USN | | | | | | | | | |

Internal Assessment Test II – April2024

Sub:	Mathematical Foundation for Computer Applications					Sub Code:	22MCA11	Branch:	MCA
Date:	08/04/2024	Duration:	90 minutes	Max Marks:	50	Sem/Sec:	I A&B		OBE

<u>Note: Answer FIVE FULL Questions, choosing ONE full question from each part.</u>

		MARKS	CO	RBT
	<u>PART I</u>			
1	What is a proposition? Let p and q be the propositions “swimming in the new jersy sea shore is allowed” and “sharks have been near the sea shore” respectively. Express each of the following as an English sentence. $i)\ p \to \neg q \quad ii)\ \neg p \to \neg q \quad iii)\ p \leftrightarrow q$	[10]	CO3	L2
	OR			
2	Let the universe be the set of all integers. Consider the following open statements. $p(x): x > 3, \quad q(x): x+1\ is\ even, \quad r(x): x \le 0$ Write down the truth values of $i)\ p(2) \quad ii)\ p(3) \vee \neg r(3) \quad iii)\ p(2) \wedge q(3)$ $iv)\ p(1) \to r(1)$	[10]	CO3	L2
	<u>PART II</u>			
3	State the following laws of logic: i) De-Morgan’s Law ii) Distributive Law iii) Identity Law	[10]	CO3	L2
	OR			
4	Check whether the following argument is valid or not. “If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore, I have a test in Mathematics.”	[10]	CO6	L3

USN | | | | | | | | | |

Internal Assessment Test II – April2024

Sub:	Mathematical Foundation for Computer Applications					Sub Code:	22MCA11	Branch:	MCA
Date:	/04/2024	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	I A&B		OBE

<u>Note: Answer FIVE FULL Questions, choosing ONE full question from each part.</u>

		MARKS	CO	RBT
	<u>PART I</u>			
1	What is a proposition? Let p and q be the propositions “swimming in the new jersy sea shore is allowed” and “sharks have been near the sea shore” respectively. Express each of the following as an English sentence. $i)\ p \to \neg q \quad ii)\ \neg p \to \neg q \quad iii)\ p \leftrightarrow q$	[10]	CO3	L2
	OR			
2	Let the universe be the set of all integers. Consider the following open statements. $p(x): x > 3, \quad q(x): x+1\ is\ even, \quad r(x): x \le 0$ Write down the truth values of $i)\ p(2) \quad ii)\ p(3) \vee \neg r(3) \quad iii)\ p(2) \wedge q(3)$ $iv)\ p(1) \to r(1)$	[10]	CO3	L2
	<u>PART II</u>			
3	State the following laws of logic: i) De-Morgan’s Law ii) Distributive Law iii) Identity Law	[10]	CO3	L2
	OR			
4	Check whether the following argument is valid or not. “If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore, I have a test in Mathematics.”	[10]	CO6	L3

	PART III			
5	Define Tautology, Contradiction and Contingency. Check whether $\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$is a tautology or not.	[10]	CO3	L3
	OR			
6	Negate and simplify the following: i) $\exists x, [p(x) \vee q(x)$ ii) $\forall x, [p(x) \wedge \neg q(x)$ iii) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$	[10]	CO3	L3
	PART IV			
7	(a) Define the logical connective 'Conditional' with its truth table. (b) Write the converse, inverse and contrapositive of "If 2 is an integer then 9 is not a multiple of 3."	[10]	CO3	L1
	OR			
8	Prove the following using laws of logic: (i) $[(p \vee q) \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$ (ii) $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$	[10]	CO3	L3
	PART V			
9	Write the negation of (i) All Americans eat cheese burgers. (ii) There is a woman who has taken a flight on every airline in the world.	[10]	CO3	L3
	OR			
10	Check the validity of following argument using rules of inference. $p \rightarrow (q \rightarrow r)$ $\neg q \rightarrow \neg p$ $\underline{p}$ $\therefore r$	[10]	CO6	L3

