



CMR INSTITUTE OF TECHNOLOGY		USN												
Internal Assessment Test – III March 2024														
Sub:	AV Mathematics-III for EC Engineering										Code:	BMATEC301		
Date:	04/03/2024	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	ECE					
Question 1 is compulsory and Answer any 6 from the remaining questions.														
											Marks	OBE CO RBT		
1	Obtain the lines of regression and hence find the coefficient of correlation for the data.											[8]	CO5	L3
	x	1	3	4	2	5	8	9	10	13	15			
	y	8	6	10	8	12	16	16	10	32	32			
2	Solve $(D^2 - 4D + 3)y = (e^x + 1)^2$.											[7]	CO4	L3
3	Solve $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = \sin(2 \log x)$.											[7]	CO4	L3
4	Solve $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 4y = 3 \sin x + 4 \cos x$.											[7]	CO4	L3

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5	Find a least square straight line for the following data.											[7]	CO5	L3
	x	1	2	3	4	5	6							
	y	6	4	3	5	4	2							
6	Determine rank correlation for the following data which shows the marks obtained in two quizzes in mathematics.											[7]	CO5	L3
	x	6	5	8	8	7	6	10	4	9	7			
	y	8	7	7	10	5	8	10	6	8	6			
7	Compute the rank correlation coefficient for the following data.											[7]	CO5	L3
	x	68	64	75	50	64	80	75	40	55	64			
	y	62	58	68	45	81	60	68	48	50	70			
8	If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$ show that $\sigma_x=2\sigma_y$ or $\sigma_y=2\sigma_x$.											[7]	CO5	L3

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① Given,

x	y	$(x-\bar{x})$	$(y-\bar{y})$	xy	x^2	y^2
1	8	-6	-7	42	36	49
3	6	-4	-9	36	16	81
4	10	-3	-5	15	9	25
2	8	-5	-7	35	25	49
5	12	-2	-3	6	4	9
8	16	1	1	16	1	16
9	16	2	1	144	4	1
10	10	3	-5	-15	9	25
13	32	6	17	102	36	289
15	32	8	17	136	64	289
				$\Sigma xy = 360$	$\Sigma x^2 = 204$	$\Sigma y^2 = 818$

As wkt, The Lines of regression are:-

"y on x" is:-

$$y - \bar{y} = \frac{\Sigma xy}{\Sigma x^2} (x - \bar{x}) \quad \text{--- (1)}$$

"x on y" is:-

$$x - \bar{x} = \frac{\Sigma xy}{\Sigma y^2} (y - \bar{y}) \quad \text{--- (2)}$$

$$\bar{x} = \frac{1+3+4+2+5+8+9+10+13+15}{10}$$

$$\bar{x} = 7$$

$$y = 15$$

$$\Sigma xy = 360, \Sigma x^2 = 204, \Sigma y^2 = 818$$

\therefore from eqn ①

$$y - 15 = \frac{360}{204} (x - 7)$$

$$y - 15 = 1.76471 (x - 7)$$

$$y - 15 = 1.76471x - 12.3529$$

$$\boxed{y = 1.76471x + 2.6471} \quad (\text{"y on x"})$$

\therefore from eqn ②

$$x - 7 = \frac{360}{818} (y - 15)$$

$$x - 7 = 0.4401 (y - 15)$$

$$x - 7 = 0.4401y - 6.60146$$

$$\boxed{x = 0.4401y + 0.39853} \quad (\text{"x on y"})$$

\therefore The above eqn's are the regression lines on "y on x" & "x on y" respectively.

