Fifth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Electromagnetic Waves

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. State and explain coulomb's law of force between two point charges in vector form. **(06 Marks)**

b. Convert point P(1, 3, 5) to cylindrical and spherical co-ordinates. Also write the equations for differential surface, differential volume for rectangular, cylindrical and spherical systems. **(06 Marks)**

c. Find electric field intensity at P(1, 1, 1) caused by 4 identical 3nc charges are located at $P_1(1, 1, 0)$, $P_2(-1, 1, 0)$, $P_3(-1, -1, 0)$ and $P_4(1, -1, 0)$. **(08 Marks)**

OR

2 a. Define electric field intensity. Derive an expression for electric field intensity due to infinite line charge. **(08 Marks)**

b. A point charge of 50nc each are located at A(1, 0, 0), B(-1, 0, 0), C(0, 1, 0) and D(0, -1, 0) in free space. Find the total force on the charge at A. Also find $\overline{E}$ at A. **(06 Marks)**

c. A uniform line charge ρ_L = 25nc/m lies on the line x = -3m, y = 4m in freespace. Find electric field intensity at a point (2, 3, 15)m. **(06 Marks)**

Module-2

3 a. State and prove Gauss's law. **(06 Marks)**

b. Evaluate both sides of the divergence theorem for the defined plane in which $1 \le x \le 2$, $2 \le y \le 3$, $3 \le z \le 4$, if $\overline{D} = 4x\,\hat{a}_x + 3y^2\,\hat{a}_y + 2z^3\,\hat{a}_z\ c/m^2$. **(10 Marks)**

c. Derive the point form of continuity of current equation. **(04 Marks)**

OR

4 a. Obtain the expression for the work done in moving a point charge in an electric field. **(06 Marks)**

b. Given that the field $\overline{D} = \dfrac{5\sin\theta\ \cos\phi}{r}\hat{a}_r\ c/m^2$. Find : i) Volume charge density ii) The tota[l] electric flux leaving the surface of the spherical volume of radius 2m. **(08 Marks**

c. Define potential difference. Derive the expression for potential field of a point charge. **(06 Marks**

Module-3

[5] a. State and prove uniqueness theorem. **(08 Marks**

b. Define Stoke's theorem. Use this theorem to evaluate both sides of the theorem for the fiel[d] $\overline{H} = 6xy\,\hat{a}_x - 3y^2\,\hat{a}_y$ A/m and the rectangular path around the region, $2 \le x \le 5$, $-1 \le y \le$ and z = 0. Let the positive direction of ds be $\hat{a}_z$. **(12 Marks**

OR

6 a. Solve the Laplace's equation for the potential field in the homogeneous region between the two concentric conducting spheres with radii 'a' and 'b' such that b > a, if potential $v = 0$ at $r = b$ and $v = v_0$ at $r = a$. Also find the capacitance between concentric spheres. **(08 Marks)**

b. Derive the expression for magnetic field intensity due to infinite long straight conductor using Biot-Savart's law. **(06 Marks)**

c. Determine whether or not the following potential fields satisfy the Laplace's equation:
i) $V = 2x^2 - 3y^2 + z^2$ ii) $V = r\cos\theta + \phi$ **(06 Marks)**

Module-4

7 a. Derive an expression for Lorentz Force equation. **(06 Marks)**

b. If $\overline{B} = 0.05x\,\hat{a}_y$ Tesla in a material for which $\chi_m = 2.5$, Find: i) μ_r ii) μ iii) $\overline{H}$ iv) $\overline{M}$ v) $\overline{J}$ vi) $\overline{J}_b$. **(08 Marks)**

c. Derive the expression for the force between two differential current elements. **(06 Marks)**

OR

8 a. Derive the expression for the boundary conditions between two magnetic medias. **(10 Marks)**

b. Calculate the magnetization in magnetic material where:
i) $\mu = 1.8 \times 10^{5}$ H/m and $M = 120$ A/m
ii) $\mu_r = 22$, there are 8.3×10^{28} Atoms/m^3 and each atom has a dipole moment of 4.5×10^{-27} A/m^2
iii) $B = 300\ \mu$T and $\chi_m = 15$. **(06 Marks)**

c. Briefly explain the forces on magnetic materials. **(04 Marks)**

Module-5

9 a. List and explain Maxwell's equations in point form and integral form. **(08 Marks)**

b. Given $\overline{E} = E_m \sin(wt - \beta z)\,\hat{a}_y$ v/m. Find: i) $\overline{D}$ ii) $\overline{B}$ iii) $\overline{H}$. Sketch $\overline{E}$ and $\overline{H}$ at $t = 0$. **(08 Marks)**

c. Find the frequency at which conduction current density and displacement current density are equal in a medium with $\sigma = 2 \times 10^{-4}$ mho/m and $\epsilon_r = 81$. **(04 Marks)**

OR

10 a. State and prove Poynting theorem. **(08 Marks)**

b. For the given medium $\epsilon = 4 \times 10^{-9}$ F/m and $\sigma = 0$, find 'K' so that $\overline{E} = (20y - kt)\hat{a}_x$ v/m and $\overline{H} = (y + 2\times10^{6}t)\,\hat{a}_z$ A/m. **(06 Marks)**

c. A uniform plane wave of frequency 10MHz travels in positive direction in a lossy medium with $\epsilon_r = 2.5$, $\mu_r = 4$ and $\sigma = 10^{-3}$ ℧/m. Calculate α, β, γ and η, λ. **(06 Marks)**

* * * * *

1.(a)State and explain Coulomb's law of force between two point charges in vector form.

Coulomb stated that the force between two very small objects separated in a vacuum or free space by a distance, which is large compared to their size, is proportional to the charge on each and inversely proportional to the square of the distance between them, or

$$F = k\frac{Q_1 Q_2}{R^2}$$

where Q_1 and Q_2 are the positive or negative quantities of charge, R is the separation, and k is a proportionality constant. If the International System of Units[1] (SI) is used, Q is measured in coulombs (C), R is in meters (m), and the force should be newtons (N). This will be achieved if the constant of proportionality k is written as

$$k = \frac{1}{4\pi\epsilon_0}$$

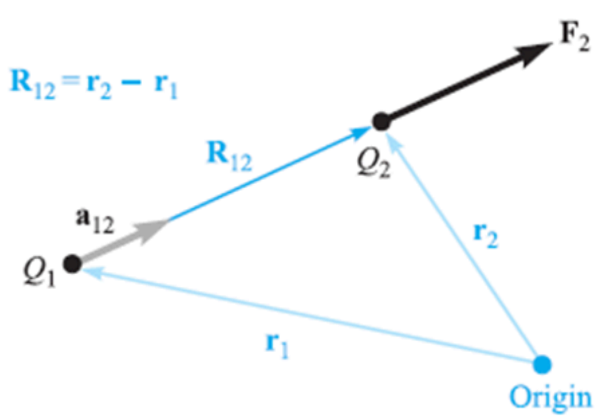
The new constant ϵ_0 is called the *permittivity of free space* and has magnitude, measured in farads per meter (F/m),

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi}10^{-9} \text{ F/m}} \qquad (1)$$

The quantity ϵ_0 is not dimensionless, for Coulomb's law shows that it has the label $C^2/N \cdot m^2$. We will later define the farad and show that it has the dimensions $C^2/N \cdot m$; we have anticipated this definition by using the unit F/m in equation (1).