	PART III			
5	Define Tautology, Contradiction and Contingency. Check whether $\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$is a tautology or not.	[10]	CO3	L3
	OR			
6	Negate and simplify the following: i) $\exists x, [p(x) \vee q(x)$ ii) $\forall x, [p(x) \wedge \neg q(x)$ iii) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$	[10]	CO3	L3
	PART IV			
7	(a) Define the logical connective 'Conditional' with its truth table. (b) Write the converse, inverse and contrapositive of "If 2 is an integer then 9 is not a multiple of 3."	[10]	CO3	L1
	OR			
8	Prove the following using laws of logic: $(i)[(p \vee q) \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$ $(ii)[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$	[10]	CO3	L3
	PART V			
9	Write the negation of (i) All Americans eat cheese burgers. (ii) There is a woman who has taken a flight on every airline in the world.	[10]	CO3	L3
	OR			
10	Check the validity of following argument using rules of inference. $p \rightarrow (q \rightarrow r)$ $\neg q \rightarrow \neg p$ $\underline{p}$ $\therefore r$	[10]	CO6	L3

IAT - II MFCA - 22MCA11

1. A proposition is a statement which is either true or false, but not both.

i) $p \rightarrow \neg q$

If swimming in the new jearsy sea shore is allowed then sharks have not been near the sea shore.

ii) $\neg p \rightarrow \neg q$

If swimming in the new jearsy sea shore is not allowed then sharks have not been near the sea shore.

iii) $p \leftrightarrow q$

Swimming in the New Jearsy sea shore is allowed if and only if sharks have been near the sea shore.

2. i) $p(2) : 2 > 3$ which is false.

Truth value is $\boxed{0.}$

ii) $p(3) \vee \neg r(3)$

$p(3) : 3 > 3$ Truth value is $\boxed{0}$

$r(3) : 3 \leq 0$

$\neg r(3) : 3 > 0$ Truth value is 1

$p(3) \vee \neg r(3)$ since one of them is true, the given open statement is true. Truth value is $\boxed{1}$.

$0 \vee 1$

1

iii) $p(2) \wedge q(3)$

Truth value is $\boxed{0}$.

$p(2) : 2 > 3$ This is a false statement.

$q(3) : 3+1$ is even. This is a true statement.

$p(2) \wedge q(3) : 0 \wedge 1 = 0$ Since one of them is false, given conjunction is false.

iv) $p(1) \rightarrow r(1)$

$p(1): 1 > 3$ This is a false statement.

$r(1): 1 \leq 0$ —— " ——

$p(1) \rightarrow r(1)$ This conditional is true as a conditional is false only when first part is 1 & second part is 0. Truth value is $\boxed{1}$

$0 \rightarrow 0$

1

3. i) De-Morgan's Law : a) $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

b) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

ii) Distributive Law : a) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

b) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

iii) Identity Law : a) $p \wedge T_0 \Leftrightarrow p$

b) $p \vee F_0 \Leftrightarrow p$

4. Let p: Today is Tuesday

q: I have a test in Mathematics.

r: I have a test in Economics.

s: My Economics Professor is sick.

Given argument is

$$\begin{array}{l} p \rightarrow (q \vee r) \\ s \rightarrow \neg r \\ p \wedge s \\ \hline \therefore q \end{array} \Rightarrow \begin{array}{l} p \rightarrow (q \vee r) \\ s \rightarrow \neg r \\ p \\ s \\ \hline \therefore q \end{array}$$

$$\Rightarrow \begin{array}{l} q \vee r \\ \neg r \\ \hline \therefore q \end{array}$$

Modus Pones for I & III premises

—— " —— II & IV premises

$$\Leftrightarrow \begin{array}{l} \neg q \rightarrow r \\ \neg r \\ \hline \therefore q \end{array}$$

This is valid in view of Modus Tollens.

5. Tautology: A compound proposition is said to be a tautology if it is always true regardless of truth values of its components.

