CBCS SCHEME

USN 1CR23MC101

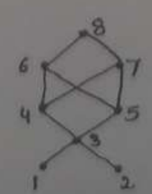
22MCA11

First Semester MCA Degree Examination, Dec.2023/Jan.2024
Mathematical Foundation for Computer Applications

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define a set, an empty set and a singleton set, with an example for each.	6	L3	CO1
	b.	For any two sets A and B, prove that (i) $\overline{A}\cap\overline{B}=\overline{A\cup B}$. (ii) $\overline{A}\cup\overline{B}=\overline{A\cap B}$.	7	L3	CO1
	c.	Find the eigen values and eigen vectors of the matrix $\begin{pmatrix}3 & 2\\ -1 & 0\end{pmatrix}$.	7	L3	CO1
		OR			
Q.2	a.	Let $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{1,2,4,6,8\}$, $B=\{2,4,5,9\}$. Compute the following : (i) $A\cup B$ (ii) $A\cap B$ (iii) $A-B$ (iv) $\overline{A}$	6	L3	CO1
	b.	A total of 1232 students have taken a course in Java, 879 in C and 114 in C++. Further, 103 have taken courses in both Java and C, 23 in both Java and C++ and 14 in both C and C++. If 2092 students have taken at least one of Java, C and C++, how many students have taken a course in all three subjects.	7	L3	CO1
	c.	State pigeon-hole principle. Show that if 50 books in a library contain a total of 27551 pages, one of the books must have atleast 552 pages.	7	L3	CO1
		Module – 2			
Q.3	a.	Write the converse, inverse and contrapositive of, "If 2 is an even number then 7 is a prime number".	6	L2	CO3
	b.	Define tautology. Show that $[(p\vee q)\wedge(p\to r)\wedge(q\to r)]\to r$ is a tautology.	7	L2	CO3
	c.	If $A=\{1,2,3,4,5\}$ is the universe of disclosure, determine the truth values of, (i) $\forall x\in A,(x+2<10)$ (ii) $\exists x\in A,(x+2=10)$ (iii) $\forall x\in A,(x^2\le 25)$	7	L2	CO3

		OR			
Q.4	a.	Show that for any three propositions p, q, r, $p \rightarrow (q \wedge r) \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$.	6	L2	CO3
	b.	Negate and simplify : (i) $\forall x, [p(x) \wedge \neg q(x)]$. (ii) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$	7	L2	CO3
	c.	Show that the following argument is valid : "If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore, I have a test in Mathematics".	7	L2	CO3
		Module – 3			
Q.5	a.	Let $A = \{1, 2, 3, 4, 6\}$ and R be a relation on A defined by 'aRb iff a is a multiple of b'. Write down R as a set of ordered pairs. Write matrix and digraph of R.	6	L3	CO1
	b.	Consider $A = \{1, 2, 3, ...11, 12\}$. The relation R on A is defined as $(x, y) \in R$ iff x – y is a multiple of 5. Verify that R is an equivalence relation.	7	L3	CO1
	c.	Let $A = \{2, 3, 4, 6, 8, 12, 24\}$ and R denote the partial order of divisibility 'xRy iff x divides y'. Find R and draw its Harse diagram.	7	L3	CO1
		OR			
Q.6	a.	Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Relations R and S from A to B are given by, $M_R = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$, $M_S = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ Find (i) $R \cup S$ (ii) $R \cap S$ (iii) $\overline{S}$	6	L3	CO1
	b.	Let $A = \{1, 2, 3, 4\}$ & $R = \{(1,1), (1,2), (2,3), (3,4)\}$, $S = \{(3,1), (4,4), (2,4), (1,4)\}$ be relations on A. Determine the relations $R \circ S$, $S \circ R$, R^2, S^2 and draw their diagraphs.	7	L3	CO1
	c.	Consider the POSET whose Harse diagram Fig. Q6 (c) is as given below. Consider $B = \{3, 4, 5\}$. Find (i) Maximal elements (ii) Minimal elements (iii) Greatest elements (iv) Least elements (v) Upper bound of B (vi) Lower Bounds of B (if they exist)  Fig. Q6 (c)	7	L2	CO4

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Module – 4

Q.7 **a.** Find the value of K such that the following distribution represents a finite probability distribution. Also find its mean and variance. *(10, L3, CO2)*