Coulomb's law is now

$$\boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}} \qquad (2)$$



1. (b)

1.(b)

$P(1, 3, 5)$

$\therefore x = 1,\ y = 3,\ z = 5$

$\therefore$
$$\boxed{\begin{aligned} \rho &= \sqrt{x^2+y^2} = \sqrt{1^2+3^2} = \sqrt{10} = 3.162 \\ \phi &= \tan^{-1}\left(\frac{3}{1}\right) = 71.56^\circ \\ z &= 5 \end{aligned}}$$

cylindrical co-ordinates

$$\boxed{\begin{aligned} r &= \sqrt{x^2+y^2+z^2} = \sqrt{1^2+3^2+5^2} = 5.916 \\ \theta &= \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\left(\frac{5}{5.916}\right) = 32.31^\circ \\ \phi &= \tan^{-1}\left(\frac{3}{1}\right) = 71.56^\circ \end{aligned}}$$

Spherical co-ordinates

Differential Area Elements in Cartesian co-ordinates

$$\vec{ds}_{front} = dy\,dz\,\hat{a}_x$$

$$\vec{ds}_{back} = dy\,dz\,(-\hat{a}_x) = -dy\,dz\,\hat{a}_x$$

$$\vec{ds}_{top} = dx\,dy\,\hat{a}_z$$

$$\rightarrow \vec{ds}_{bottom} = dx\,dy\,(-\hat{a}_z) = -dx\,dy\,\hat{a}_z$$

$$\vec{ds}_{left} = dx\,dz\,(-\hat{a}_y) = -dx\,dz\,\hat{a}_y$$

$$\vec{ds}_{right} = dx\,dz\,\hat{a}_y$$

Volume Element in Cartesian co-ordinate System

$$dv = dx\,dy\,dz$$

Cylindrical Coordinates

$$\vec{ds}_{curvature} = \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$\vec{ds}_{top} = d\rho \cdot \rho\, d\phi\, \hat{a}_z = \rho\, d\rho\, d\phi\, \hat{a}_z$$

$$\vec{ds}_{bottom} = \rho\, d\rho\, d\phi\, (-\hat{a}_z) = -\rho\, d\rho\, d\phi\, \hat{a}_z$$

Differential Volume Element

$$dv = dz \cdot \rho\, d\phi \cdot d\rho$$

$$\boxed{dv = \rho\, d\rho\, d\phi\, dz}$$

Spherical Coordinates

$$\vec{ds} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r$$

$$dv = r^2 \sin\theta\, d\theta\, d\phi\, dr$$

1. c)

Find $\vec{E}$ at (1, 1, 1) caused by 4 identical 3nC charges located at P1 (1, 1, 0), P2 (−1, 1, 0), P3 (−1, −1, 0) and P4 (1, −1, 0).

Solution

$$\left.\begin{array}{l} P1\,(1,1,0) \\ P2\,(-1,1,0) \\ P3\,(-1,-1,0) \\ P4\,(1,-1,0) \end{array}\right\} 3\,nC$$

$\vec{E}$ at $P(1,1,1)$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 |\vec{R}|^2}\hat{a}_R \text{ V/m}$$

$$3nC \underset{(1,1,0)}{P1} \xrightarrow{\vec{R}_1} P(1,1,1)$$

$$\vec{R}_1 = (1-1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z = \hat{a}_z$$

$$|\vec{R}_1| = \sqrt{1^2} = 1$$

$$\hat{a}_R = \frac{\hat{a}_z}{|\hat{a}_z|} = \hat{a}_z$$

$$\vec{E}_1 = \frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\cdot\frac{\hat{a}_z}{1^2} = \frac{3\times10^{-9}\,\hat{a}_z}{4\pi\times8.854\times10^{-12}}$$

$$=$$

$$3nC \underset{(-1,1,0)}{P_2} \xrightarrow{\vec{R}_2} \underset{P}{(1,1,1)}$$

$$\vec{R}_2 = (2\hat{a}_x + \hat{a}_z)$$

$$|\vec{R}_2| = \sqrt{5}, \quad \hat{a}_R = \frac{(2\hat{a}_x + \hat{a}_z)}{\sqrt{5}}$$

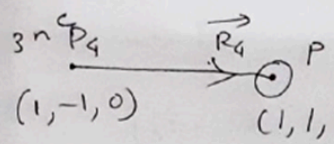
$$\therefore \vec{E}_2 = \left(\frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\right)\frac{1}{(\sqrt{5})^2}\cdot\frac{(2\hat{a}_x + \hat{a}_z)}{\sqrt{5}}$$

$$\underset{P_3}{(-1,-1,0)} \xrightarrow{\vec{R}_3} P(1,1,1)$$

$$\vec{R}_3 = 2\hat{a}_x + 2\hat{a}_y + \hat{a}_z$$

$$\therefore |\vec{R}_3| = \sqrt{2^2+2^2+1} = \sqrt{9} = 3$$

$$\therefore \vec{E}_3 = \frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\cdot\frac{1}{3^2}\cdot\frac{(2\hat{a}_x + 2\hat{a}_y + \hat{a}_z)}{3}$$



$$\vec{R}_4 = (2\hat{a}_y + \hat{a}_z)$$

$$\therefore |\vec{R}_4| = \sqrt{5}$$

$$\vec{E}_4 = \left(\frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\right)\cdot\frac{1}{(\sqrt{5})^2}\cdot\frac{(2\hat{a}_y + \hat{a}_z)}{\sqrt{5}}$$

$\therefore$ The total intensity at P,

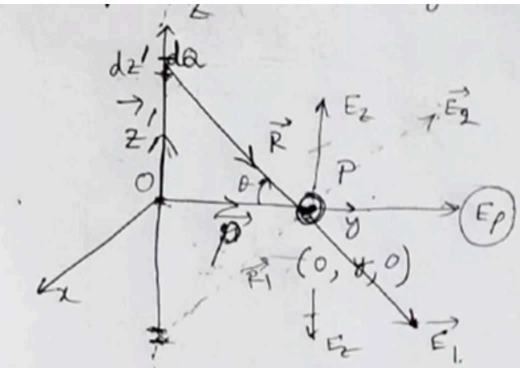
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$= \frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\left[\hat{a}_z + \frac{1}{5\sqrt{5}}(2\hat{a}_x + \hat{a}_z) + \frac{(2\hat{a}_x + 2\hat{a}_y + \hat{a}_z)}{27} + \frac{(2\hat{a}_y + \hat{a}_z)}{5\sqrt{5}}\right]$$

$$= 26.96\,[\quad\quad]$$

$$= 6.82\,\hat{a}_x + 6.82\,\hat{a}_y + 32.8\,\hat{a}_z \text{ V/m}$$

2. (a)



$\frac{z'}{\rho} = \tan\theta$

$z' = \rho\tan\theta$

ρ_L C/m

Find $\vec{E}$ at $P(0, y, 0)$ because of the line charge along z-axis.

We consider incremental charge

$dQ = \rho_L dz'$ . . . (1) [charge on length dz']

$\vec{R} = \vec{\rho} - \vec{z}'$ $\quad (\vec{z}' + \vec{R} = \vec{\rho})$

$\vec{R} = (\rho\hat{a}_\rho - z'\hat{a}_z)$ . . (2) $\quad |\vec{R}| = \sqrt{\rho^2 + z'^2}$

∴ The field at P, because of dQ,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 |\vec{R}|^2}\hat{a}_R = \frac{\rho_L dz'}{4\pi\epsilon_0(\rho^2+z'^2)} \cdot \frac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{(\rho^2+z'^2)^{1/2}}$$

$[\because |\vec{R}| = \sqrt{\rho^2 + z'^2}]$

$$= \frac{\rho_L dz' \cdot (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi\epsilon_0(\rho^2+z'^2)^{3/2}}$$

We know from the symmetry of the problem - (3)

$$\vec{dE} = \frac{\rho_L dz' \rho\hat{a}_\rho}{4\pi\epsilon_0(\rho^2+z'^2)^{3/2}}$$

∴ The field at P,

$$\vec{E} = \int d\vec{E} = \int_{z'\to-\infty}^{\infty} \frac{\rho_L\, dz'\, \rho\, \hat{a}_\rho}{4\pi\epsilon_0(\rho^2+z'^2)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2+z'^2)^{3/2}} \hat{a}_\rho$$

