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Internal Assessment Test – III March 2024

Sub:	AV Mathematics-III for EC Engineering							Code:	BMATEC301
Date:	04/03/2024	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	**ECE**

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Obtain the lines of regression and hence find the coefficient of correlation for the data.	[8]	CO5	L3

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

2	Solve $(D^2 - 4D + 3)y=(e^x + 1)^2$.	[7]	CO4	L3
3	Solve $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y=\sin(2\log x)$.	[7]	CO4	L3
4	Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y=3\sin x + 4\cos x$.	[7]	CO4	L3

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4	Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y=3\sin x + 4\cos x$.	[7]	CO4	L3

5	Find a least square straight line for the following data. x: 1, 2, 3, 4, 5, 6 y: 6, 4, 3, 5, 4, 2	[7]	CO5	L3
6	Determine rank correlation for the following data which shows the marks obtained in two quizzes in mathematics. x: 6, 5, 8, 8, 7, 6, 10, 4, 9, 7 y: 8, 7, 7, 10, 5, 8, 10, 6, 8, 6	[7]	CO5	L3
7	Compute the rank correlation coefficient for the following data. x: 68, 64, 75, 50, 64, 80, 75, 40, 55, 64 y: 62, 58, 68, 45, 81, 60, 68, 48, 50, 70	[7]	CO5	L3
8	If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$ show that $\sigma_x=2\sigma_y$ or $\sigma_y=2\sigma_x$.	[7]	CO5	L3

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7	Compute the rank correlation coefficient for the following data. x: 68, 64, 75, 50, 64, 80, 75, 40, 55, 64 y: 62, 58, 68, 45, 81, 60, 68, 48, 50, 70	[7]	CO5	L3
8	If the coefficient of correlation between two variables x and y is 0.5 and the acute angle between their lines of regression is $\tan^{-1}\left(\frac{3}{5}\right)$ show that $\sigma_x=2\sigma_y$ or $\sigma_y=2\sigma_x$.	[7]	CO5	L3

① Given,

x	y	$(x-\bar{x})$	$(y-\bar{y})$	xy	x^2	y^2
1	8	-6	-7	42	36	49
3	6	-4	-9	36	16	81
4	10	-3	-5	15	9	25
2	8	-5	-7	35	25	49
5	12	-2	-3	6	4	9
8	16	1	1	1	1	1
9	16	2	1	2	4	1
10	10	3	-5	-15	9	25
13	32	6	17	102	36	289
15	32	8	17	136	64	289
				$\Sigma xy=360$	$\Sigma x^2=204$	$\Sigma y^2=818$

As WKT, The Lines of regression are:-

"y on x" is:-

$$y-\bar{y} = \frac{\Sigma xy}{\Sigma x^2}(x-\bar{x}) \quad -①$$

"x on y" is:-

$$x-\bar{x} = \frac{\Sigma xy}{\Sigma y^2}(y-\bar{y}) \quad -②$$

$$\bar{x} = \frac{1+3+4+2+5+8+9+10+13+15}{10}$$

$$\bar{x} = 7$$

$\bar{y} = 15$

$\Sigma xy = 360, \quad \Sigma x^2 = 204, \quad \Sigma y^2 = 818$

$\therefore$ from eqn ①

$$y - 15 = \frac{360}{204}(x - 7)$$

$$y - 15 = 1.76471(x - 7)$$

$$y - 15 = 1.76471x - 12.3529$$

$$\boxed{y = 1.76471x + 2.64471} \quad (\text{"}y \text{ on } x\text{"})$$

$\therefore$ from eqn ②

$$x - 7 = \frac{360}{818}(y - 15)$$

$$x - 7 = 0.4401(y - 15)$$

$$x - 7 = 0.4401y - 6.60146$$

$$\boxed{x = 0.4401y + 0.39853} \quad (\text{"}x \text{ on } y\text{"})$$

$\therefore$ The above eqn's are the regression lines on "y on x" & "x on y" respectively.