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

b. When a coin is tossed 4 times, find the probability of getting, (i) exactly one head (ii) atmost 3 heads (iii) atleast two heads. *(10, L3, CO2)*

OR

Q.8 **a.** Find K such that $f(x)=\begin{cases} kx^2, & -3 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$ is a probability density function. *(6, L3, CO2)*

Also find (i) $P(1 \le x \le 2)$ (ii) $P(x>1)$

b. In a certain town, the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that the shower will last for, *(7, L3, CO2)*
- (i) 10 minutes or more
- (ii) less than 10 minutes
- (iii) between 10 and 12 minutes.

c. The marks of 1000 students in an exam follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be, (i) less than 65 (ii) more than 75 (iii) 65 to 75. *(7, L3, CO2)*

Given that $\phi(1)=0.3413$, where $z=\dfrac{x-\mu}{\sigma}$.

Module – 5

Q.9 **a.** Define the following with an example for each : *(6, L2, CO4)*
- (i) Simple graph
- (ii) Regular graph
- (iii) Spanning subgraph

b. Check whether the following graphs in Fig. Q9 (b) are isomorphic. *(7, L2, CO4)*

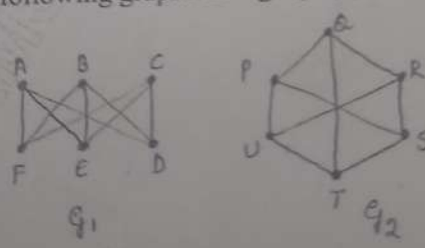

Fig. Q9 (b)

c. Explain Konigsberg Bridge Problem. *(7, L2, CO4)*

OR

Q.10 **a.** Define Euler circuit, Hamilton cycle and Planar graphs with an example for each. *(6, L2, CO4)*

			6	L2	CO3
		OR			
		any three propositions p, q, r,			

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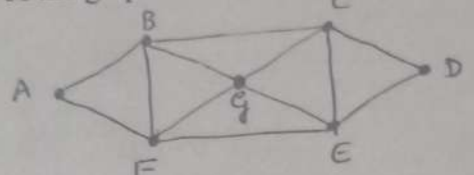
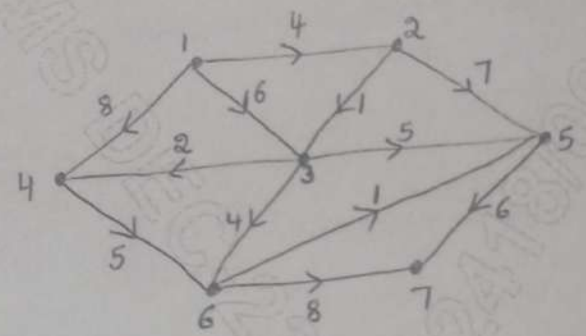
	b.	Give graph coloring of the graph F.  Fig. Q10 (b)	7	L2	CO4
	c.	Using the Dijkstra's algorithm. Obtain the shortest path from vertex 1 to each of the other vertices in the weighted, directed network shown below in Fig. Q10 (c)  Fig. Q10 (c)	7	L2	CO4

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Mathematical Foundation for Computer Applications: Dec 23/Jan 24

1a) A set is a collection of well defined objects.

Eg: $\{2, 4, 6, 8, \ldots\}$

Empty set: A set that contains no elements is called an empty set.

Eg: set of all prime numbers from 8 to 10.

Singleton set: A set containing a single element is called a singleton set.

Eg: set of all multiples of 3 from 7 to 10.

1b. $\overline{A} \cap \overline{B} = \{x / x \in \overline{A} \cap \overline{B}\}$

$= \{x / x \in \overline{A} \text{ and } x \in \overline{B}\}$

$= \{x / x \notin A \text{ and } x \notin B\}$

$= \{x / x \notin A \cup B\}$

$= \{x / x \in \overline{A \cup B}\} = \overline{A \cup B}$ $\therefore \boxed{\overline{A} \cap \overline{B} = \overline{A \cup B}}$

$\overline{A} \cup \overline{B} = \{x / x \in \overline{A} \text{ or } x \in \overline{B}\}$

$= \{x / x \notin A \text{ or } x \notin B\}$

$= \{x / x \notin A \cap B\}$

$= \{x / x \in \overline{A \cap B}\}$ $\therefore \boxed{\overline{A} \cup \overline{B} = \overline{A \cap B}}$