Note: $\tan\theta = \frac{z'}{\rho}$

$z' = \rho \tan\theta$ | $z' \to \infty,\ \theta = \pi/2$

$dz' = \rho \sec^2\theta\, d\theta$ | $z' \to -\infty,\ \theta = -\pi/2$

$$\therefore \vec{E} = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta\, d\theta}{(\rho^2+\rho^2\tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta\, d\theta}{(\rho^2)^{3/2}(1+\tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \cancel{\rho}}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\cancel{\rho} \sec^2\theta\, d\theta}{\cancel{\rho^3}\rho\,(\sec^2\theta)^{3/2}} \hat{a}_\rho$$

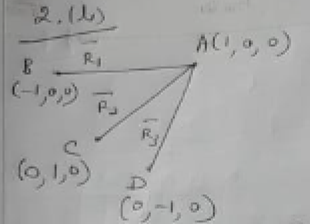
$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta\, d\theta}{\sec^3\theta} \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta}\, d\theta\, \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta\, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} [1+1]\, \hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho$$

2.(b)

2.(b)



$\bar{R}_1 = 2\,\hat{a}_x$

$\bar{R}_2 = +\hat{a}_y + \hat{a}_x$

$\bar{R}_3 = \hat{a}_x - \hat{a}_y$

$\bar{R}_2$

$= \sqrt{1^2+1^2}$

$= \sqrt{2}$

$\hat{a}_{R_2} = \dfrac{\hat{a}_x + \hat{a}_y}{\sqrt{2}}$

$$\therefore \bar{F}_1 = \frac{(50\times10^{-9})^2}{4\pi\epsilon_0\,|2|^2}\,\frac{\cancel{2}\,\hat{a}_x}{\cancel{2}}$$

$$= \frac{(50\times10^{-9})^2}{4\pi\epsilon_0}\,\frac{\hat{a}_x}{4}\ \text{N}$$

$$\bar{F}_2 = \frac{(50\times10^{-9})^2}{4\pi\epsilon_0\ 2}\cdot\frac{(\hat{a}_x + \hat{a}_y)}{\sqrt{2}}\ \text{N}$$

$$\bar{F}_3 = \frac{(50\times10^{-9})^2}{4\pi\epsilon_0\cdot 2}\cdot\frac{(\hat{a}_x - \hat{a}_y)}{\sqrt{2}}\ \text{N}$$

$$\therefore \bar{F}_{total} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

$$= \frac{(50\times10^{-9})^2}{4\pi\epsilon_0}\left[\frac{\hat{a}_x}{4} + \frac{\hat{a}_x}{2\sqrt{2}} + \cancel{\frac{\hat{a}_y}{2\sqrt{2}}} + \frac{\hat{a}_x}{2\sqrt{2}} - \cancel{\frac{\hat{a}_y}{2\sqrt{2}}}\right]$$

$$= \frac{(50\times10^{-9})^2}{4\pi\epsilon_0}\left[\frac{\hat{a}_x}{4} + \frac{\hat{a}_x}{\sqrt{2}}\right]$$

$$= \frac{(50\times10^{-9})^2}{4\pi\epsilon_0}\left(0.25\,\hat{a}_x + 0.707\,\hat{a}_x\right)$$

$$= 22.48\times10^{-6}\times0.957$$

$$= 21.51\times10^{-6}\,\hat{a}_x\ \text{N}$$

$$\therefore \bar{E} = \frac{\bar{F}}{Q} = 430.06\,\hat{a}_x\ \text{V/m}$$

2. c)

2.(c)

$\rho_L = 25\,nC/m$

Intensity at point P (2, 3, 15)

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho$$

Here, $\rho = \sqrt{(2+3)^2+(3-4)^2} = \sqrt{5^2+1^2} = \sqrt{2}$

$\because \bar{\rho} = (2+3)\hat{a}_x + (3-4)\hat{a}_y$

$= 5\hat{a}_x - \hat{a}_y$

$$\therefore \bar{E} = \frac{25\times10^{-9}}{2\pi\times8.854\times10^{-12}\times\sqrt{26}}\frac{(5\hat{a}_x-\hat{a}_y)}{\sqrt{26}}$$

$= 17.29(5\hat{a}_x - \hat{a}_y)$

$= 86.45\,\hat{a}_x - 17.29\,\hat{a}_y$ V/m

3. a)

2.(a) Derive Mathematical form of Gauss's law, and ...

Gauss's law : The electric flux passing through any closed surface is equal to the total charge enclosed by the surface

$D_s \rightarrow$ flux density at the surface

↓

varies from one point to another on the surface

$\Delta S \rightarrow$ a small portion of the surface.

$\overrightarrow{\Delta S} \rightarrow$ mag. and direction

direction normal at that point.

outward +ve for any surface.

At any pt. P consider an incremental surface ΔS.

$\overrightarrow{D_s}$ makes an angle θ with $\overrightarrow{\Delta S}$.

Then flux crossing $\overrightarrow{\Delta S}$ is then,

$$\Delta \psi = \text{flux crossing } \Delta S$$

$$= \overrightarrow{D_s} \cdot \overrightarrow{\Delta S}.$$

$\therefore$ Total flux passing through entire closed surface,

$$\psi = \int d\psi = \oint_{\text{closed surface}} \overrightarrow{D_s} \cdot d\vec{S} = \text{charge enclosed} = Q$$

This type of closed surface is a //

3. b)

3.(b) $\bar{D} = 4x\,\hat{a}_x + 3y^2\,\hat{a}_y + 2z^3\,\hat{a}_z$ C/m^2

$$\therefore \bar{\nabla}\cdot\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$= 4 + 6y + 6z^2$$

$$\therefore \iiint \bar{\nabla}\cdot\bar{D}\,dv = \int_{x=1}^{2}\int_{y=2}^{3}\int_{z=3}^{4} 4\,dx\,dy\,dz + 6\iiint y\,dx\,dy\,dz + 6\iiint z^2\,dx\,dy\,dz$$

$$= 4\cdot[x]_1^2\,[y]_2^3\,[z]_3^4 + 6\,[x]_1^2\left[\frac{y^2}{2}\right]_2^3\cdot[z]_3^4 + 6\cdot[x]_1^2\cdot[y]_2^3\cdot\left[\frac{z^3}{3}\right]_3^4$$

$$= 4\times1\times1\times1 + \overset{3}{\cancel{6}}\cdot1\cdot\frac{(9-4)}{\cancel{2}}\,1 + \overset{2}{\cancel{6}}\cdot1\cdot1\,\frac{(64-27)}{\cancel{3}}$$

$$= 4 + 15 + 2\times37$$

$$= 19 + 74 = 93\ \text{C}.$$

$$\iint_{\text{front}} \bar{D}\cdot\bar{ds} = \iint \bar{D}\cdot dy\,dz\,\hat{a}_x$$

$$= \iint 4x\,dy\,dz$$

$$= 4\times2\times[y]_2^3\,[z]_3^4$$

$$= 8\times1\times1 = 8$$

$$\iint_{\text{back}} \bar{D}\cdot\bar{ds} = \iint \bar{D}\cdot dy\,dz\,(-\hat{a}_x)$$

$$= -\iint 4x\,dy\,dz$$

$$= -4\times1\times1\times1 = -4$$

$$\iint_{right} \bar{D}\cdot\bar{ds} = \iint \bar{D}\cdot dx\,dz\,\hat{a}_y$$

$$= \iint 3y^2\,dx\,dz$$

$$= 3\cdot 3^2 \times 1 \times 1 = 27$$

$$\iint_{left} \bar{D}\cdot\bar{ds} = \iint \bar{D}\cdot dx\,dz\,(-\hat{a}_y)$$

$$= -\iint 3y^2\,dx\,dz$$

$$= -3 \times 2^2 \times 1 \times 1$$

$$= -12$$

$$\iint_{top} \bar{D}\cdot\bar{ds} = \iint \bar{D}\cdot dx\,dy\,\hat{a}_z$$

$$= \iint 2z^3\,dx\,dy$$

$$= 2\cdot 4^3\cdot 1 \times 1 = 128$$

$$\iint_{down} \bar{D}\cdot\bar{ds} = \iint \bar{D}\cdot dx\,dy\,(-\hat{a}_z)$$

$$= -\iint 2z^3\,dx\,dy$$

$$= -2\cdot 3^3 \times 1 \times 1 = -54$$

$$\therefore \oiint \bar{D}\cdot\bar{ds} = 8-4+27-12+128-54$$

$$= 4+15+74$$

$$= 93\ C$$

$$\therefore \boxed{\oiint \bar{D}\cdot\bar{ds} = \iiint \bar{\nabla}\cdot\bar{D}\,dv} \text{ (verified)}$$

3. c)

Continuity of current → Applicable when we are considering closed surface

Charges can be neither created nor destroyed.

