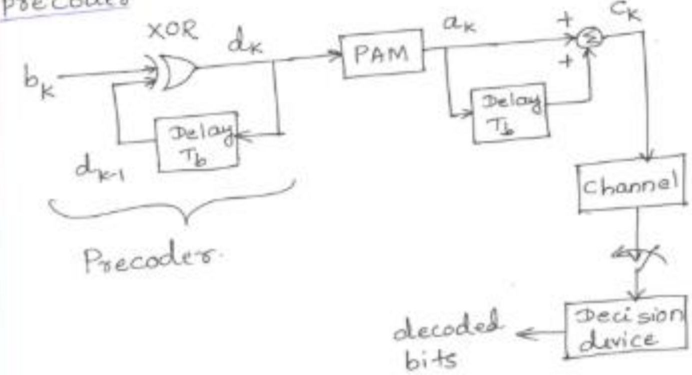
$$\hat{b}_k = \begin{cases} 1 & \text{if } \hat{a}_k = 1 \\ 0 & \text{if } \hat{a}_k = -1 \end{cases} \quad \dots (8)$$

Note that the decision on the current bit depends on previous decision.

∴ In the detection process, once an error is made, the error tends to propagate.

This can be avoided using a precoder before duobinary coder.

Block diagram of duobinary coder with precoder



Precoder output, $d_k = b_k + d_{k-1}$ ...(9)

$$a_k = \text{PAM}(d_k)$$

$$= \begin{cases} +1V & \text{if } d_k = 1 \\ -1V & \text{if } d_k = 0 \end{cases} \quad \ldots (10)$$

$$c_k = a_k + a_{k-1} \quad \ldots (11)$$

$$c_k = \begin{cases} \pm 2V & \text{if } b_k = 0 \\ 0V & \text{if } b_k = 1 \end{cases} \quad \ldots (12)$$

(see problem#8)

Decision rule @ receiver.

$$\hat{b}_k = \begin{cases} 0 & \text{if } |c_k| > 1V \\ 1 & \text{if } |c_k| \leq 1V \end{cases} \quad \ldots (13)$$

Note that present decision depends only on present amplitude but not on previous decision.

∴ If an error is made, it will not affect the future decisions

Hence, error propagation can be prevented

Note: Duobinary coding is also called Partial Response Signalling

Module-3

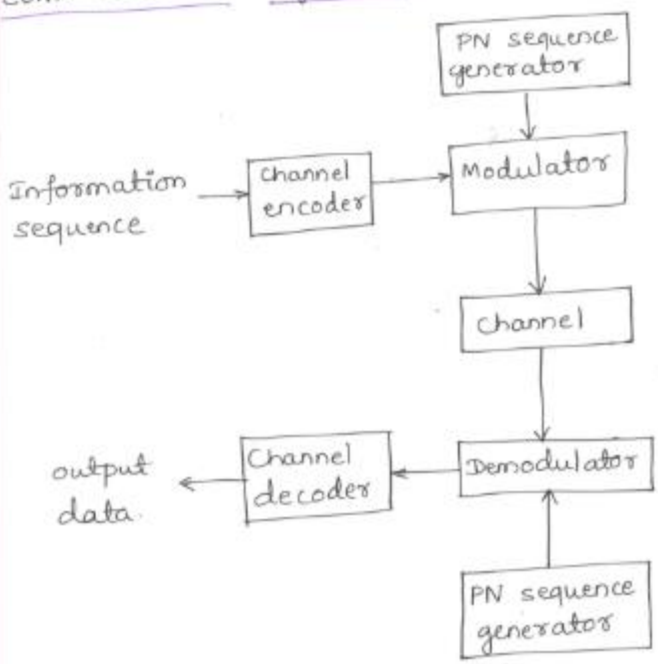
5 (a) Model of spread spectrum digital communication system.

spread spectrum communication is a means of communication in which the transmitted signal occupies a bandwidth in excess of minimum bandwidth necessary to send the data.

The spectrum spreading is accomplished before transmission through the use of a code called Pseudo-Noise (PN)

sequence.
The same code is used in the receiver to despread the received signal so that the original data may be recovered.

Model of a spread spectrum digital communication system



Prior to the transmission of information

a fixed PN sequence is transmitted ③
so as to achieve synchronization of PN sequence generators at the transmitter and at the receiver.
Transmission of information commences only after that.

Advantages of spread spectrum communication

The primary advantage of spread spectrum communication is its ability to reject interference, be it the unintentional interference of another user simultaneously attempting to transmit through the channel or intensional interference of a hostile transmitter attempting to jam the transmission.
This technique can also provide multipath rejection in a ground based mobile radio environment.

Also, this technique can provide (4)
multiple access communication in which
a number of independent users can sha-
-re a common channel without an
external synchronizing mechanism.
There are 2 types of spread spectrum
techniques.