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2} \sqrt{\Sigma y^2}}$$

$$r = \frac{360}{\sqrt{204} \sqrt{818}}$$

$$\boxed{r = 0.8813}$$



∴ The coefficient of correlation for the given data is: $r = 0.8813$

2

Given,

$$(D^2 - 4D + 3)y = (e^x + 1)^2 \quad \text{--- (1)}$$

$$f(D)y = (e^x + 1)^2$$

∴ By considering the Homogenous part of eqⁿ (1), we

get,

$$(D^2 - 4D + 3)y = 0 \quad \text{--- (2)}$$

∴ The Auxillary equation for eqⁿ (2) is:-

$$(m^2 - 4m + 3)y = 0$$

$$m^2 - 4m + 3 = 0$$

$$m^2 - 3m - m + 3 = 0$$

$$m(m-3) - 1(m-3) = 0$$

$$(m-1)(m-3) = 0$$

$$\boxed{m = 1, 3}$$

∴ The Complementary function for eqⁿ (1) is:-

$$\boxed{CF = C_1 e^x + C_2 e^{3x}}$$

$$PQ = \frac{(e^x + 1)^2}{f(D)}$$

$$PQ = \frac{e^{2x} + 1 + 2e^x}{f(D)}$$

$$(\because (a+b)^2 = a^2 + b^2 + 2ab)$$

$$PQ = \frac{e^{2x} + 1 + 2e^x}{(D^2 - 4D + 3)}$$

$$P_2 = \frac{e^{2x}}{D^2 - 4D + 3} + \frac{2e^x}{D^2 - 4D + 3} + \frac{1 \cdot e^{0x}}{D^2 - 4D + 3}$$

$$(\because e^{0x} = 1)$$

$$\text{let } P_1 = \frac{e^{2x}}{D^2 - 4D + 3}, \quad P_2 = \frac{2e^x}{D^2 - 4D + 3}, \quad P_3 = \frac{1 \cdot e^{0x}}{D^2 - 4D + 3}$$

$$P_2 = \cancel{P_1} + P_2 + P_3 = \textcircled{3}$$

let

$$P_1 = \frac{e^{2x}}{D^2 - 4D + 3}$$

(\because replace 'D' by 'a')

$$P_1 = \frac{e^{2x}}{(2)^2 - 4(2) + 3}$$

$$P_1 = \frac{e^{2x}}{4 - 8 + 3}$$

$$P_1 = \frac{e^{2x}}{4 - 8} \Rightarrow \frac{e^{2x}}{-4}$$

$$P_1 = -e^{2x}$$

$$P_2 = \frac{2e^x}{D^2 - 4D + 3}$$

$$P_2 = \frac{2e^x \cdot x}{f'(D)} \quad (\because \text{if we substitute 'D' with 'a=1' then we get denominator as zero})$$

$$P_2 = \frac{2x e^x}{2D - 4}$$

$$(\because \frac{d}{dx}(x) = 1, \quad \frac{d}{dx}(kx) = k, \quad \frac{d}{dx}(k) = 0)$$

$$P_2 = \frac{2x \cdot e^x}{2(1) - 4}$$

$$P_2 = \frac{2x \cdot e^x}{-2}$$

$$P_2 = -x e^x$$

$$P_3 = \frac{1 \cdot e^{0x}}{D^2 - 4D + 3}$$

∴ Replace 'D' by 'a=0'

$$P_3 = \frac{1}{3}$$

∴ substitute P_1, P_2, P_3 in eqⁿ (3)

$$∴ P_2 = -e^{2x} - x \cdot e^x + \frac{1}{3}$$

General solution is:-

$$y = CF + P_2$$

$$y = C_1 e^x + C_2 e^{3x} + \frac{1}{3} - e^{2x} - x e^x$$

(2)

Given,

$$x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = \sin(2 \log x) \quad \text{--- (1)}$$

∴ From Cauchy's D.E :-

$$x^2 \frac{d^2 y}{dx^2} = D(D-1)y$$

$$x = e^t$$

$$t = \log x$$

$$x \frac{dy}{dx} = Dy$$

where, $D = \frac{d}{dt}$

∴ substitute the above values in eqⁿ (1)

$$D(D-1)y - 4Dy + 6y = \sin(2 \log x)$$

$$D(D-1)y - 4Dy + 6y = \sin(2t)$$

$$D^2 y - Dy - 4Dy + 6y = \sin 2t$$

$$D^2 y - 5Dy + 6y = \sin 2t \quad \text{--- (2)}$$

$$(D^2 - 5D + 6)y = \sin 2t \quad \text{--- (2)}$$

∴ Consider the Homogeneous part of eqn (2)