- Equal amounts of +ve and -ve charge may be simultaneously created, obtained by separation, destroyed or lost by recombination.

Current through the closed surface,

$$I = \oint_s \vec{J} \cdot \vec{ds}$$

Outward flow of +ve charge balanced by a decrease of +ve charge within the closed surface.

Let, Q_i → be the charge inside the closed surface.

$$\therefore I = \oint_s \vec{J} \cdot \vec{ds} = -\frac{dQ_i}{dt}$$ → reduction in charge. ∴ negative sign

Using Divergence theorem,

$$\oint_s \vec{J} \cdot \vec{ds} = \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv$$

Now, $Q = \int_{vol} \rho_v \, dv$.

$$\therefore \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv = -\frac{d}{dt} \int_{vol} \rho_v \, dv$$

If the ~~de~~ surface is constant, derivative becomes a partial derivative.

$$\therefore \int_{vol} (\vec{\nabla} \cdot \vec{J})\, dv = \int_{vol} -\frac{\partial \rho_v}{\partial t}\, dv.$$

This expression is true for any volume, however small, it is true for an incremental volume,

$$(\vec{\nabla} \cdot \vec{J})\, \Delta v = -\frac{\partial \rho_v}{\partial t}\, \Delta v.$$

$\therefore$ Point form of continuity equation,

$$\boxed{(\vec{\nabla} \cdot \vec{J}) = -\frac{\partial \rho_v}{\partial t}}$$

4. a)

$\vec{E} \rightarrow$ Force on unit +ve test charge.

$\vec{F}_E = Q\vec{E} \rightarrow$ Force on charge Q.

- To move the charge we have to apply equal and opposite force on the charge.

Let, $\vec{dL} \rightarrow$ distance over which the charge is moved.

Let $\vec{dL} = dL\,\hat{a}_L$, i.e. $\hat{a}_L$ is unit vector along $\vec{dL}$.

$$F_{applied} = -Q\vec{E}\cdot\hat{a}_L$$

The component of force along $\vec{dL}$,

$$F_{EL} = \vec{F}_E \cdot \hat{a}_L = Q\vec{E}\cdot\hat{a}_L$$

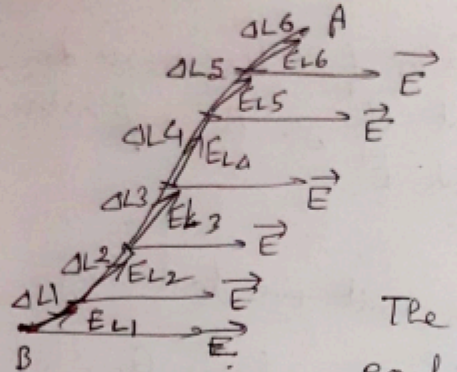
$\therefore$ The work done to move the charge over a ~~finite~~ distance $\vec{dL}$

$$dW = -Q\vec{E}\cdot\vec{dL}$$

$$dW = -Q\vec{E}\cdot(\hat{a}_L\, dL)$$

$\therefore$ The work done to move the charge over a finite distance

$$\boxed{W = -Q\int_{init}^{final} \vec{E}\cdot\vec{dL}}$$



chosen six line segments,

$\Delta L_1, \Delta L_2 \; -- \; \Delta L_6.$

The component of $\vec{E}$ along each segment,

$EL_1, EL_2 \; -- \; EL_6.$

∴ The work done to move the charge from B to A,

$$W = -Q\left(E_{L_1}\Delta L_1 + E_{L_2}\Delta L_2 \cdots E_{L_6}\Delta L_6\right)$$

$$= -Q\left(\vec{E}\cdot\vec{\Delta L_1} + \vec{E}\cdot\vec{\Delta L_2} + \cdots \vec{E}\cdot\vec{\Delta L_6}\right)$$

[considering uniform electric field]

$$\therefore W = -Q\vec{E}\cdot\left(\vec{\Delta L_1} + \vec{\Delta L_2} + \cdots \vec{\Delta L_6}\right)$$

$$= -Q\vec{E}\cdot\vec{L_{BA}}$$

i.e. $\boxed{W = -Q\int_B^A \vec{E}\cdot\vec{dL}}$

4. b)

$$\bar{D} = \frac{5\sin\theta\cos\phi}{r}\hat{a}_r \ \ c/m^2.$$

$$\therefore \rho_v = \bar{\nabla}\cdot\bar{D}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\, D_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{5\sin\theta\cos\phi}{r}\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r \cdot 5\sin\theta\cos\phi\right)$$

$$= \frac{5\sin\theta\cos\phi}{r^2} \ \ c/m^3$$

Total flux leaving the surface of the spherical volume of radius 2m,

$$\Psi = \iiint \bar{\nabla}\cdot\bar{D}\, dv$$

$$= \iiint \frac{5\sin\theta\cos\phi}{\cancel{r^2}} \cdot \cancel{r^2}\sin\theta\, d\theta\, d\phi\, dr$$

$$= 5\int_{r=0}^{2} dr \int_{\theta=0}^{\pi} \sin^2\theta\, d\theta \int_{\phi=0}^{2\pi} \cos\phi\, d\phi$$

$$= 5 \times 2 \times \pi/2 \times 0 = 0 \ C \text{ (Ans)}$$

4. c)

Potential Difference (V) :—

This is the work done to move a unit +ve charge from one point to another in an electric field.

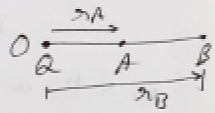
$$\therefore \text{potential diff, } V = -\int_{init}^{final} \vec{E}\cdot\vec{dl} \ \text{ Volt}$$

If initial point is B and final point is A

$$\boxed{V_{AB} = -\int_{B}^{A} \vec{E}\cdot\vec{dl} \ \text{ volt}}$$

Potential field of point charge: $= \frac{x^{n+1}}{n+1}$

$$V_{AB} = -\int_B^A \vec{E}\cdot\vec{dl}$$



$$= -\int_B^A \left(\frac{Q}{4\pi\epsilon_0 r^2}\right)\hat{a}_r\cdot(dr)\hat{a}_r$$

$$= -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon_0 r^2}\,dr = -\frac{Q}{4\pi\epsilon_0}\int_{r_B}^{r_A}\frac{dr}{r^2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_B}^{r_A} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_{r_B}^{r_A}$$

$$\therefore \boxed{V_{AB} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)} \text{ volt}$$

$r_B > r_A$, V_{AB} is +ve

i.e. energy is expended by external source to move the charge from B to A.

$r_B < r_A$, V_{AB} is −ve.

If we consider potential at point $A \rightarrow V_A$

" " " $B \rightarrow V_B$.

$V_{AB} = (V_A - V_B)$.