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}}$$

$$r = \frac{360}{\sqrt{204}\sqrt{818}}$$

$$\boxed{r = 0.8813}$$

∴ The coefficient of correlation for the given data is:- r = 0.8813

(2) Given,

$$(D^2-4D+3)y=(e^x+1)^2 \quad -①$$

$$f(D)\,y=(e^x+1)^2$$

∴ By considering the Homogenous Part of eqn ① we get,

$$(D^2-4D+3)\,y=0 \quad -②$$

∴ The Auxillary Equation for eqn ② is:-

$$(m^2-4m+3)y=0$$

$$m^2-4m+3=0$$

$$m^2-3m-m+3=0$$

$$m(m-3)-1(m-3)=0$$

$$(m-1)(m-3)=0$$

$$\boxed{m=1,3}$$

∴ The complementary function for eqn ① is:-

$$\boxed{CF = C_1e^x+C_2e^{3x}}$$

$$PI=\frac{(e^x+1)^2}{f(D)}$$

$$PI=\frac{e^{2x}+1+2e^x}{f(D)} \qquad (\because (a+b)^2=a^2+b^2+2ab)$$

$$PI=\frac{e^{2x}+1+2e^x}{(D^2-4D+3)}$$

$$P_2 = \frac{e^{2x}}{D^2-4D+3} + \frac{2e^x}{D^2-4D+3} + \frac{1\cdot e^{0x}}{D^2-4D+3} \quad (\because e^{0x} = 1)$$

let $P_1 = \frac{e^{2x}}{D^2-4D+3}$, $P_2 = \frac{2e^x}{D^2-4D+3}$, $P_3 = \frac{1\cdot e^{0x}}{D^2-4D+3}$

$P_2 = P_1 + P_2 + P_3$ — (3)

let

$$P_1 = \frac{e^{2x}}{D^2-4D+3} \quad (\because \text{Replace } 'D' \text{ by } 'a')$$

$$P_1 = \frac{e^{2x}}{(2)^2-4(2)+3}$$

$$P_1 = \frac{e^{2x}}{4-8+3}$$

$$P_1 = \frac{e^{2x}}{7-8} \Rightarrow \frac{e^{2x}}{-1}.$$

$$\boxed{P_1 = -e^{2x}}$$

$$P_2 = \frac{2e^x}{D^2-4D+3}$$

$$P_2 = \frac{2e^x \cdot x}{f'(D)'} \quad (\because \text{If we substitute } 'D' \text{ with } 'a=1' \text{ then we get Denominator as zero})$$

$$P_2 = \frac{2xe^x}{2D-4} \quad \left(\because \frac{d}{dx}(x) = 1,\ \frac{d}{dx}(kx) = k,\ \frac{d}{dx}(k) = 0\right)$$

$$P_2 = \frac{2x\cdot e^x}{2(1)-4}$$

$$P_2 = \frac{2x\cdot e^x}{-2}$$

$$\boxed{P_2 = -x\cdot e^x}$$

$$P_3 = \frac{1 \cdot e^{0x}}{D^2 - 4D + 3}$$

∴ Replace 'D' by 'a = 0'

$$\boxed{P_3 = \frac{1}{3}}$$

∴ Substitute P_1, P_2, P_3 in eqn (3)

$$\therefore \boxed{P_2 = -e^{2x} - x \cdot e^x + \frac{1}{3}}$$

General solution is:-

$$y = CF + PI$$

$$\boxed{y = C_1 e^x + C_2 e^{3x} + \frac{1}{3} - e^{2x} - xe^x}$$

(3) Given,

$$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = \sin(2\log x) \quad \text{--- (1)}$$

∴ From Cauchy's D.E :-

$$x^2 \frac{d^2y}{dx^2} = D(D-1)y \qquad x = e^t$$

$$t = \log x \qquad \text{where, } D = \frac{d}{dt}$$

$$x \frac{dy}{dx} = Dy$$

∴ Substitute the above values in eqn (1)

$$\therefore D(D-1)y - 4Dy + 6y = \sin(2\log x)$$

$$D(D-1)y - 4Dy + 6y = \sin(2t)$$

$$D^2y - Dy - 4Dy + 6y = \sin 2t$$

$$D^2y - 5Dy + 6y = \sin 2t \quad \text{--- (2)}$$

$(D^2-5D+6)y = \sin 2t$ ... (2)

∴ Consider the Homogeneous Part of eqn (2)

$(D^2-5D+6)y = 0$

∴ Auxillary eqn is:-

$m^2-5m+6=0$

$m^2-3m-2m+6=0$

$m(m-3)-2(m-3)=0$

$(m-2)(m-3)=0$

$m = 2, 3$

∴ The complementary function is:- $CF = C_1e^{2t} + C_2e^{3t}$

$PI = \frac{\phi(x)}{f(D)}$

$PI = \frac{\sin 2t}{D^2-5D+6}$

$PI = \frac{\sin 2t \cdot t}{2D-5}$

$PI = \frac{\sin 2t}{-4-5D+6}$ (∵ Replace D^2 by $-a^2$, $a=2$, $a^2=4$)