$$(D^2 - 5D + 6)y = 0$$

∴ Auxiliary eqn is:-

$$m^2 - 5m + 6 = 0$$

$$m^2 - 3m - 2m + 6 = 0$$

$$m(m-3) - 2(m-3) = 0$$

$$(m-2)(m-3) = 0$$

$$m = 2, 3$$

∴ The Complementary function is:- $CF = C_1 e^{2t} + C_2 e^{3t}$

$$P_2 = \frac{Q(x)}{f(D)}$$

$$P_2 = \frac{\sin 2t}{D^2 - 5D + 6}$$

$$P_2 = \frac{\sin 2t \cdot t}{2D - 5}$$

$$P_2 = \frac{\sin 2t}{-4 - 5D + 6}$$

$$P_2 = \frac{\sin 2t}{2 - 5D} \times \frac{2 + 5D}{2 + 5D}$$

$$P_2 = \frac{2\sin 2t + 5D\sin 2t}{(2)^2 - (5D)^2}$$

$$P_2 = \frac{2\sin 2t + 5 \cos 2t \cdot 2}{4 - 25D^2}$$

(∴ Replace 'D' by '-a')

$$a = 2$$

$$a^2 = 4$$

(∴ $\frac{d}{dx} (\sin ax) = a \cos ax$)

$$PZ = \frac{2\sin 2t + 10\cos 2t}{4 - 25D^2}$$

$$PZ = \frac{2\sin 2t}{4 - 25D^2} + \frac{10\cos 2t}{4 - 25D^2}$$

$$PZ = \frac{2\sin 2t}{4 - 25(-4)} + \frac{10\cos 2t}{4 - 25(-4)}$$

$$PZ = \frac{2\sin 2t}{104} + \frac{10\cos 2t}{104}$$

$$PZ = \frac{2\sin 2t + 10\cos 2t}{104}$$

$$PZ = \frac{2(\sin 2t + 5\cos 2t)}{104 \cdot 52}$$

$$PZ = \frac{\sin 2t + 5\cos 2t}{52}$$

General solution is:-

$$y = CF + PZ$$

$$y = C_1 e^{2t} + C_2 e^{3t} + \frac{1}{\sqrt{2}} [\sin 2t + 5\cos 2t]$$

$$y = C_1 e^{2(\log x)} + C_2 e^{3(\log x)} + \frac{1}{\sqrt{2}} [\sin 2(\log x) + 5\cos 2(\log x)]$$

($\because t = \log x$)

$$y = C_1 x^2 + C_2 x^3 + \frac{1}{\sqrt{2}} [\sin(2\log x) + 5\cos(2\log x)]$$

($\because e^{2\log x} = x^2$)

4) Given,

$$\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 3\sin x + 4\cos x \quad \text{--- (1)}$$

As WKT

$$D = \frac{d}{dx}$$

∴ we can write eqn (1) as:

$$D^2y + 4Dy + 4y = 3\sin x + 4\cos x$$

$$(D^2 + 4D + 4)y = 3\sin x + 4\cos x \quad \text{--- (2)}$$

∴ By considering the homogeneous part of eqn (2)

$$(D^2 + 4D + 4)y = 0$$

∴ Auxiliary eqn is:

$$m^2 + 4m + 4 = 0$$

$$m^2 + 2m + 2m + 4 = 0$$

$$m(m+2) + 2(m+2) = 0$$

$$(m+2)(m+2) = 0$$

$$\boxed{m = -2, -2}$$

$$CF = \cancel{e^{-2x}} (C_1 + C_2x)e^{-2x}$$

∴ The complementary function is:

$$CF = (C_1 + C_2x)e^{-2x}$$

$$PZ = \frac{\phi(x)}{f(D)}$$

$$PZ = \frac{3\sin x + 4\cos x}{D^2 + 4D + 4}$$

$$PZ = \frac{3\sin x}{D^2 + 4D + 4} + \frac{4\cos x}{D^2 + 4D + 4}$$

$$PZ = \frac{3\sin x}{-1 + 4D + 4} + \frac{4\cos x}{-1 + 4D + 4}$$

$$PZ = \frac{3\sin x}{4D + 3} + \frac{4\cos x}{4D + 3}$$

$$PZ = \frac{3\sin x \times 4D - 3}{4D + 3} + \frac{4\cos x \times 4D - 3}{4D + 3}$$

