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Internal Assessment Test III – May 2024

Sub:	Mathematical Foundation for Computer Applications					Sub Code:	22MCA11	Branch:	MCA	
Date:	20/05/2024	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	I A&B		OBE	

Note: Answer FIVE FULL Questions, choosing ONE full question from each part.

		MARKS	CO	RBT
	PART I			
1	Find the value of k such that the following distribution represents a finite probability distribution. Hence find its mean, variance and standard deviation.	[10]	CO2	L3

x	-3	-2	-1	0	1	2	3
P(x)	k	2k	3k	4k	3k	2k	k

OR

		MARKS	CO	RBT
2	The probability that a pen manufactured by a factory be defective is $1/_{10}$. If 12 such pens are manufactured, what is the probability that (i) exactly two are defective (ii) at least two are defective (iii) none of them are defective.	[10]	CO2	L3
	PART II			
3	Given that 2% of the fuses manufactured by a firm are defective. Find by using Poisson distribution, the probability that a box containing 200 fuses has (i) no defective fuses (ii) 3 or more defective fuses (iii) at least one defective fuse.	[10]	CO2	L3
	OR			
4	A random variable x has the density function $f(x) = \begin{cases} kx^2 & -3 \leq x \leq 3 \\ 0 & elsewhere \end{cases}$. Evaluate k and find (i) $P(x \leq 2)$ (ii) Mean	[10]	CO2	L3

4m

	PART III		CO2	L3
5	In a certain town, the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that the shower will last for (i) 10 minutes or more (ii) less than 10 minutes (iii) between 10 and 12 minutes?	[10]		
	OR			
6	The marks of 1000 students in an exam follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) 65 to 75. Given that $\varphi(1) = 0.3413$ *where* φ *carries the usual meaning.*	[10]	CO2	L2
	PART IV			
7	For what value of k, the following distribution represents a finite probability distribution.	[10]	CO2	L3

x	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

	Find $P(x \geq 5)$ *and* $P(3 < x \leq 6)$.			
	OR			
8	Define the following with an example for each (i) complete graph (ii) regular graph (iii) planar graph.	[10]	CO4	L1
	PART V			
9	Determine the order $\lvert V \rvert$ of the graph $G = (V, E)$ in the following cases: (i)G is a cubic graph of 9 edges. (ii) G is regular with 15 edges. (iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.**OR**	[10]	CO4	L3
10	Define Euler Circuit and Write a note on Konigsberg Bridge Problem.	[10]	CO4	L2

(2 m)

Q.1

Sol: We must have $p(x) \geq 0 \ \forall x$ & $\Sigma p(x) = 1$.

$\Rightarrow k + 2k + 3k + 4k + 3k + 2k + k = 1$

$\Rightarrow 16k = 1$

$\therefore k = \frac{1}{16}$

$\therefore$ The discrete probability distribution is:

x	-3	-2	-1	0	1	2	3
$p(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

Mean $\mu = \Sigma x\, p(x) = (-3)\left(\frac{1}{16}\right) - 2\left(\frac{2}{16}\right) - 1\left(\frac{3}{16}\right) + 0\left(\frac{4}{16}\right) + 1\left(\frac{3}{16}\right) + 2\left(\frac{2}{16}\right) + 3\left(\frac{1}{16}\right)$

$= \frac{1}{16}(-3 - 4 - 3 + 0 + 3 + 4 + 3)$

$= 0$

Variance $= V = \Sigma (x-\mu)^2 . p(x)$

$= (-3)^2\left(\frac{1}{16}\right) + (-2)^2\left(\frac{2}{16}\right) + (-1)^2\left(\frac{3}{16}\right) + 1^2\left(\frac{2}{16}\right) + 2^2\left(\frac{2}{16}\right) + 3^2\left(\frac{1}{16}\right)$

$= \frac{5}{2}$.