$PI = \frac{\sin 2t}{2-5D} \times \frac{2+5D}{2+5D}$

$PI = \frac{2\sin 2t + 5D\sin 2t}{(2)^2-(5D)^2}$

$PI = \frac{2\sin 2t + 5\cos 2t \cdot 2}{4-25D^2}$ (∵ $\frac{d}{dx}(\sin ax) = a\cos ax$)

$$PI = \frac{2\sin 2t + 10\cos 2t}{4 - 2D^2}$$

$$PI = \frac{2\sin 2t}{4 - 2D^2} + \frac{10\cos 2t}{4 - 2D^2}$$

$$PI = \frac{2\sin 2t}{4 - 2(-4)} + \frac{10\cos 2t}{4 - 2(-4)}$$

$$PI = \frac{2\sin 2t}{12} + \frac{10\cos 2t}{12}$$

$$PI = \frac{2\sin 2t + 10\cos 2t}{12}$$

$$PI = \frac{2(\sin 2t + 5\cos 2t)}{12}$$

$$\boxed{PI = \frac{\sin 2t + 5\cos 2t}{52}}$$

General solution is:-

$$y = CF + PI$$

$$y = c_1 e^{2t} + c_2 e^{3t} + \frac{1}{52}[\sin 2t + 5\cos 2t]$$

$$y = c_1 e^{2(\log x)} + c_2 e^{3(\log x)} + \frac{1}{52}[\sin 2(\log x) + 5\cos 2(\log x)] \quad (\because t = \log x)$$

$$\boxed{y = c_1 x^2 + c_2 x^3 + \frac{1}{52}[\sin(2\log x) + 5\cos(2\log x)]} \quad (\because e^{\log x} = x)$$

(4) Given,

$$\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 4y = 3\sin x + 4\cos x \quad -①$$

As WKT

$$D = \frac{d}{dx}$$

∴ we can write eqn ① as :-

$$D^2y + 4Dy + 4y = 3\sin x + 4\cos x$$

$$(D^2 + 4D + 4)y = 3\sin x + 4\cos x \quad -②$$

∴ By considering the Homogeneous Part of eqn ②

$$(D^2 + 4D + 4)y = 0$$

∴ Auxiliary eqn is:-

$$m^2 + 4m + 4 = 0$$

$$m^2 + 2m + 2m + 4 = 0$$

$$m(m+2) + 2(m+2) = 0$$

$$(m+2)(m+2) = 0$$

$$\boxed{m = -2, -2}$$

$$CF = (C_1 + C_2x)e^{-2x}$$

∴ The complementary function is:-

$$CF = (C_1 + C_2x)e^{-2x}$$

$$PI = \frac{\phi(x)}{f(D)}$$

$$PI = \frac{3\sin x + 4\cos x}{D^2+4D+4}$$

$$PI = \frac{3\sin x}{D^2+4D+4} + \frac{4\cos x}{D^2+4D+4}$$

$$PI = \frac{3\sin x}{-1+4D+4} + \frac{4\cos x}{-1+4D+4}$$

$$PI = \frac{3\sin x}{4D+3} + \frac{4\cos x}{4D+3}$$

$$PI = \frac{3\sin x}{4D+3}\times\frac{4D-3}{4D-3} + \frac{4\cos x}{4D+3}\times\frac{4D-3}{4D-3}$$

$$PI = \frac{12D\sin x - 9\sin x}{(4D)^2-(3)^2} + \frac{16D\cos x - 12\cos x}{(4D)^2-(3)^2}$$

$$PI = \frac{12\cos x - 9\sin x}{16D^2-9} + \frac{(-16\sin x - 12\cos x)}{16D^2-9}$$

$$PI = \frac{12\cos x - 9\sin x}{16(-1)-9} + \frac{(-16\sin x - 12\cos x)}{16(-1)-9} \Rightarrow \frac{12\cos x - 9\sin x}{-25} + \frac{(-16\sin x - 12\cos x)}{-25}$$

$$PI = \frac{12\cos x - 9\sin x - 16\sin x - 12\cos x}{-25}$$

$$PI = \frac{-25\sin x}{-25}$$

$$\boxed{PI = \sin x}$$

$$\left[\begin{array}{l} D^2 = -a^2 \\ \therefore a = 1 \\ a^2 = 1 \\ D^2 = -1 \end{array}\right]$$

General solution is:- $y = CF + PI$

$$\boxed{y = (C_1 + C_2 x)e^{-2x} + \sin x}$$

5) Given,

x	y	xy	x^2
1	6	6	1
2	4	8	4
3	3	9	9
4	5	20	16
5	4	20	25
6	2	12	36
$\Sigma x = 21$	$\Sigma y = 24$	$\Sigma xy = 75$	$\Sigma x^2 = 91$

∴ As WKT,

The eqn of straight line is:- $y = ax + b$ - ①

∴ $\Sigma y = a\Sigma x + bn$ - ②

$\Sigma xy = a\Sigma x^2 + b\Sigma x$ - ③

} (The normal equations of the straight line equation.)