5. a)

Uniqueness Theorem:

Statement: If a solution to Laplace's equation can be found that satisfies the boundary conditions, then the solution is unique.

Let's assume Laplace's equation has two solutions, V_1 and V_2.

$$\nabla^2 V_1 = 0$$

$$\nabla^2 V_2 = 0$$

$$\therefore \quad \nabla^2 (V_1 - V_2) = 0 \quad \text{--- (1)}$$

Value of V_1 on the boundary be $V_{1b} = V_b$

Value of V_2 " " " be $V_{2b} = V_b$.

$V_b \rightarrow$ give potential on the boundary

Vector identity,

$$\vec{\nabla}(V\vec{D}) = V(\vec{\nabla}\cdot\vec{D}) + \vec{D}\cdot(\vec{\nabla}V) \quad \text{--- (2)}$$

Let the scalar be $(V_1 - V_2)$

and the vector be $\vec{\nabla}(V_1 - V_2)$

$\therefore$ From (2),

$$\vec{\nabla}\cdot\left((V_1 - V_2)\,\vec{\nabla}(V_1 - V_2)\right) = (V_1 - V_2)\left(\vec{\nabla}\cdot\vec{\nabla}(V_1 - V_2)\right) + \vec{\nabla}(V_1 - V_2)\cdot\vec{\nabla}(V_1 - V_2)$$

$$\vec{\nabla}\cdot\left((V_1 - V_2)\,\vec{\nabla}(V_1 - V_2)\right) = (V_1 - V_2)\underbrace{\nabla^2 (V_1 - V_2)}_{=0} + \left[\vec{\nabla}(V_1 - V_2)\right]^2$$

Integrating over the volume enclosed by the boundary surface area,

$$\iiint \vec{\nabla}\cdot\left((V_1 - V_2)\,\vec{\nabla}(V_1 - V_2)\right) dv = \iiint \left[\vec{\nabla}(V_1 - V_2)\right]^2 dv$$

[According to divergence theorem, $\iiint \vec{\nabla}\cdot\vec{A}\, dv = \oiint \vec{A}\cdot d\vec{s}$]

Surface integral over the surface at which the boundaries are specified,

$$\oiint (V_1 - V_2)\,\vec{\nabla}(V_1 - V_2)\cdot d\vec{s} = \oiint (V_{1b} - V_{2b})\,\vec{\nabla}(V_{1b} - V_{2b})\cdot d\vec{s} = 0$$

$$\therefore \quad \iiint \left[\vec{\nabla}(V_1 - V_2)\right]^2 dv = 0$$

If an integral is zero,
- either the integrand is zero or
- it is half +ve and half -ve, so that the overall contribution cancels each other.

$\left[\vec{\nabla}(V_1 - V_2)\right]^2 \rightarrow$ can't be -ve.

$$\left[\vec{\nabla}(V_1 - V_2)\right]^2 = 0$$

or $\quad \nabla(V_1 - V_2) = 0$

or $\quad \text{grad}\,(V_1 - V_2) = 0$

$\therefore \; (V_1 - V_2) = \text{constant}$

At a point on the boundary,

$$V_1 - V_2 = V_{1b} - V_{2b} = 0$$

$$\boxed{V_1 = V_2}$$

$\therefore$ Laplace's solution is unique, if that satisfies the same boundary conditions.

5. b)

Use Stoke's theorem to evaluate both sides of the theorem for the field $\bar{H} = 6xy\,\hat{a}_x - 3y^2\,\hat{a}_y$ A/m and the rectangular path around the region $2 \le x \le 5$, $-1 \le y \le 1$ and $z = 0$. Let the +ve direction of $d\bar{s}$ be $\hat{a}_z$.

<u>Soln.</u>

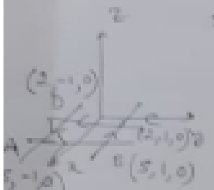
$$\bar{\nabla} \times \bar{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix} = -\hat{a}_z\, 6x$$

$$\therefore \iint (\bar{\nabla} \times \bar{H})\, \overline{ds} = -\iint 6x\, \hat{a}_z \cdot dx\, dy\, \hat{a}_z$$

$$= -\iint 6x\, dx\, dy = -6 \int_{x=2}^{5} x\, dx \int_{y=-1}^{1} dy$$

$$= -\overset{3}{\cancel{6}} \cdot \left[\frac{x^2}{\cancel{2}}\right]_2^5 \left[y\right]_{-1}^{1} = -3 \cdot (25-4)(1+1)$$

$$= -\cancel{6} \times 21 = -63 \times 2 = -126$$



<u>Along the path AB</u>

$$\int \bar{H} \cdot \overline{dL} = \int (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y) \cdot dy\,\hat{a}_y = -3 \int_{y=-1}^{1} y^2\, dy$$

$$= -\cancel{3} \left[\frac{y^3}{\cancel{3}}\right]_{-1}^{1} = 2$$

<u>Along the path BC</u>

$$\int \bar{H} \cdot \overline{dL} = \int (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y) \cdot dx\,\hat{a}_x = \int_{x=5}^{2} 6xy\, dx = \overset{3}{\cancel{6}} \cdot \left[\frac{x^2}{\cancel{2}}\right]_5^2$$

$$= 3 \cdot (4 - 25) = -63$$

<u>Along the path CD</u>

$$\int \bar{H} \cdot \overline{dL} = \int (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y) \cdot dy\,\hat{a}_y = -3 \int_{y=1}^{-1} y^2\, dy$$

$$= -\cancel{3} \left[\frac{y^3}{\cancel{3}}\right]_1^{-1} = [-1-1] = -2$$

<u>Along the path DA</u>

$$\int \bar{H} \cdot \overline{dL} = \int (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y) \cdot dx\,\hat{a}_x = \int_{x=2}^{5} 6xy\, dx = \overset{3}{\cancel{6}} \cdot (-1) \cdot \left[\frac{x^2}{\cancel{2}}\right]_2^5$$

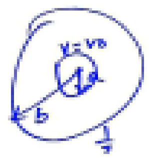
$$= -3\,(25-4) = -3 \times 21 = -63$$

$$\therefore \oint \bar{H} \cdot \overline{dL} = +2 - 63 - 2 - 63 = -126 = \iint \bar{\nabla} \times \bar{H} \cdot \overline{ds} \text{ (proved)}$$

6.a) Capacitance between two concentric spheres:

Spherical co-ordinates:

(i) $V(r)$



$$\nabla^2 V = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

Integrate,

$$r^2 \frac{dV}{dr} = C_1$$

$$\frac{dV}{dr} = \frac{C_1}{r^2}$$

Integrate,

$$\boxed{V = -\frac{C_1}{r} + C_2}$$

(i) At $r = a$, $V = V_0$

$$V_0 = -\frac{C_1}{a} + C_2$$

(ii) At $r = b$, $V = 0$

$$0 = -\frac{C_1}{b} + C_2$$

$$C_1 = \frac{-V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$C_2 = \frac{-V_0}{b\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$\therefore \boxed{V(r) = \frac{V_0}{r\left(\frac{1}{a} - \frac{1}{b}\right)} - \frac{V_0}{b\left(\frac{1}{a} - \frac{1}{b}\right)}}$$

$$\vec{E} = -\vec{\nabla} V = -\frac{dV}{dr}\vec{a_r}$$

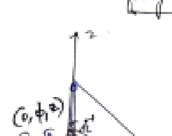
$$\vec{E} = +\frac{V_0}{r^2\left(\frac{1}{a} - \frac{1}{b}\right)}\vec{a_r}$$

$$Q = \iint_S \varepsilon \vec{E} \cdot \vec{ds_r} = \varepsilon \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{V_0}{r^2\left(\frac{1}{a} - \frac{1}{b}\right)} \cdot r^2 \sin\theta \, d\theta \, d\phi = \frac{\varepsilon V_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \cdot 4\pi$$

$$\boxed{C = \frac{Q}{V_0} = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}} \; F.$$