1c) $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$

characteristic eqⁿ of $A = |A - \lambda I| = 0$

$\begin{vmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda(3-\lambda) + 2 = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda(\lambda - 2) - 1(\lambda - 2) = 0$

$(\lambda - 2)(\lambda - 1) = 0$

$\lambda = 1, 2$

To find Eigen vector consider

$(A - \lambda I)X = 0$

$$\begin{pmatrix} 3-\lambda & 2 \\ -1 & -\lambda \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \left.\begin{array}{l} (3-\lambda)x + 2y = 0 \\ \cancel{(-1-\lambda)x} \\ -x - \lambda y = 0 \end{array}\right\} - (1)$$

Case (i) Put $\lambda = 1$ in (1)

$2x + 2y = 0 \Rightarrow x = -y$

$-x - y = 0$

$(1, -1)$ is the eigenvector

<u>Case (ii)</u> Put $\lambda = 2$ in (1)

$x + 2y = 0 \Rightarrow x = -2y$

$-x - 2y = 0$

$\frac{x}{-2} = \frac{y}{1}$

$(-2, 1)$ is the eigenvector.

2 (a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 4, 6, 8\}$

$B = \{2, 4, 5, 9\}$

(i) $A \cup B = \{1, 2, 4, 5, 6, 8, 9\}$

(ii) $A \cap B = \{2, 4\}$

(iii) $A - B = \{1, 6, 8\}$

(iv) $\bar{A} = U - A = \{3, 5, 7, 9\}$

(b) Let A, B, C be the set of all students who have taken a course in Java, C, C^{++} respectively.

Given $|A| = 1232$, $|B| = 879$, $|C| = 114$, $|A \cap B| = 103$, $|A \cap C| = 23$,

$|B \cap C| = 14$, $|A \cup B \cup C| = 2092$, $|A \cap B \cap C| = ?$

From the principle of Inclusion - Exclusion,

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$2092 = 1232 + 879 + 114 - 103 - 23 - 14 + |A \cap B \cap C|$

$\Rightarrow |A \cap B \cap C| = 7$

$\therefore$ 7 students have taken all three courses.

(c) Let us treat 27551 pages as pigeons & 50 books as pigeon holes.

From Pigeon hole Principle, at least one pigeon hole must contain

p+1 or more pigeons in it. Here, $m = 27551$, $n = 50$

$$p = \left\lfloor \frac{m-1}{n} \right\rfloor = \left\lfloor \frac{27551-1}{50} \right\rfloor = 551$$

$$p+1 = 552.$$

Hence, atleast one book must contain 552 pages in it.

3(a) Let p: 2 is an even no

q: 7 is a prime no

Given statement is $p \rightarrow q$

Converse: $q \rightarrow p$

ie., If 7 is a prime no then 2 is an even no.

Inverse: $\neg p \rightarrow \neg q$

ie., If 2 is not an even no then 7 is not a prime no.

Contrapositive: $\neg q \rightarrow \neg p$

ie., If 7 is not a prime no then 2 is not an even no.

3(b) If a compound statement is always true regardless of truth values of its components then it is called a tautology.

p	q	r	$p\vee q$	$p\to r$	$q\to r$	$[(p\vee q)\wedge(p\to r)\wedge(q\to r)]$	$x\to r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	1	1	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Since all the entries of the last column are 1's, it is a tautology

3(c) $A = \{1, 2, 3, 4, 5\}$

(i) $\forall x \in A,\ (x+2<10)$

$1+2<10,\quad 2+2<10,\quad 3+2<10,\quad 4+2<10,\quad 5+2<10$

Hence the statement is true.

(ii) $\exists x \in A,\ (x+2=10)$

False $x+2 \neq 10$ for any $x \in A$.

(iii) $\forall x \in A,\ (x^2 \leq 25)$

$1^2 \leq 25,\ 2^2 \leq 25,\ 3^2 \leq 25,\ 4^2 \leq 25,\ 5^2 \leq 25$

True.