∴ From the given data,

$n = 6$

$\Sigma x = 21$, $\Sigma y = 24$, $\Sigma xy = 75$, $\Sigma x^2 = 91$

∴ Substitue the values in eqn ②, ③ we get

$24 = a(21) + b(6)$ - ②a

$75 = a(91) + b(21)$ - ③a

∴ $21a + 6b = 24$ - (2a)

$91a + 21b = 75$ - (3a)

∴ By solving the above eqn's we get

$a = -0.51429$, $b = 5.8$

∴ By Substituting "a & b" values in eqn ① we get

$y = (-0.51429)x + 5.8$

∴ The Least Square line is :- $\boxed{y = 5.8 - (0.51429)x}$

(6) Given

x	y	r_x	r_y	$d = r_x - r_y$	d^2
6	8	7.5	4	3.5	12.25
5	7	9	6.5	2.5	6.25
8	7	3.5	6.5	-3	9
8	10	3.5	1.5	2	4
7	5	5.5	10	-4.5	20.25
6	8	7.5	4	3.5	12.25
10	10	1	1.5	-0.5	0.25
4	6	10	8.5	1.5	2.25
9	8	2	4	-2	4
7	6	5.5	8.5	-3	9

$\Sigma d^2 = 79.5$

$n = 10$

$m_1 = 2$
$m_2 = 2$
$m_3 = 2$
$m_4 = 2$
$m_5 = 3$
$m_6 = 2$
$m_7 = 2$

$$R = 1 - \left[\frac{6\Sigma d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \frac{m_3(m_3^2-1)}{12} + \frac{m_4(m_4^2-1)}{12} + \frac{m_5(m_5^2-1)}{12} + \frac{m_6(m_6^2-1)}{12} + \frac{m_7(m_7^2-1)}{12}}{n(n^2-1)} \right]$$

$$R = 1 - \left[\frac{6(79.5) + \frac{2\times3}{12} + \frac{2\times3}{12} + \frac{2\times3}{12} + \frac{2\times3}{12} + \frac{3\times8}{12} + \frac{2\times3}{12} + \frac{2\times3}{12}}{10(100-1)} \right]$$

$$R = 1 - \left[\frac{477 + 0.5 + 0.5 + 0.5 + 0.5 + 2 + 0.5 + 0.5}{10(99)}\right]$$

$$R = 1 - \left[\frac{477 + 5}{990}\right]$$

$$R = 1 - \left[\frac{482}{990}\right]$$

$$R = \frac{254}{495}$$

$$\boxed{R = 0.5131}$$

∴ The rank correlation for the given data is :- $\boxed{R = 0.5131}$

7) Given,

x	y	r_x	r_y	$d = r_x - r_y$	d^2
68	62	4	5	-1	1
64	58	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	1	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
40	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16

$\Sigma d^2 = 72$

$m_1 = 2$

$m_2 = 3$

$m_3 = 2$

$n = 10$

$$R = 1 - \left[\frac{6\Sigma d^2 + \frac{m_1(m_1^2-1)}{12} + \frac{m_2(m_2^2-1)}{12} + \frac{m_3(m_3^2-1)}{12}}{n(n^2-1)} \right]$$

$$R = 1 - \left[\frac{6(72) + \frac{2(4-1)}{12} + \frac{3(8)}{12} + \frac{2(5-1)}{12}}{10(100-1)} \right]$$

$$R = 1 - \left[\frac{432 + 0.5 + 2 + 0.5}{10(99)} \right]$$

$$R = 1 - \left[\frac{435}{990} \right]$$

$$R = 0.5606$$

∴ The Rank correlation coefficient for the following data is:-

$R = 0.561$

(8) Given.

$r = 0.5$

$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

T.P:-

$\sigma_x = 2\sigma_y$ (or) $\sigma_y = 2\sigma_x$.

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}\left(\frac{1-r^2}{r}\right)$$

$$\therefore \tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$$

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-(0.5)^2}{0.5}\right)$$

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{0.75}{0.5}\right)$$

$$\frac{3}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{3}{2}\right)$$

$$\frac{2}{5} = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

The lines of Regression are respectively:-

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$2x^2 + 2y^2 = 5xy$

$2x^2 - xy + 2y^2 = 0$

$2x^2 - 4xy - xy + 2y^2 = 0$

$2x(x - 2y) - y(x - 2y) = 0$

$(2x - y)(x - 2y) = 0$

$\therefore 2x = y$ (or) $x = 2y$

$\therefore x = 2y$ (or) $y = 2x$

Hence Proved.