6.b) Magnetic field due to infinitely long conductor:

Infinitely long straight filament:



$d\vec{l} = dz\vec{a_z}$

$$\vec{H} = \int_{z=z_1}^{z_2} \frac{I \, \rho \, dz \, \vec{a_\phi}}{4\pi \left[\rho^2 + z^2\right]^{3/2}}$$

$$\tan\alpha = \frac{z}{\rho}$$

$$\therefore z = \rho \tan\alpha$$

z	z_1	z_2
α	α_1	α_2

$$dz = \rho \sec^2\alpha \, d\alpha$$

$$\vec{H} = \frac{I}{4\pi} \int_{\alpha=\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\alpha \, d\alpha \, \vec{a_\phi}}{\left[\rho^2 + \rho^2 \tan^2\alpha\right]^{3/2}}$$

$$= \frac{I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\alpha \, d\alpha}{\left(\rho^2 \sec^2\alpha\right)^{3/2}} \vec{a_\phi}$$

$$= \frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha \, \vec{a_\phi}$$

Finite conductor. $\boxed{\vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right] \vec{a_\phi}}$ A/m

$\therefore z_1 \to -\infty$ & $z_2 \to \infty$

$\alpha_1 = -\pi/2$ & $\alpha_2 = \pi/2$

$$\vec{H} = \frac{I}{4\pi\rho}\left[1 - (-1)\right] \vec{a_\phi}$$

Infinite conductor. $\boxed{\vec{H} = \frac{I}{2\pi\rho} \vec{a_\phi}}$ A/m

6Problem:

6) c) (i) $V = 2x^2 - 3y^2 + z^2$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= 4 - 6 + 2$$

$$\nabla^2 V = 0$$

V satisfies Laplace's eqn

ii) $V = r\cos\theta + \phi$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cos\theta\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta(-r\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}(1)$$

$$\nabla^2 V = \frac{1}{r^2}\cos\theta\,(2r) - \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial \theta}(\sin^2\theta) + 0$$

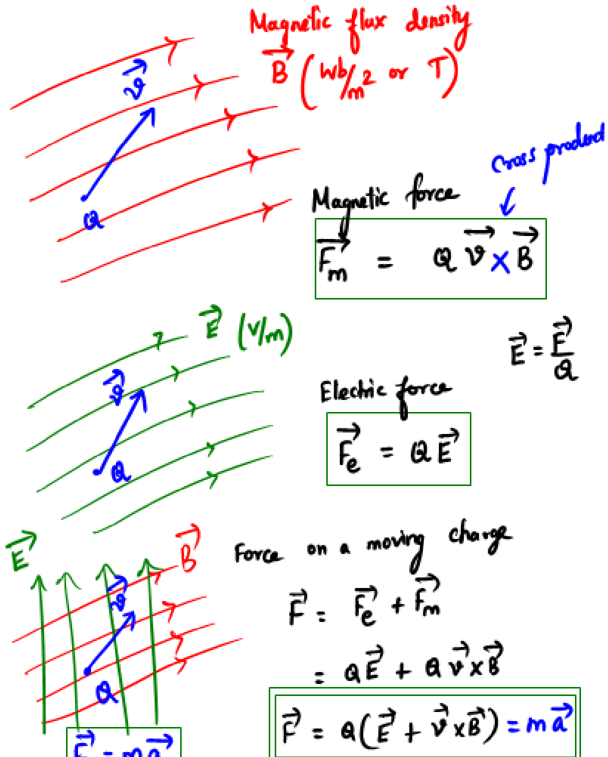
$$\nabla^2 V = \frac{2\cos\theta}{r} - \frac{1}{r\cancel{\sin\theta}}\, 2\cancel{\sin\theta}\cos\theta$$

$$\nabla^2 V = 0$$

V satisfies Laplace's eqn

7. a) Lorentz force equation:

Force on a moving charge;



Lorentz Force Equation

7.b) Problem

:

7) a) $\vec{B} = 0.05x\,\vec{a_y}$ T

$\chi_m = 2.5$

1. $\mu_r = 1 + \chi_m = 3.5$

2. $\mu = \mu_0\mu_r = 4\pi\times10^{-7} \times 3.5 = 4.3982\times10^{-6}$ H/m

3. $\vec{B} = \mu\vec{H}$.

$\vec{H} = \frac{\vec{B}}{\mu} = 11.3682\times10^{3}x\,\vec{a_y}$ A/m

4. $\vec{M} = \chi_m\vec{H} = 28.42\times10^{3}x\,\vec{a_y}$ A/m

5. $\vec{J} = \vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 11.3682\times10^{3}x & 0 \end{vmatrix} = 11.3682\times10^{3}\,\hat{a_z}$ A/m^2

6. $\vec{J_b} = \vec{\nabla}\times\vec{M} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 28.42\times10^{3}x & 0 \end{vmatrix} = 28.42\times10^{3}\,\hat{a_z}$ A/m^2

7. c) Force between differential current elements;

Force between differential current elements:

Consider two current elements, to find force b/w two current elements

W.K.T: Magnetic field at point 2 due to current element at point 1.

$$d\vec{H_2} = \frac{I_1\, d\vec{L_1} \times \vec{a_{R_{12}}}}{4\pi\, |\vec{R_{12}}|^2}$$

Differential force on a differential current element

$$d\vec{F} = I\, d\vec{L} \times \vec{B}$$

Differential Flux density $(d\vec{B_2})$ at point 2 caused by current element 1 $(I\,d\vec{L_1})$

Differential amount of differential force on element 2,

$$d(d\vec{F_2}) = I_2\, d\vec{L_2} \times d\vec{B_2}$$

where

$$d\vec{B_2} = \mu_0\, d\vec{H_2} = \frac{\mu_0 I_1\, d\vec{L_1} \times \vec{a_{R_{12}}}}{4\pi\, |\vec{R_{12}}|^2}.$$

$$\therefore \boxed{d(d\vec{F_2}) = \mu_0 \frac{I_1 I_2}{4\pi\, |\vec{R_{12}}|^2} \cdot d\vec{L_2} \times (d\vec{L_1} \times \vec{a_{R_{12}}})} \rightarrow (13)$$

$d(d\vec{F_1}) \neq d(d\vec{F_2})$ because of the non physical nature of the current element.

From the differential force, we can get

Total force between two filamentary circuits

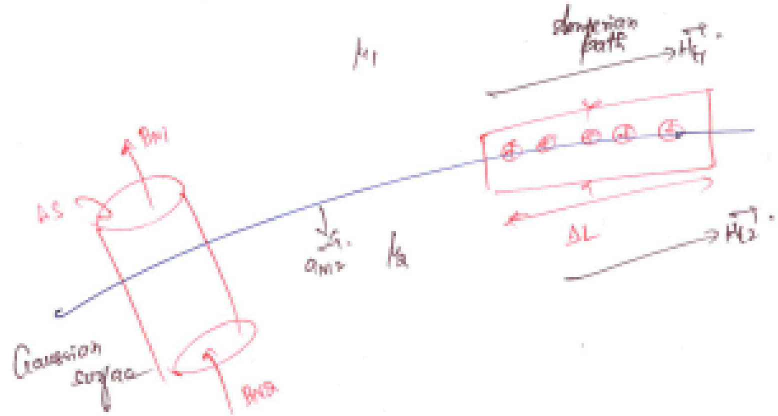
$$\boxed{\vec{F_2} = \frac{\mu_0 I_1 I_2}{4\pi} \oint \left[d\vec{L_2} \times \oint \frac{d\vec{L_1} \times \vec{a_{R_{12}}}}{|\vec{R_{12}}|^2} \right]}$$

(or)

$$\boxed{\vec{F_2} = \frac{\mu_0 I_1 I_2}{4\pi} \oint \left[\oint \frac{\vec{a_{R_{12}}} \times d\vec{L_1}}{|\vec{R_{12}}|^2} \right] \times d\vec{L_2}}$$

8.a) Magnetic Boundary Conditions:

Magnetic boundary conditions:



1) Apply Gauss's law for magnetic fields

$$\oint_S \vec{B}\cdot d\vec{s} = 0$$

$$B_{N1}\,\Delta s - B_{N2}\,\Delta s = 0$$

$$\boxed{B_{N1} = B_{N2}} \Rightarrow \boxed{H_{N2} = \frac{\mu_1}{\mu_2} H_{N1}}$$

Normal component of $\vec{B}$ is continuous

Normal component of $\vec{H}$ is discontinuous by the ratio μ_1/μ_2.