4(a) $LHS = p \to (q \wedge r)$

$\Leftrightarrow \neg p \vee (q \wedge r)$ $\qquad p\to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow (\neg p \vee q)\wedge(\neg p \vee r)$

$\Leftrightarrow (p\to q)\wedge(p\to r)$

$= RHS.$

4(b) (i) Negation $\quad \neg\left[\forall x,\ \{p(x) \land \neg q(x)\}\right]$

$\iff \exists x,\ \neg\{p(x) \land \neg q(x)\}$

$\iff \exists x,\ \{\neg p(x) \lor q(x)\}$

(ii) Neg^n: $\neg\left\{\exists x, \left[\{p(x) \lor q(x)\} \rightarrow r(x)\right]\right\}$

$\iff \forall x,\ \neg\left[\{p(x) \lor q(x)\} \rightarrow r(x)\right]$

$\iff \forall x,\ \{p(x) \lor q(x)\} \land \neg r(x)$

4(c) Let p: Today is Tuesday.

q: I have test in Maths

r: I have a test in Economics.

s: My Economics prof is sick

~~t: I will have a test in Economics~~

Given
$$\begin{array}{l} p \rightarrow (q \lor r) \\ s \rightarrow \neg r \\ p \land s \\ \hline \therefore q \end{array} \implies \begin{array}{l} p \rightarrow (q \lor r) \\ s \rightarrow \neg r \\ p \\ s \\ \hline \therefore q \end{array}$$

$$\implies \begin{array}{l} (q \lor r) \\ \neg r \\ \hline \therefore q \end{array} \quad \text{Modus pones} \implies \begin{array}{l} \neg q \rightarrow r \\ \neg r \\ \hline \therefore q \end{array}$$

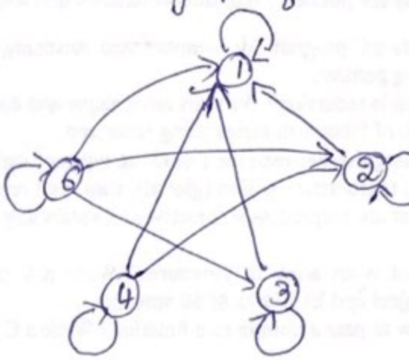
$$\implies \begin{array}{l} \neg(\neg q) \\ \hline \therefore q \end{array} \quad \text{Modus Tollens} \implies \begin{array}{l} q \\ \hline \therefore q \end{array}$$

It is a valid argument.

5 (a) $A = \{1,2,3,4,6\}$ & 'aRb iff a is a multiple of b'.

$R = \{(1,1), (2,1), (2,2), (4,1), (4,2), (4,4), (6,1), (6,2), (6,3), (6,6)\}$

Digraph of R:



$$M(R) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

5(b) $A = \{1, 2, 3, \ldots, 11, 12\}$

$R = \{(1,1), (1,6), (1,11), (2,2), (2,7), (2,12), (3,3), (3,8), (4,4), (4,9), (5,5), (5,10), (6,1), (6,6), (6,11), (7,2), (7,7), (7,12), (8,3), (8,8), (9,4), (9,9), (10,5), (10,10), (11,1), (11,6), (11,11), (12,2), (12,7), (12,12)\}$

Reflexive: wkt $x - x = 0$ is a multiple of 5, $\forall x \in A$

$\Rightarrow (x,x) \in R$, $\forall x \in A$

$\Rightarrow$ R is reflexive

Symmetric: Suppose $(x,y) \in R$

$\Rightarrow x-y$ is a multiple of 5.

$\Rightarrow -(x-y)$ is a multiple of 5.

$\Rightarrow (y-x)$ is a multiple of 5.

$\Rightarrow (y,x) \in R$

$\therefore$ R is symmetric.

Anti-Symmetric: Suppose $(x,y) \in R$ & $(y,z) \in R$

$\Rightarrow x-y$ is a multiple of 5 & $y-z$ is a multiple of 5.

$\Rightarrow x-y = 5k_1$ & $y-z = 5k_2$; $k_1, k_2 \in Z$

$\Rightarrow (x-y)+(y-z) = x-z = 5(k_1-k_2)$ which is a multiple of 5.

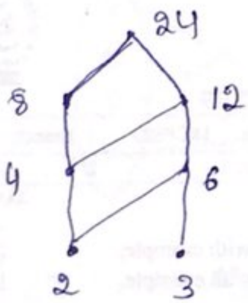
$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence R is an equivalence relⁿ.

5(c) $A = \{2, 3, 4, 6, 8, 12, 24\}$

$R = \{(2,2), (2,4), (2,6), (2,8), (2,12), (2,24), (3,3), (3,6), (3,12), (3,24), (4,4), (4,8), (4,12), (4,24), (6,6), (6,12), (6,24), (8,8), (8,24), (12,12), (12,24), (24,24)\}$.