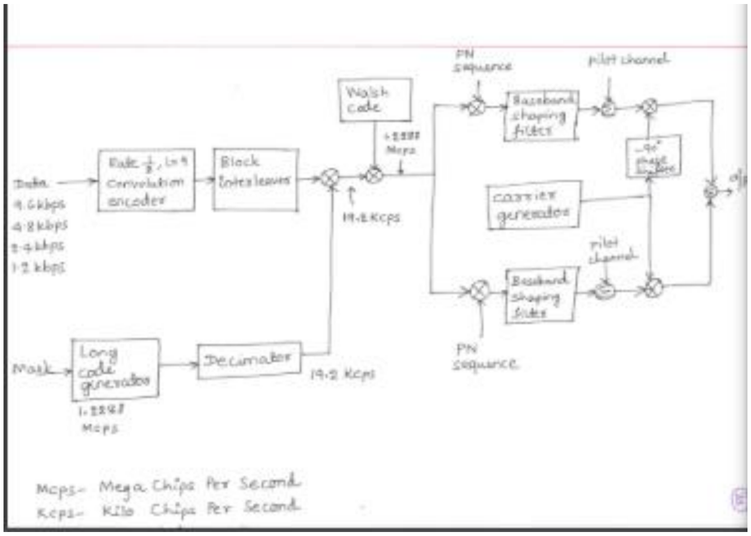
1. Direct Sequence Spread Spectrum (DSSS)

In DSSS technique, two stages of modul-
-ation are used
First, the incoming data sequence is
used to modulate a wideband code
called PN sequence.
This code transforms the narrowband
data sequence into a noise-like wide
-band signal.
The resulting wideband signal under-
-goes a second modulation using
phase shift keying technique.

2. Frequency Hopped Spread Spectrum (FHSS)

In FHSS technique, spectrum spreading
is achieved by changing the carrier (5)
frequency in a pseudo random manner.

5(b) CDMA system on IS-95



(54)

The speech coder is a code excited linear predictive (CELP) coder that generates data at the variable rates of 9.6, 4.8, 2.4 and 1.2 kbps.

This data is encoded by rate 1/2, constraint length L=9 convolutional code.

The encoded bits are transmitted through block interleaver to overcome the effect of burst error.

Each user is assigned a Walsh (Hadamard) code of length 64.

There are 64 orthogonal Walsh codes assigned to each base station, hence, there are 64 channels available.

The encoded data is multiplied with Walsh code.

The resulting sequence is used to modulate PN sequences of length 2^{15}, creating in-phase and quadrature compo-

35

-nents.

At the receiver, a RAKE demodulator resolves major multipath signal compo-nents.

These components are passed to the Viterbi soft decision decoder.

Note: The block diagram shown on page 33 is the block diagram of IS-95 forward link.

Forward link means channel from base station to mobile phone.

The channel from mobile phone to base station is called reverse link.

Module-4

7 (a)

MEASURE OF INFORMATION :

The source encoder replaces the symbols by bits. Hence, the goal here is to evaluate the rate of info. at the source, and to evaluate the max. transmission rate. They will not be same, when varied length coding is utilized by the source encoder. Hence, we make use of a term called "Entropy," which is the avg. info. content / symbol. It is also called as the avg. unpredictability in a random variable. The avg. info. rate is defined as the minimum avg. no. of bits /sec., needed to represent the o/p of the source.

Entropy, H = bits / symbol

Source info. rate = symbol-rate × Entropy

Source info. rate = symbol-rate × Entropy

i.e, $R = r_s \times H = \frac{\text{symbol}}{\text{sec}} \times \frac{\text{bits}}{\text{symbol}}$

$$\therefore \boxed{R = r_s \times H} \text{ bits / sec.}$$

Note: Channel capacity is defined as the rate of data transfer over the channel, with an arbitrarily small probability of error.

Shannon's formula for the same is-

$$\text{ch. capacity, } C = B. \log_2\left(1 + S/N\right)$$

where
B = Bandwidth of ch.
S = Avg. signal power
N = Avg. Noise power

The total info. content is given by,

$$I_{total} = N \sum_{i=1}^{M} p_i \cdot \lg(1/p_i)$$

Hence, the avg. info. per symbol is given by –

$$\frac{I_{total}}{N} = \sum_{i=1}^{M} p_i \cdot \lg(1/p_i) \text{ bits/symbol.}$$

This is called as "Average self-info." or "Source entropy", which is utilized to predict the info. rate from the given source.

$$\therefore \boxed{\text{Entropy, } H = \sum_{i=1}^{M} p_i \cdot \lg(1/p_i)}$$

Entropy represents the avg. uncertainty of the source, and hence it is also called as "avg. self-information". It is used to obtain the avg. no. of bits per symbol, in long messages. The avg. info. rate is given by –

$$\boxed{R = r_s \times H}$$

where r_s is the symbol rate.

PROPERTIES OF ENTROPY:

① The entropy function is continuous for every independent variable p, in the interval (0,1).

② The entropy function is a symmetrical fn of its arguments. i.e, $H[P,(1-P)] = H[(1-P),P]$.

③ The entropy attains a maximum value when all the source symbols become equiprobable. i.e, $H_{max} = \lg(M)$ bits/symbol.

④ The entropy does not get reduced, when the symbols are split into sub-symbols.

Note: The third property is called as "Extremal property", and the fourth property is called as the "Additivity property".

The efficiency of the source is given by, $\boxed{\eta_s = \frac{H}{H_{max}}}$. Therefore, the source redundancy is given by, $\boxed{R_{\eta_s} = 1 - \eta_s}$.