Also $$\boxed{M_{N2} = \chi_{m2}\frac{\mu_1}{\mu_2} H_{N1} = \frac{\chi_{m2}}{\chi_{m1}}\frac{\mu_1}{\mu_2} M_{N1}}$$

2) Ampere's circuital law

$$\oint \vec{H}\cdot d\vec{l} = I.$$

$$H_{t1}\,\Delta L - H_{t2}\,\Delta L = K\cdot\Delta L$$

$$\boxed{H_{t1} - H_{t2} = K.}$$

Directions can be written as

$$\boxed{(\vec{H_1} - \vec{H_2}) \times \vec{a_{N12}} = \vec{K}.}$$

(or) $$\boxed{\vec{H_{t1}} - \vec{H_{t2}} = \vec{a_{N12}} \times \vec{K}}$$ X

8. b)Problem:

D8.6. Find the magnetization in a magnetic material where: (*a*) $\mu = 1.8 \times 10^{-5}$ H/m and $H = 120$ A/m; (*b*) $\mu_r = 22$, there are 8.3×10^{28} atoms/m^3, and each atom has a dipole moment of 4.5×10^{-27} A $\cdot$ m^2; (*c*) $B = 300\,\mu$T and $\chi_m = 15$.

Ans. 1599 A/m; 374 A/m; 224 A/m

a) $\mu = 1.8 \times 10^{-5}$ H/m $= \mu_0 \mu_r$

$\mu_r = \dfrac{\mu}{\mu_0} = 14.32$

$H = 120$ A/m

$\chi_m = \mu_r - 1 = 13.32$

$\boxed{M = \chi_m H = 1598.4 \text{ A/m}}$

b) $\mu_r = 22$

$\dfrac{n}{\Delta V} = 8.3 \times 10^{28}$ atoms/m^3

$m_i = 4.5 \times 10^{-27}$ Am^2

$\boxed{M = \dfrac{\sum_n m_i}{\Delta V}}$

$\boxed{M = \dfrac{n m_i}{\Delta V} = 373.5 \text{ A/m}}$

c) $B = 300\,\mu T$, $\chi_m = 15$

$H = \dfrac{B}{\mu_0 \mu_r}$; $\mu_r = 1 + \chi_m$; $M = \chi_m H$

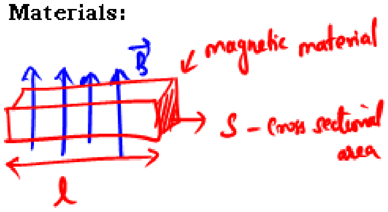
$\mu_r = 16$

$H = 14.920$ A/m

$\boxed{M = 223.8 \text{ A/m}}$

8.c)Forces on Magnetic materials:

Potential energy and Force on magnetic Materials:



Volume of material $= Sl$

$\vec{F} = ?$ Relation between Energy and Force

$dW = \vec{F}\cdot d\vec{l}$

$\boxed{dW = F\,dl} \Rightarrow F = \frac{dW}{dl}$

$\vec{B} = \mu_0\mu_r\vec{H}$

w.k.t. Potential Energy in Magnetic Field

$$W_H = \frac{1}{2}\iiint_V \vec{B}\cdot\vec{H}\,dv = \frac{1}{2}\iiint_V \frac{\vec{B}\cdot\vec{B}}{\mu_0\mu_r}\,dv$$

$$= \frac{1}{2}\frac{B^2}{\mu_0\mu_r}\iiint_V dv$$

$$\boxed{W_H = \frac{1}{2}\frac{B^2}{\mu_0\mu_r}Sl}$$

Force on a magnetic material, $F = \frac{dW_H}{dl}$

$$\boxed{F = \frac{1}{2}\frac{B^2 S}{\mu_0\mu_r}}$$

9.a) Maxwell's equations:

List the Maxwell's Equations in Integral form and differential (point) form:

Four Equations

	Integral form	Point form
Faraday's law	$\oint_L \vec{E}\cdot d\vec{l} = 0$	$\vec{\nabla}\times\vec{E} = 0$
Ampere's circuital Law	$\oint_L \vec{H}\cdot d\vec{l} = \iint_S \sigma\vec{E}\cdot d\vec{s}$ $\oint_L \vec{H}\cdot d\vec{l} = \iint_S \vec{J}\cdot d\vec{s}$	$\vec{\nabla}\times\vec{H} = \sigma\vec{E}$ $\vec{\nabla}\times\vec{H} = \vec{J}$
Gauss's law ($\vec{E}$- field)	$\oint_S \vec{D}\cdot d\vec{s} = Q_{enc} = \iiint_V \rho_v\,dv$	$\vec{\nabla}\cdot\vec{D} = \rho_v$
Gauss's law ($\vec{H}$- field)	$\oint_S \vec{B}\cdot d\vec{s} = 0$	$\vec{\nabla}\cdot\vec{B} = 0$

9.b) Problem:

Given **E** = E_m sin ($\omega t - \beta z$) $\mathbf{a}_y$ V/m. Find **D**, **B**, **H**. Also Sketch **E** and **H** at t=0.

Freespace

$$\boxed{\vec{E} = E_m \sin(\omega t - \beta z)\,\hat{a_y}} \ V/m$$

$$\vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0 E_m \sin(\omega t - \beta z)\,\hat{a_y} \ C/m^2$$

Faraday's law: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\vec{B} = \int \frac{\partial \vec{B}}{\partial t} dt = \int -\vec{\nabla}\times\vec{E}\, dt$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_m \sin(\omega t - \beta z) & 0 \end{vmatrix}$$

$$\vec{\nabla}\times\vec{E} = \hat{a_x}\left[0 - \frac{\partial}{\partial z}\left(E_m \sin(\omega t - \beta z)\right)\right] - \hat{a_y}(0-0) + \hat{a_z}\left[\frac{\partial}{\partial x}\left(E_m \sin(\omega t - \beta z)\right) - 0\right]$$

$\to 0$

$$\vec{\nabla}\times\vec{E} = \hat{a_x}\left[-E_m \cos(\omega t - \beta z)(-\beta)\right]$$

$$\vec{\nabla}\times\vec{E} = \beta E_m \cos(\omega t - \beta z)\,\hat{a_x}$$

$$\vec{B} = -\int(\vec{\nabla}\times\vec{E})\,dt$$

$$\vec{B} = -\int \beta E_m \cos(\omega t - \beta z)\,dt\,\hat{a_x}$$

$$\boxed{\vec{B} = -\frac{\beta E_m}{\omega}\sin(\omega t - \beta z)\,\hat{a_x}} \ Wb/m^2 \text{ (or T)}$$