6(a) $R = \{(1,1), (1,3), (2,4), (3,1), (3,2), (3,3)\}$

$S = \{(1,1), (1,2), (1,3), (1,4), (2,4), (3,2), (3,4)\}$

(i) $R \cup S = \{(1,1), (1,2), (1,3), (1,4), (2,4), (3,1), (3,2), (3,3), (3,4)$

(ii) $R \cap S = \{(1,1), (1,3), (2,4), (3,2)\}$

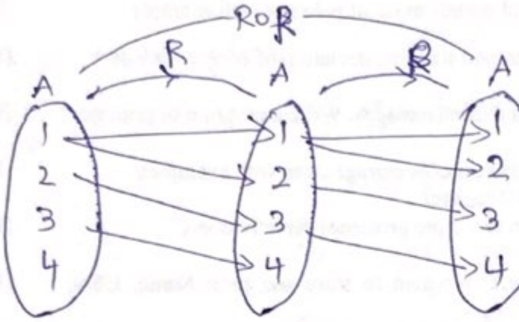
(iii) $\bar{S} = (A \times B) - S = \{(2,1), (2,2), (2,3), (3,1), (3,3)\}$

6(b) $A = \{1, 2, 3, 4\}$

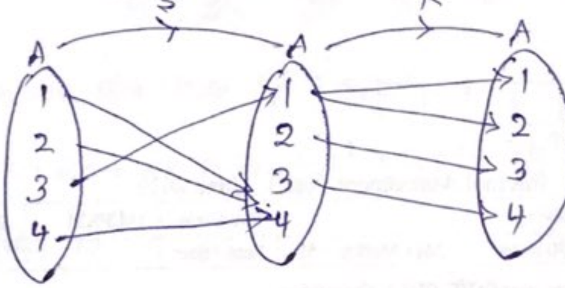
$R = \{(1,1), (1,2), (2,3), (3,4)\}$

$S = \{(3,1), (4,4), (2,4), (1,4)\}$

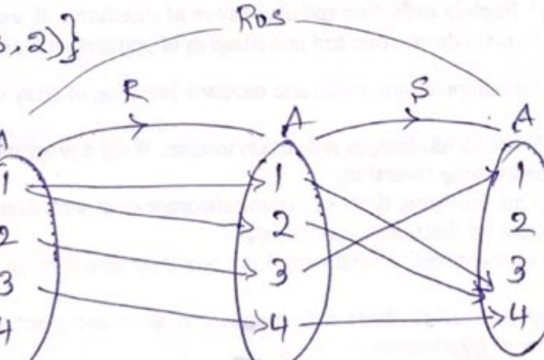
$\overset{2}{\text{Ro}\cancel{S}}$:



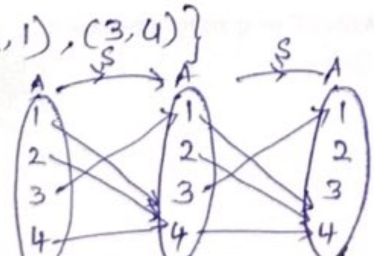
$R^2 = R \circ R = \{(1,1), (1,3), (1,2), (2,4)\}$



$S \circ R = \{(3,1), (3,2)\}$

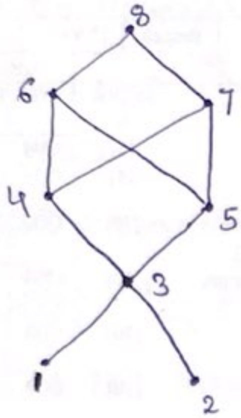


$R \circ S = \{(1,4), (2,1), (3,4)\}$



$S^2 = S \circ S = \{(1,4), (2,4), (3,4), (4,4)\}$

6(c)



$B = \{3, 4, 5\}$

(i) Maximal Elements $= \{8\}$

(ii) Minimal Elements $= \{1, 2\}$

(iii) Greatest element $= \{8\}$

(iv) Least element = Doesn't exist

(v) Upper Bound of $B = \{6, 7, 8\}$

(vi) Lower Bound of $B = \{1, 2, 3\}$

7(a)

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

We must have $P(x) \geq 0$ & $\sum p(x) = 1$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$ $\qquad \Rightarrow k = -1$ & $k = 1/10$

$\Rightarrow 10k^2 + 10k - k - 1 = 0$ $\qquad$ If $k = -1$ then the condition

$\Rightarrow 10k(k+1) - 1(k+1) = 0$ $\qquad p(x) \geq 0$ fails.