$S.D = \sqrt{V} = \sqrt{5/2} = 1.58$

Thus, $k = 1/16$

Mean $= 0$

& S.D $= 1.58$

Also, $p(x \leq 1) = p(-3) + p(-2) + p(-1) + p(0) + p(1)$

$= \cancel{13/7}\ 1/16 + 2/16 + 3/16 + 4/16 + 3/16$

$= 13/16$

$p(x > 1) = p(2) + p(3) = 2/16 + 1/16 = 3/16$

$p(-1 < x \leq 2) = p(0) + p(1) + p(2)$

$= 4/16 + 3/16 + 2/16$

$= 9/16.$

Q2

Sol^n: prob of a def pen is $p = 1/10 = 0.1$

— " — non-def pen is $q = 1 - p$

$= 1 - 0.1$

$= 0.9$

we have $P(x) = {}^nC_x \, p^x q^{n-x}$

Here $n = 12$

(i) Prob (exactly two defec) $= P(x=2)$

$= {}^{12}C_2 \, (0.1)^2 (0.9)^{10}$

$= 0.2301$

(ii) Prob (atleast two defec) $= 1 - P(x<2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \left[{}^{12}C_0 (0.1)^0 (0.9)^{12} + {}^{12}C_1 (0.1)^1 (0.9)^{11}\right]$

$= 0.341$

Prob (no defec) $= P(x=0)$

$= {}^{12}C_0 \, (0.1)^0 \, (0.9)^{12}$

$= (0.9)^{12}$

$= 0.2824$

Q3

Sol^n: $p =$ prob of def fuse $= 2/100 = 0.02$

mean number of defectives $m = np = 200 \times 0.02 = 4$

The Poisson distribution is given by

$$P(x) = \frac{m^x e^{-m}}{x!}$$

$$= \frac{4^x e^{-4}}{x!}$$

(i) Prob of no def fuse $= P(0)$ $\boxed{0! = 1}$

$$= \frac{4^0 e^{-4}}{0!} = e^{-4}$$

$$= 0.0183$$

(ii) Prob of 3 or more def fuses

$$= 1 - [P(0) + P(1) + P(2)] \quad (e^{-4} = 0.0183)$$

$$= 1 - \left[0.0183 + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!}\right]$$

$$= 1 - 0.0183\left[1 + 4 + \frac{16}{2}\right]$$

$$= 0.7621$$

(iii) Prob of atleast one def fuse $= 1 - P(x<1)$

$= 1 - P(0)$

$= 1 - 0.0183 = 0.9817$

Q4

$f(x) \geq 0$ if $k \geq 0$

& we must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-3}^{3} kx^2\,dx = 1$$

$$\Rightarrow k\left[\frac{x^3}{3}\right]_{-3}^{3} = 1$$

$$\Rightarrow \frac{k}{3}\left[(3)^3 - (-3)^3\right] = 1$$

$$\Rightarrow \frac{k}{3}\left[27+27\right] = 1$$

$$\Rightarrow 18k = 1 \quad \Rightarrow \boxed{k = \frac{1}{18}}$$

(i) $P(1 \leq x \leq 2) = \int_{1}^{2} \frac{x^2}{18}\,dx = \left[\frac{x^3}{54}\right]_{1}^{2}$

$$= \frac{1}{54}\left[8-1\right] = \frac{7}{54}.$$

(ii) $P(x \leq 2) = \int_{-3}^{2} \frac{x^2}{18}\,dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2} = \frac{1}{54}\left[8+27\right] = \frac{35}{54}$