$$\left[\begin{array}{l} D^2 = -a^2 \\ \therefore a = 1 \\ a^2 = -1 \\ D^2 = -1 \end{array} \right]$$

$$PZ = \frac{12D\sin x - 9\sin x}{(4D)^2 - (3)^2} + \frac{16D\cos x - 12\cos x}{(4D)^2 - (3)^2}$$

$$PZ = \frac{12\cos x - 9\sin x}{16D^2 - 9} + \frac{-16\sin x - 12\cos x}{16D^2 - 9}$$

$$PZ = \frac{12\cos x - 9\sin x}{16(-D)^2 - 9} + \frac{(-16\sin x - 12\cos x)}{16(-D)^2 - 9} \Rightarrow \frac{12\cos x - 9\sin x}{-25} + \frac{-16\sin x - 12\cos x}{-25}$$

$$PZ = \frac{12\cos x - 9\sin x - 16\sin x - 12\cos x}{-25}$$

$$PZ = \frac{-27\sin x}{-25}$$

$$PZ = \sin x$$

General solution is: $y = CF + PZ$

$$y = (C_1 + C_2 x) e^{-2x} + \sin x$$

5) Given,

x	y	xy	x ²
1	6	6	1
2	4	8	4
3	3	9	9
4	5	20	16
5	4	20	25
6	2	12	36
$\Sigma x = 21$	$\Sigma y = 24$	$\Sigma xy = 75$	$\Sigma x^2 = 91$

∴ As wkt,

The eqn of straight line is: $y = ax + b$ — (1)

$$\therefore \Sigma y = a \Sigma x + b n \text{ — (2)}$$

$$\Sigma xy = a \Sigma x^2 + b \Sigma x \text{ — (3)}$$

} (The normal equations of the straight line equation.)

∴ From the given data,

$$n = 6$$

$$\Sigma x = 21, \Sigma y = 24, \Sigma xy = 75, \Sigma x^2 = 91$$

∴ substitute the values in eqn (2), (3) we get

$$24 = a(21) + b(6) \text{ — (2)}$$

$$75 = a(91) + b(21) \text{ — (3)}$$

$$\therefore 21a + 6b = 24 \text{ — (2a)}$$

$$91a + 21b = 75 \text{ — (3a)}$$

∴ By solving the above eqn's we get

$$a = -0.51429, b = 5.8$$

∴ By substituting "a & b" values in eqn (1) we get

$$y = (-0.51429)x + 5.8$$

∴ The Least square line is: $y = 5.8 - (0.51429)x$

6) Given

x	y	x	y	d = x - y	d ²
6	8	7.5	4	3.5	12.25
5	7	9	6.5	2.5	6.25
8	7	3.5	6.5	-3	9
8	10	3.5	1.5	2	4
7	5	5.5	10	-4.5	20.25
6	8	7.5	4	3.5	12.25
10	10	1	6.5	-0.5	0.25
4	6	10	8.5	1.5	2.25
9	8	2	4	-2	4
7	6	5.5	8.5	-3	9

$$m_1 = 2$$

$$m_2 = 2$$

$$m_3 = 2$$

$$m_4 = 2$$

$$m_5 = 3$$

$$m_6 = 2$$

$$m_7 = 2$$

$$n = 10$$

$$\sum d^2 = 79.5$$

$$R = 1 - \frac{6 \sum d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \frac{m_3(m_3^2-1)}{12} + \frac{m_4(m_4^2-1)}{12} + \frac{m_5(m_5^2-1)}{12} + \frac{m_6(m_6^2-1)}{12} + \frac{m_7(m_7^2-1)}{12}}{n(n^2-1)}$$

$$R = 1 - \frac{6(79.5) + \frac{2 \times 3}{12} + \frac{2 \times 3}{12} + \frac{2 \times 3}{12} + \frac{2 \times 3}{12} + \frac{3 \times 8}{12} + \frac{2 \times 3}{12} + \frac{2 \times 3}{12}}{10(100-1)}$$