$\Rightarrow (k+1)(10k-1) = 0$ $\qquad$ Hence $\boxed{k = 1/10}$

x	0	1	2	3	4	5	6	7
$p(x)$	0	$1/10$	$1/5$	$1/5$	$3/10$	$1/100$	$1/50$	$17/100$

Mean, $\mu = \sum x p(x) = 0(0) + 1(1/10) + 2(1/5) + 3(1/5) + 4(3/10) + 5(1/100)$

$$+ 6(1/50) + 7(17/100) = \frac{183}{50} = 3.66$$

Variance, $V = \sum (x-\mu)^2 \cdot p(x) = (1-3.66)^2 \, 1/10 + (2-3.66)^2 \, 1/5 +$
$(3-3.66)^2 \, 1/5 + (4-3.66)^2 \, 3/10 + (5-3.66)^2 \, 1/100 + (6-3.66)^2 \, 1/50 + (7-3.66)^2 \, 17/100$

7(b) $p = P(\text{head}) = \frac{1}{2} = 0.5$

$q = 1 - p = 0.5$

$n = 4$

Wkt probability of x successes out of n trials is given by

$P(x) = {}^{n}C_x \, p^x q^{n-x}$

(i) $P(1 \text{ head}) = P(x=1) = {}^4C_1 (0.5)^1 (0.5)^3 = 0.25$

(ii) $P(\text{atmost } 3 \text{ heads}) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= {}^4C_0 (0.5)^0 (0.5)^4 + {}^4C_1 (0.5)^1 (0.5)^3 + {}^4C_2 (0.5)^2 (0.5)^2 + {}^4C_3 (0.5)^3 (0.5)$

$= 0.9375$

(iii) $P(\text{atleast } 2 \text{ heads}) = 1 - [P(x=0) + P(x=1)]$

$= 1 - [(0.5)^4 + {}^4C_1 (0.5)(0.5)^3]$

$= 0.6875$

8(a) $f(x) = \begin{cases} kx^2, & -3 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$

Wkt $f(x) \ge 0$ & $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{-3}^{3} kx^2\,dx = 1 \Rightarrow k\left[\frac{x^3}{3}\right]_{-3}^{3} = 1 \Rightarrow \frac{k}{3}\left[3^3 - (-3)^3\right] = 1$

$\Rightarrow \frac{k}{3}[27 + 27] = 1 \quad \Rightarrow k = \frac{1}{18}$

(i) $P(1 \le x \le 2) = \int_{1}^{2} \frac{1}{18} x^2 \cdot dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{1}^{2} = \frac{7}{54}$

(ii) $P(x > 1) = \int_{1}^{3} \frac{1}{18} x^2 \cdot dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{1}^{3} = \frac{13}{27}$

8(b) The p.d.f of exp distribution is given by

$f(x) = \alpha e^{-\alpha x}, \quad x > 0$

Mean, $\mu = 1/\alpha = 5 \Rightarrow \alpha = 1/5$

$\therefore f(x) = \frac{1}{5} e^{-x/5}$

(i) $P(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx = \frac{1}{5}\left[\frac{e^{-x/5}}{-1/5}\right]_{10}^{\infty} = \frac{1}{e^2} - 0 = 0.1353$

(ii) $P(x < 10) = \int_{0}^{10} \frac{1}{5} e^{-x/5} dx = -\left[e^{-x/5}\right]_{0}^{10} = 1 - e^{-2} = 0.8647$

(iii) $P(10 < x < 12) = \int_{10}^{12} \frac{1}{5} e^{-x/5} dx =$

8(c) Here Mean, $\mu = 70$, $\sigma = 5$

$z = \frac{x-\mu}{5} = \frac{x-70}{5}$

(i) If $x = 65$, $z = -1$

$P(z < -1) = P(z > 1) = P(z \geq 0) - P(0 \leq z \leq 1)$

$= 0.5 - \phi(1) = 0.1587$

(ii) If $x = 75$, $z = 1$

$P(z > 1) = 0.1587$

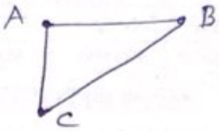
(iii) $P(-1 < z < 1) = 2P(0 < z < 1) = 2\phi(1) = 2(0.3413) = 0.6826$

$\therefore$ No of students scoring marks less than $65 = 0.1587 \times 1000$
$= 158.7 \simeq 159$

——— " ——————————— more than $75 = 0.1587 \times 1000$
$\simeq 159$

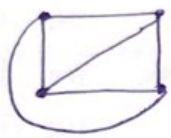
——— " ——— b/w 65 to 75 $= 0.6826 \times 1000 \simeq 683$

9(a) (i) Simple graph: A graph that has no loops or no multiple edges is called a simple graph.