(iii) $P(x>1) = \int_1^3 \frac{1}{18} x^2 dx = \frac{1}{18}\left[\frac{x^3}{3}\right]_1^3 = \frac{1}{54}[27-1]$

$$= \frac{26}{54} = \frac{13}{27}$$

(iv) Mean $= \int_{-\infty}^{\infty} x f(x)\, dx$

$$= \int_{-3}^{3} x\left(\frac{x^2}{18}\right) dx = \frac{1}{18}\left[\frac{x^4}{4}\right]_{-3}^{3}$$

$$= \frac{1}{72}\left[81-81\right] = 0$$

v) Variance $= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

$$= \int_{-3}^{3} x^2\left(\frac{x^2}{18}\right) \cdot dx = \frac{1}{18}\left[\frac{x^5}{5}\right]_{-3}^{3}$$

$$= \frac{1}{90}\left[243 + 243\right] = \frac{25}{7}$$

Q5

Sol^n: The p.d.f of exp dist^n is given by

$$f(x) = \alpha e^{-\alpha x}, \quad x > 0$$

$$\text{Mean} = 1/\alpha = 5 \quad \Rightarrow \quad \alpha = 1/5$$

$$\therefore f(x) = \frac{1}{5} e^{-x/5}$$

(i) $$P(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx = -\left[e^{-x/5}\right]_{10}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-2}\right] = \frac{1}{e^2} - 0 = 0.1353$$

(ii) $$P(x < 10) = \int_{0}^{10} \frac{1}{5} e^{-x/5} = -\left[e^{-x/5}\right]_{0}^{10} = -(e^{-2} - 1)$$

$$= 1 - e^{-2} = 0.8647$$

(iii) $$P(10 < x < 12) = \int_{10}^{12} \frac{1}{5} e^{-x/5} dx = -\left[e^{-x/5}\right]_{10}^{12}$$

$$= -(e^{-12/5} - e^{-2}) = 0.0446$$

[Q6]

Sol^n: Let x represent the marks of students.

Given $\mu = 70$, $\sigma = 5$,

$$z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$$

(i) If $x = 65$, $z = -1$

$$P(z<-1) = P(z>1)$$
$$= P(z \geq 0) - P(0 \leq z \leq 1)$$
$$= 0.5 - \phi(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

No of students scoring less than 65 $\approx 0.1587 \times 1000 \approx 159$

(ii) If $x = 75$, $z = 1$

$$P(z>1) = P(z \geq 0) - P(0 \leq z \leq 1)$$
$$= 0.5 - \phi(1)$$
$$= 0.5 - 0.3413 = 0.1587$$

Req ans $= 0.1587 \times 1000 = 158.7 \approx 159$

(iii) $P(-1 < z < 1) = 2P(0<z<1)$

$$= 2\phi(1)$$
$$= 2(0.3413)$$
$$= 0.6826$$

$\therefore$ no of students scoring marks b/w 65 & 75

$$= 1000 \times 0.6826 \approx 682.6$$
$$\approx 683$$

Q7

Sol^n: The prob dist is valid if $P(x) \geq 0$ & $\Sigma P(x) = 1$

$\Rightarrow k = 1/49$

$P(x \geq 5) = P(5) + P(6) = 24/49$

$P(3 < x \leq 6) = P(4) + P(5) + P(6) = 33/49$

4. The prob dist of a finite random variable X is given by the foll^g table :

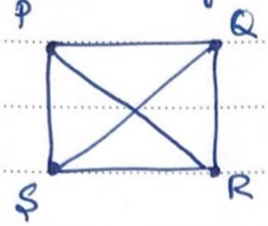
x_i	-2	-1	0	1	2	3
$P(x_i)$	0.1	K	0.2	$2K$	0.3	K

Find the value of K, mean & variance.

Ans : $K = 0.1$, Mean, $\mu = 0.8$
Variance, $V = 2.16$

Q.8. (i) Complete Graph - A complete graph is a simple graph of order $n \geq 2$ ~~when~~ in which there is an edge between every pair of vertices. It is denoted by K_n.

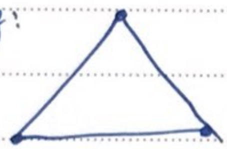
Eg:



K_4

(ii) Regular Graph - A graph in which degree of every vertex is same, say k, is called a k-regular graph.