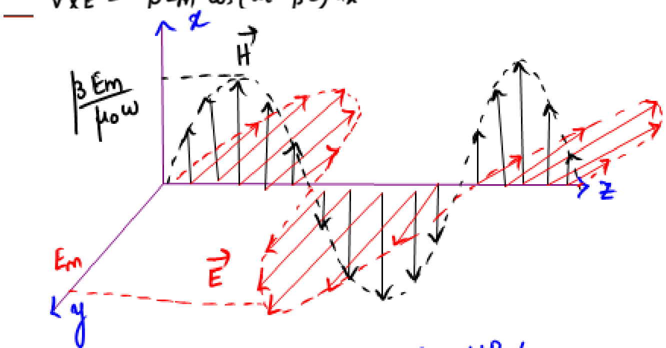
$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{-\beta E_m}{\mu_0 \omega}\sin(\omega t - \beta z)\,\hat{a_x}} \ A/m$$

At $\boxed{t=0}$ $\Rightarrow$ $\vec{E} = E_m \sin(-\beta z)\,\hat{a_y}$

$$\boxed{\vec{E} = -E_m \sin(\beta z)\,\hat{a_y}}$$

$$\vec{H} = \frac{-\beta E_m}{\mu_0 \omega}\sin(-\beta z)\,\hat{a_x}$$

$$\boxed{\vec{H} = \frac{\beta E_m}{\mu_0 \omega}\sin(\beta z)\,\hat{a_x}}$$



Eg: TEM (or UPW (Uniform Plane wave)

9.c) Problem:

9)c) $\frac{|J_c|}{|J_D|} = \frac{\sigma}{\omega\varepsilon} = 1$

$\sigma = 2\times10^{-4}$ ℧/m

$\varepsilon_r = 81$.

$\sigma = \omega\varepsilon$

$\omega = \frac{\sigma}{\varepsilon_0\varepsilon_r} = \frac{2\times10^{-4}}{8.854\times10^{-12}\times 81} = 0.2788\times10^{6}$ rad/s.

$f = \frac{\omega}{2\pi} = 44.372$ kHz

10 a) Poynting's theorem:

Poynting's theorem & Wave power:

Poynting's theorem.

It states that the net power flowing out of a given volume is equal to the time rate of decrease in stored energy within the volume minus conduction losses.

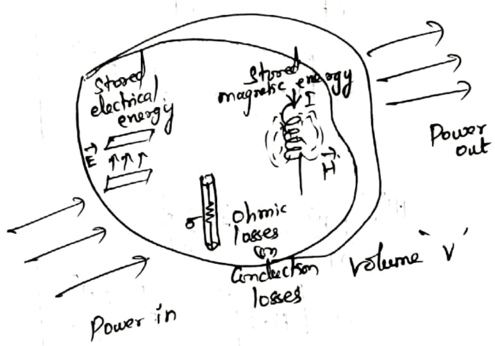


Fig. Illustration of power balance for EM fields.

Proof:

Maxwell's equations

$$\vec{\nabla}\cdot\vec{D} = \rho_v$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t}$$

Vector identity:

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\vec{\nabla}\times\vec{E} - \vec{E}\cdot\vec{\nabla}\times\vec{H}$$

Apply Maxwell's equations into this vector identity,

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\vec{H}\cdot\mu\frac{\partial \vec{H}}{\partial t} - \vec{E}\cdot\left[\sigma\vec{E} + \frac{\varepsilon\,\partial\vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\mu\,\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} - \sigma E^2 - \varepsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}.$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\sigma E^2 - \varepsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} - \mu\cdot\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}.$$

$\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = ?$

$\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = ?$

Let

$$\frac{\partial\vec{E}^2}{\partial t} = 2\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\Rightarrow \boxed{\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\frac{\partial\vec{E}^2}{\partial t}}$$

Ill$^{\text{rly}}$

$$\frac{\partial H^2}{\partial t} = 2\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}$$

$$\Rightarrow \boxed{\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{1}{2}\frac{\partial H^2}{\partial t}.}$$

Point form.

$$\boxed{\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\sigma E^2 - \frac{1}{2}\cdot\varepsilon\frac{\partial E^2}{\partial t} - \frac{1}{2}\mu\frac{\partial H^2}{\partial t}.}$$

Integrating over the given volume,

$$\underbrace{\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dv}_{} = -\iiint_V \sigma E^2\,dv - \frac{1}{2}\iiint_V \varepsilon\frac{\partial E^2}{\partial t}\,dv - \frac{1}{2}\iiint_V \mu\frac{\partial H^2}{\partial t}\,dv$$

Apply Divergence theorem

Divergence theorem, $\iiint_V (\vec{\nabla}\cdot\vec{A})\,dv = \oint\!\!\oint_S \vec{A}\cdot\vec{ds}$.

∴ The equation becomes **Poynting theorem.**

$$\boxed{\underbrace{\oint\!\!\oint_S (\vec{E}\times\vec{H})\cdot\vec{ds}}_{\text{net power flowing out of the volume}} = \underbrace{-\sigma\iiint E^2 dv}_{\text{Ohmic losses or Conduction losses}} \underbrace{-\frac{d}{dt}\left[\frac{1}{2}\iiint_V \varepsilon E^2 dv\right]}_{\text{Rate of decrease in stored electric energy}} \underbrace{-\frac{d}{dt}\left[\frac{1}{2}\iiint_V \mu H^2 dv\right]}_{\text{Rate of decrease in stored magnetic energy.}}}$$

Hence proved ∴

w.k.t

Electric potential energy, $\boxed{W_E = \frac{1}{2}\iiint_V \varepsilon E^2 dv}$

and

Magnetic potential energy, $\boxed{W_H = \frac{1}{2}\iiint_V \mu H^2 dv}$

Power flow of an electromagnetic wave

$$\boxed{P = \oint\!\!\oint_S (\vec{E}\times\vec{H})\cdot\vec{ds}} \quad W$$

where $\boxed{\vec{E}\times\vec{H} = \vec{P}}$ = Poynting vector = Power density vector

or

$\boxed{\vec{E}\times\vec{H} = \vec{S}}$ = Poynting vector (W/m^2)

10. b) Problem:

b) $\vec{E} = (20y - kt)\,\hat{a}_x$ V/m

$\vec{H} = (y + 2\times10^{6}t)\,\hat{a}_z$ A/m

$$\nabla \times \vec{H} = \cancel{\sigma \vec{E}}^{\,0} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix}$$

$$\nabla \times \vec{H} = \hat{a}_x\left[\frac{\partial}{\partial y}\left(y + 2\times10^{6}t\right) - 0\right] - \hat{a}_y\left[0 - 0\right] + \hat{a}_z\left[0 - 0\right]$$

$$\boxed{\nabla \times \vec{H} = \hat{a}_x\left[1 + 0\right]}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{D} = 4\times10^{-9}(20y - kt)\,\hat{a}_x$$

$$\frac{\partial \vec{D}}{\partial t} = 4\times10^{-9}(0 - k)\,\hat{a}_x$$

$$\boxed{\frac{\partial \vec{D}}{\partial t} = -4\times10^{-9}k\,\hat{a}_x}$$

$$\hat{a}_x = -4\times10^{-9}k\,\hat{a}_x$$

$$\boxed{k = -0.25\times10^{9}\ \frac{V}{ms}}$$

10. c) Problem:

10) c) $f = 10$ MHz

$\epsilon_r = 2.5$

$\mu_r = 4$

$\sigma = 10^{-3}$ ℧/m

Loss tangent

$$\tan\theta = \frac{\sigma}{\omega\epsilon} = \frac{10^{-3}}{2\pi\times10\times10^{6}\times8.854\times10^{-12}\times2.5} < 1$$

Propagation constant, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

$\gamma = 0.7355\angle 72.14^\circ$

$\gamma = 0.2255 + j0.7 = \alpha + j\beta$

$\alpha = 0.2255$ Np/m

$\beta = 0.7$ rad/m

Intrinsic Impedance, $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = 429.39\angle 17.86^\circ\ \Omega$

Wave length, $\lambda = \frac{2\pi}{\beta} = 8.975$ m.