(ii) Regular graph:

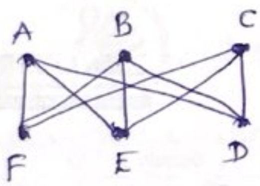
A graph in which the degree of every vertex is same, say k, is called a k-regular graph.



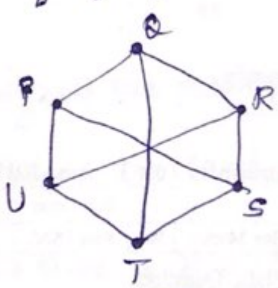
3-regular graph

(iii) A subgraph of $G_1(V_1, E_1)$ of $G(V, E)$ is said to be a spanning subgraph of G if $V_1 = V$.

9(b)



G_1



G_2

- Both G_1 & G_2 have 6 vertices.
- Both G_1 & G_2 have 9 edges.
- Both G_1 & G_2 are cubic graphs.

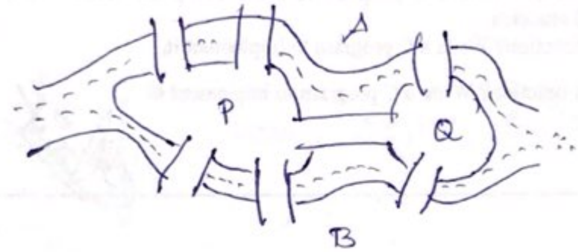
There exists an 1-1 correspondence b/w the vertices of G_1 & G_2.

Hence they are Isomorphic.

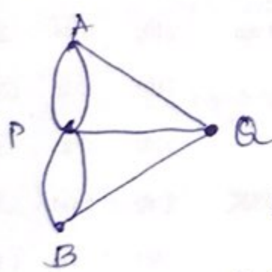
- Mapping b/w the vertices: $A \leftrightarrow P$, $B \leftrightarrow T$, $C \leftrightarrow R$, $D \leftrightarrow S$, $E \leftrightarrow Q$, $F \leftrightarrow U$.

- Mapping b/w the edges: $\{A, F\} \leftrightarrow \{P, U\}$, $\{A, E\} \leftrightarrow \{P, Q\}$, $\{A, D\} \leftrightarrow \{P, S\}$
$\{B, F\} \leftrightarrow \{T, U\}$, $\{B, E\} \leftrightarrow \{T, Q\}$, $\{B, D\} \leftrightarrow \{T, S\}$, $\{C, F\} \leftrightarrow \{R, U\}$,
$\{C, E\} \leftrightarrow \{R, Q\}$, $\{C, D\} \leftrightarrow \{R, S\}$

9(c) In 18^{th} Century, in a city named Konigsberg, a river flowed named Piegel river which divides city into 4 parts - 2 banks of the river and 2 islands. There 4 land areas were connected to each other by 7 bridges. Citizens of the city posed a problem - "by starting at any of the four land areas, can we return to that area after crossing each bridge exactly once?". This problem is known as Konigsberg bridge problem.



In 1736, Euler analyzed the problem with the help of a graph & gave the solution. Denote the land areas as A, B, P, Q as shown in the diagram. Construct a graph by treating the four land areas as four vertices & 7 bridges as edges connecting the vertices.

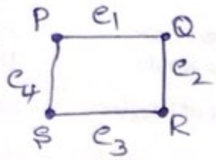


We note that in this graph

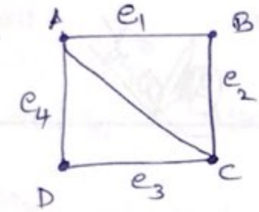
$deg(A) = deg(B) = deg(Q) = 3$ & $deg(P) = 5$

which are not even. $\therefore$ the graph doesn't have an Euler circuit. Hence, it is not possible to walk over each of the seven bridges exactly once & return to the starting point.