Contradiction: —— " —— if it is always false —— " ——

Contingency: —— " —— if it is neither a tautology nor a contradiction.

p	q	r	$p \vee q$	① $(p \vee q) \rightarrow r$	$\neg r$	$\neg(p \vee q)$	② $\neg r \rightarrow \neg(p \vee q)$	① $\leftrightarrow$ ②
0	0	0	0	1	1	1	1	1
0	0	1	0	1	0	1	1	1
0	1	0	1	0	1	0	0	1
0	1	1	1	1	0	0	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	0	1	0	0	1
1	1	1	1	1	0	0	1	1

Since all the entries of the last column are 1s, given compound proposition is a tautology.

6. i) Negation is: $\neg\{\exists x, [p(x) \vee q(x)]\}$

$$\Leftrightarrow \forall x, \neg[p(x) \vee q(x)]$$

$$\Leftrightarrow \forall x, \neg p(x) \wedge \neg q(x)$$

ii) Negation: $\neg\{\forall x, [p(x) \wedge \neg q(x)]\}$

$$\Leftrightarrow \exists x, \neg[p(x) \wedge \neg q(x)]$$

$$\Leftrightarrow \exists x, \neg p(x) \vee q(x)$$

iii) Negation: $\neg\{\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$

$$\Leftrightarrow \forall x, \neg[\{p(x) \vee q(x)\} \rightarrow r(x)]$$

$$\Leftrightarrow \forall x, \{p(x) \vee q(x)\} \wedge \neg r(x)$$

7. (a) A compound proposition obtained by inserting the words if ...then ... in appropriate places is called a conditional.

If p then q is denoted by $p \to q$. We read it as 'if p then q'.

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

(b) Let p: 2 is an integer.

q: 9 is a multiple of 3.

Given $p \to \neg q$

converse: $\neg q \to p$

ie., If 9 is not a multiple of 3 then 2 is an integer.

Inverse: $\neg p \to q$

ie., If 2 is not an integer then 9 is a multiple of 3.

Contrapositive: $q \to \neg p$

ie., If 9 is a multiple of 3 then 2 is not an integer.

8. (i) LHS $= [(p \vee q) \vee (\neg p \wedge \neg q \wedge r)]$

$\Leftrightarrow (p \vee q) \vee (\neg(p \vee q) \wedge r)$ De-Morgan's Law

$\Leftrightarrow [(p \vee q) \vee \neg(p \vee q)] \wedge [p \vee q \vee r]$ Distributive law

$\Leftrightarrow T_0 \wedge (p \vee q \vee r)$ Inverse Law

$\Leftrightarrow p \vee q \vee r$ Identity Law

$=$ RHS

ii) $LHS = p \to (q \wedge r)$

$\Leftrightarrow \neg p \vee (q \wedge r)$ $\qquad$ $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ $\qquad$ Distributive Law

$\Leftrightarrow (p \to q) \wedge (p \to r)$ $\qquad$ $\neg p \vee q \Leftrightarrow p \to q$

$= RHS$.

9. (i) Let A: set of all Americans, be the Universe.

$p(x)$: x eats cheese burgers.

Given $\forall x \in A, p(x)$

Negation: $\neg [\forall x \in A, p(x)]$

$\Leftrightarrow \exists x \in A, \neg p(x)$

i.e., Some Americans do not eat cheese burgers.

(ii) Let the Universe be U: set of all women.

$p(x)$: x has taken a flight on every airline in the world.

Given $\exists x \in U, p(x)$

Negation: $\neg [\exists x \in U, p(x)]$

$\Leftrightarrow \forall \cancel{\exists} x \in U, \neg p(x)$

~~Some~~ No woman has taken every airline in the world.

10. $p \to (q \to r)$ $\qquad$ $q \to r$ $\quad$ Modus Pones

$$\begin{array}{l} \neg q \to \neg p \\ p \\ \hline \therefore r \end{array} \Rightarrow \begin{array}{l} \neg q \to \neg p \\ p \\ \hline \therefore r \end{array}$$

$$q \to r$$

$\Leftrightarrow$ $p \to q$ A conditional is logically equivalent to its contrapositive.

$$\begin{array}{l} p \\ \hline \therefore r \end{array}$$

$$\Leftrightarrow \quad \begin{array}{r} p \to r \\ p \\ \hline \therefore r \end{array}$$

Rule of Syllogism.

This is a valid argument in view of Modus Pones.