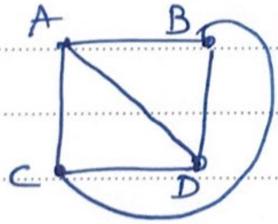
Eg:



2-regular graph.

(iii) Planar graph - A graph which can be drawn atleast in one way that the edges meet only at vertices is called a planar graph.

Eg:



5) Determine the order $|V|$ of the graph $G = (V, E)$ in the following cases:

(1) G is a cubic graph with 9 edges

(2) G is regular graph with 15 edges

(3) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.

Ans

(1) Let the order of G be n.

Since G is a cubic graph, all vertices of G have degree 3. Sum of the degrees of vertices $= 3n$

Since G has 9 edges, we should have

$$3n = 2 \times 9$$

$$\Rightarrow n = 6.$$

Thus, the order of $G = 6$.

(2) Since G is regular, all the vertices must have same degree say k. Let the order of G be n.

$\therefore$ Sum of ~~all~~ the deg of all vertices $= kn$.

$\therefore$ $kn = 15 \times 2$ since G has 15 edges.

$\Rightarrow k = 30/n$.

Since k has to be a +ve int, n must be a divisor of 30

$n = 30, 15, 10, 6, 5, 3, 2, 1$

③ G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3.

Let the order of G be n.

~~Since two vertices of G are of~~

Sum of the degrees of vertices $= 4+4+3(n-2)$

$8+3(n-2) = 10 \times 2$ from hand-shaking prop.

$\Rightarrow 3(n-2) = 20-8 = 12$

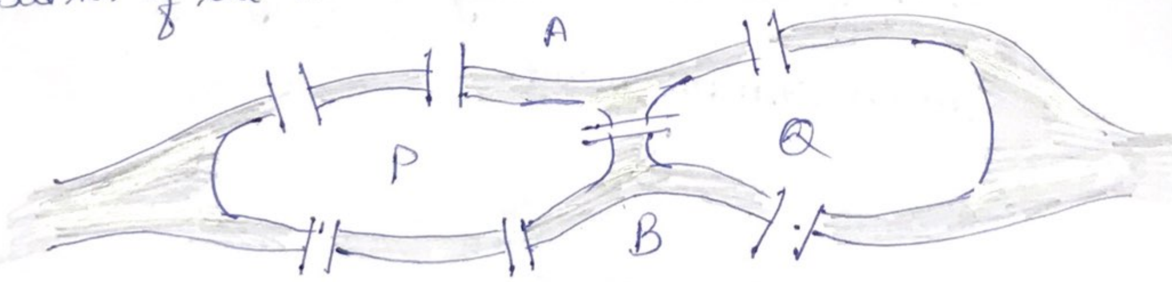
$\Rightarrow n-2 = 4$

$\Rightarrow n = 6$

Thus, order of G is 6.

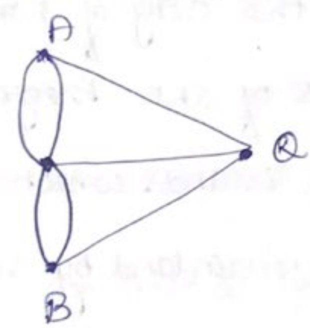
Q10) In 18th century, the city of Konigsberg in Prussia was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. The problem was, by starting at any of the four land areas, can we return to that area after crossing each of the seven bridges exactly once?

[four land areas - two of these parts are the banks of the river and two are islands]



This is the starting point for the development of graph theory. In 1736, Euler analyzed this problem with the help of a graph and gave the sol^n. This problem is known as Konigsberg Bridge Problem.

Let the land areas be denoted by A, B, P, Q. A, B are banks of the river & P, Q are islands. Treat four land areas as four vertices and 7 bridges as 7 edges. So, we get the graph –



We note that, in this graph deg (A) = 3
deg (B) = 3, deg (P) = 5, deg (Q) = 3
which are not even. ∴ graph doesn't have an Euler circuit. ∴ It is not possible to walk over each of the seven bridges exactly once and return to the starting point.