$$R = 1 - \frac{477 + 0.5 + 0.5 + 0.5 + 0.5 + 2 + 0.5 + 0.5}{10(99)}$$

$$R = 1 - \left[\frac{477 + 5}{990} \right]$$

$$R = 1 - \left[\frac{482}{990} \right]$$

$$R = \frac{254}{495}$$

$$R = 0.5131$$

∴ The rank correlation for the given data is: $R = 0.5131$

15.7	18.7	23.5	27.1	30.2	33.1	35.7	38.2	40.5	42.8	45.1	47.5	49.8	52.1	54.5	56.8	59.2	61.5	63.8	66.1	68.5	70.8	73.1	75.5	77.8	80.1	82.5	84.8	87.1	89.5	91.8	94.1	96.5	98.8	101.1	103.5	105.8	108.1	110.5	112.8	115.1	117.5	119.8	122.1	124.5	126.8	129.1	131.5	133.8	136.1	138.5	140.8	143.1	145.5	147.8	150.1	152.5	154.8	157.1	159.5	161.8	164.1	166.5	168.8	171.1	173.5	175.8	178.1	180.5	182.8	185.1	187.5	189.8	192.1	194.5	196.8	199.1	201.5	203.8	206.1	208.5	210.8	213.1	215.5	217.8	220.1	222.5	224.8	227.1	229.5	231.8	234.1	236.5	238.8	241.1	243.5	245.8	248.1	250.5	252.8	255.1	257.5	259.8	262.1	264.5	266.8	269.1	271.5	273.8	276.1	278.5	280.8	283.1	285.5	287.8	290.1	292.5	294.8	297.1	299.5	301.8	304.1	306.5	308.8	311.1	313.5	315.8	318.1	320.5	322.8	325.1	327.5	329.8	332.1	334.5	336.8	339.1	341.5	343.8	346.1	348.5	350.8	353.1	355.5	357.8	360.1	362.5	364.8	367.1	369.5	371.8	374.1	376.5	378.8	381.1	383.5	385.8	388.1	390.5	392.8	395.1	397.5	399.8	402.1	404.5	406.8	409.1	411.5	413.8	416.1	418.5	420.8	423.1	425.5	427.8	430.1	432.5	434.8	437.1	439.5	441.8	444.1	446.5	448.8	451.1	453.5	455.8	458.1	460.5	462.8	465.1	467.5	469.8	472.1	474.5	476.8	479.1	481.5	483.8	486.1	488.5	490.8	493.1	495.5	497.8	500.1	502.5	504.8	507.1	509.5	511.8	514.1	516.5	518.8	521.1	523.5	525.8	528.1	530.5	532.8	535.1	537.5	539.8	542.1	544.5	546.8	549.1	551.5	553.8	556.1	558.5	560.8	563.1	565.5	567.8	570.1	572.5	574.8	577.1	579.5	581.8	584.1	586.5	588.8	591.1	593.5	595.8	598.1	600.5	602.8	605.1	607.5	609.8	612.1	614.5	616.8	619.1	621.5	623.8	626.1	628.5	630.8	633.1	635.5	637.8	640.1	642.5	644.8	647.1	649.5	651.8	654.1	656.5	658.8	661.1	663.5	665.8	668.1	670.5	672.8	675.1	677.5	679.8	682.1	684.5	686.8	689.1	691.5	693.8	696.1	698.5	700.8	703.1	705.5	707.8	710.1	712.5	714.8	717.1	719.5	721.8	724.1	726.5	728.8	731.1	733.5	735.8	738.1	740.5	742.8	745.1	747.5	749.8	752.1	754.5	756.8	759.1	761.5	763.8	766.1	768.5	770.8	773.1	775.5	777.8	780.1	782.5	784.8	787.1	789.5	791.8	794.1	796.5	798.8	801.1	803.5	805.8	808.1	810.5	812.8	815.1	817.5	819.8	822.1	824.5	826.8	829.1	831.5	833.8	836.1	838.5	840.8	843.1	845.5	847.8	850.1	852.5	854.8	857.1	859.5	861.8	864.1	866.5	868.8	871.1	873.5	875.8	878.1	880.5	882.8	885.1	887.5	889.8	892.1	894.5	896.8	899.1	901.5	903.8	906.1	908.5	910.8	913.1	915.5	917.8	920.1	922.5	924.8	927.1	929.5	931.8	934.1	936.5	938.8	941.1	943.5	945.8	948.1	950.5	952.8	955.1	957.5	959.8	962.1	964.5	966.8	969.1	971.5	973.8	976.1	978.5	980.8	983.1	985.5	987.8	990.1	992.5	994.8	997.1	999.5	1001.8	1004.1	1006.5	1008.8	1011.1	1013.5	1015.8	1018.1	1020.5	1022.8	1025.1	1027.5	1029.8	1032.1	1034.5	1036.8	1039.1	1041.5	1043.8	1046.1	1048.5	1050.8	1053.1	