10 a) Euler Circuit - If there is a circuit in G that contains all the edges of G, then the circuit is called an Euler circuit in the graph G.

Eg :-  Here, $Pe_1Qe_2Re_3Se_4P$ is an Euler circuit.

Hamilton Cycle: In a connected graph G, if there is a cycle that contains all the vertices of G, then that cycle is called a Hamilton cycle in G.

Eg :-  Here, $Ae_1Be_2Ce_3De_4A$ is a Hamilton cycle.

Planar Graphs: A graph which can be represented as one plane drawing in which edges meet only at vertices is called a planar graph. Eg: 

10b) Let's assign the colour C_1 to G. Then let's assign a colour C_2 to B & E which are adjacent to G. C & F are adjacent to B & E. So, let's assign a colour C_3 to C & F. Vertices A & D are not adjacent to G. So, we can assign the colour C_1 to them. We have done the colouring with only 3 colours. ∴ chromatic number is 3.

10c) For the first iteration, consider the set $P = \{1\}$ & $t_j = a_{1j}$,
$t_2 = a_{12} = 4$, $t_3 = a_{13} = 6$, $t_4 = 8$, $t_5 = \infty$, $t_6 = \infty$, $t_7 = \infty$.

Step1: t_j is minimum for $j=2$ ie., $t_2=4$. Label the arc $(1,2)$ as P_1.

Also we adjoin 2 to P. $\Rightarrow$ $P=\{1,2\}$

Step 2: $P=\{1,2\}$, $t_2=4$. Choose new $t_3 = \min\{t_3, t_2+q_{23}\} = \min\{6,5\}$
$= 5$

new $t_4 = \min\{t_4, t_2+q_{24}\} = \min\{8, 4+\infty\} = 8$

new $t_5 = 11$, new $t_6 = \infty$, new $t_7 = \infty$.

Second Iteration: $P=\{1,2\}$, $t_2=4$, $\underbrace{t_3=5,\ t_4=8,\ t_5=11,\ t_6=\infty,\ t_7=\infty}_{\text{new } t_j\text{'s}}$

Step1: Among new t_j's, min $t_j = 5$ for $j=3$.

Among the vertices from P, arc $(2,3)$ has least weight. $(2,3)$ is P_2.

$P=\{1,2,3\}$

Step 2: new $t_4=7$, new $t_5=10$, new $t_6=9$, new $t_7=\infty$

Third itrn: $P=\{1,2,3\}$, $t_2=4$, $t_3=5$ New $t_4=7$, $t_5=10$, $t_6=9$, $t_7=\infty$

Step1: Among new t_j's $t_4=7$ is min. for $j=4$. Label $(3,4)$ as P_3 that has least weight among the arcs from vertices of P to vertex 4. $P=\{1,2,3,4\}$.

Step2: new $t_5=10$, new $t_6=9$, new $t_7=\infty$

Fourth Itrn.

Step 1: Min t_j is for $j=6$. Label $(3,6)$ as P_4. $P=\{1,2,3,4,6\}$

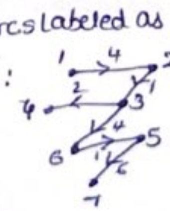
Step 2: new $t_5=10$, $t_7=17$

Fifth Itrn: Step1: Among new t_j's, $t_5=10$ is min for $j=5$. Label $(6,5)$ as P_5.

$P=\{1,2,3,4,6,5\}$

Final Step: Only one t_j is left. $(5,7)$ has least weight. Label $(5,7)$ as P_6.

$P=\{1,2,3,4,5,6,7\}$ which is the vertex set. Arcs labeled as P_1 to P_6 are $(1,2)$, $(2,3)$, $(3,4)$, $(3,6)$, $(6,5)$, $(5,7)$. Shortest Path arborescence is:



(1) Path from 1 to 2 : $1\to2$; weight 4

(2) — " — 1 to 3 : $1\to2\to3$; weight 5

(3) — " — 1 to 5 : $1\to2\to3\to6\to5$; weight 10

(4) — " — 1 to 6 : $1\to2\to3\to6$; weight 9

(5) — " — 1 to 4 : $1\to2\to3\to4$; weight 7

(6) — " — 1 to 7 : $1\to2\to3\to6\to5\to7$; weight 16