1055.5	1057.8	1060.1	1062.5	1064.8	1067.1	1069.5	1071.8	1074.1	1076.5	1078.8	1081.1	1083.5	1085.8	1088.1	1090.5	1092.8	1095.1	1097.5	1099.8	1102.1	1104.5	1106.8	1109.1	1111.5	1113.8	1116.1	1118.5	1120.8	1123.1	1125.5	1127.8	1130.1	1132.5	1134.8	1137.1	1139.5	1141.8	1144.1	1146.5	1148.8	1151.1	1153.5	1155.8	1158.1	1160.5	1162.8	1165.1	1167.5	1169.8	1172.1	1174.5	1176.8	1179.1	1181.5	1183.8	1186.1	1188.5	1190.8	1193.1	1195.5	1197.8	1200.1	1202.5	1204.8	1207.1	1209.5	1211.8	1214.1	1216.5	1218.8	1221.1	1223.5	1225.8	1228.1	1230.5	1232.8	1235.1	1237.5	1239.8	1242.1	1244.5	1246.8	1249.1	1251.5	1253.8	1256.1	1258.5	1260.8	1263.1	1265.5	1267.8	1270.1	1272.5	1274.8	1277.1	1279.5	1281.8	1284.1	1286.5	1288.8	1291.1	1293.5	1295.8	1298.1	1300.5	1302.8	1305.1	1307.5	1309.8	1312.1	1314.5	1316.8	1319.1	1321.5	1323.8	1326.1	1328.5	1330.8	1333.1	1335.5	1337.8	1340.1	1342.5	1344.8	1347.1	1349.5	1351.8	1354.1	1356.5	1358.8	1361.1	1363.5	1365.8	1368.1	1370.5	1372.8	1375.1	1377.5	1379.8	1382.1	1384.5	1386.8	1389.1	1391.5	1393.8	1396.1	1398.5	1400.8	1403.1	1405.5	1407.8	1410.1	1412.5	1414.8	1417.1	1419.5	1421.8	1424.1	1426.5	1428.8	1431.1	1433.5	1435.8	1438.1	1440.5	1442.8	1445.1	1447.5	1449.8	1452.1	1454.5	1456.8	1459.1	1461.5	1463.8	1466.1	1468.5	1470.8	1473.1	1475.5	1477.8	1480.1	1482.5	1484.8	1487.1	1489.5	1491.8	1494.1	1496.5	1498.8	1501.1	1503.5	1505.8	1508.1	1510.5	1512.8	1515.1	1517.5	1519.8	1522.1	1524.5	1526.8	1529.1	1531.5	1533.8	1536.1	1538.5	1540.8	1543.1	1545.5	1547.8	1550.1	1552.5	1554.8	1557.1	1559.5	1561.8	1564.1	1566.5	1568.8	1571.1	1573.5	1575.8	1578.1	1580.5	1582.8	1585.1	1587.5	1589.8	1592.1	1594.5	1596.8	1599.1	1601.5	1603.8	1606.1	1608.5	1610.8	1613.1	1615.5	1617.8	1620.1	1622.5	1624.8	1627.1	1629.5	1631.8	1634.1	1636.5	1638.8	1641.1	1643.5	1645.8	1648.1	1650.5	1652.8	1655.1	1657.5	1659.8	1662.1	1664.5	1666.8	1669.1	1671.5	1673.8	1676.1	1678.5	1680.8	1683.1	1685.5	1687.8	1690.1	1692.5	1694.8	1697.1	1699.5	1701.8	1704.1	1706.5	1708.8	1711.1	1713.5	1715.8	1718.1	1720.5	1722.8	1725.1	1727.5	1729.8	1732.1	1734.5	1736.8	1739.1	1741.5	1743.8	1746.1	1748.5	1750.8	1753.1	1755.5	1757.8	1760.1	1762.5	1764.8	1767.1	1769.5	1771.8	1774.1	1776.5	1778.8	1781.1	1783.5	1785.8	1788.1	1790.5	1792.8	1795.1	1797.5	1799.8	1802.1	1804.5	1806.8	1809.1	1811.5	1813.8	1816.1	1818.5	1820.8	1823.1	1825.5	1827.8	1830.1	1832.5	1834.8	1837.1	1839.5	1841.8	1844.1	1846.5	1848.8	1851.1	1853.5	1855.8	1858.1	1860.5	1862.8	1865.1	1867.5	1869.8	1872.1	1874.5	1876.8	1879.1	1881.5	1883.8	1886.1	1888.5	1890.8	1893.1	1895.5	1897.8	1900.1	1902.5	1904.8	1907.1	1909.5	1911.8	1914.1	1916.5	1918.8	1921.1	1923.5	1925.8	1928.1	1930.5	1932.8	1935.1	1937.5	1939.8	1942.1	1944.5	1946.8	1949.1	1951.5	1953.8	1956.1	1958.5	1960.8	1963.1	1965.5	1967.8	1970.1	1972.5	1974.8	1977.1	1979.5	1981.8	1984.1	1986.5	1988.8	1991.1	1993.5	1995.8	1998.1	2000.5
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7 Given,

x	y	r ₁	r ₂	r ₄	d = r _x - r _y	d ²
68	62	4	4	5	-1	1
64	58	4	6	7	-1	1
75	68	2.5	2.5	3.5	-1	1
80	45		9	10	-1	1
64	81		6	1	5	25
80	60		1	6	-5	25
75	68	2.5	2.5	3.5	-1	1
40	48		10	9	1	1
55	50	8	8	8	0	0
64	70	6	6	2	4	16

m₁ = 2
 m₂ = 3
 m₃ = 2

Σd² = 72

n = 10

$$R = 1 - \left[\frac{6 \Sigma d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \frac{m_3(m_3^2-1)}{12}}{n(n^2-1)} \right]$$

$$R = 1 - \left[\frac{6(72) + \frac{2(4)}{12} + \frac{3(8)}{12} + \frac{2(4)}{12}}{10(100-1)} \right]$$

$$R = 1 - \left[\frac{432 + 0.5 + 2 + 0.5}{10(99)} \right]$$

$$R = 1 - \left[\frac{435}{990} \right]$$

R = 0.5606

∴ The Rank correlation coefficient for the following data is:

$$R = 0.561$$

8 Given,

$$r = 0.5$$

$$\theta = \tan^{-1}\left(\frac{3}{5}\right)$$

Q.P:-

$$\sigma_x = 2\sigma_y \text{ (or) } \sigma_y = 2\sigma_x$$

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2} \left(\frac{1 - r^2}{r} \right)$$

$$\therefore \tan \theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{1 - r^2}{r} \right)$$

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{1 - (0.5)^2}{0.5} \right)$$

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{0.75}{0.5} \right)$$

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \left(\frac{3}{2} \right)$$

$$\frac{2}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

The lines of Regression are respectively :-

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$2x^2 + 2y^2 = 5xy$$

$$2x^2 - 5xy + 2y^2 = 0$$

$$2x^2 - 4xy - xy + 2y^2 = 0$$

$$2x(x - 2y) - y(x - 2y) = 0$$

$$(2x - y)(x - 2y) = 0$$

$$\therefore 2x = y \text{ (or) } x = 2y$$

$$\therefore x = 2y \text{ (or) } y = 2x$$

Hence Proved.