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Internal Assessment Test I – March 2024

Sub:	Mathematical Foundation for Computer Applications			Sub Code:	22MCA11	Branch:	MCA
Date:	12/03/2024	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	I A&B

Note: Answer FIVE FULL Questions, choosing ONE full question from each part.

		MARKS	OBE CO	OBE RBT
	PART I			
1	Let A={1, 2, 3, 4} and R be a relation on A defined by 'xRy iff x divides y'. Write down R as a set of ordered pairs. Find the matrix of R and draw the digraph of R. Find the in-degree and out-degree of each vertex.	[10]	CO1	L2
	OR			
2	Let A={1, 2, 3, 4}, B={2, 5}, C={3, 4, 7}. Find $(i)\ A \times B$ $(ii)\ B \times A$ $(iii)\ A \times A$ $(iv)\ (A \times C) \cup (B \times C)$ $(v)\ (A \cup B) \times C$	[10]	CO1	L2
	PART II			
3	Let $M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $M_S = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ be the matrix of the relations R and S on A. Find $i)\ R \cup S$ $ii)\ R \cap S$ $iii)\ \bar{R}$ $iv)\ S^c$ and write their matrix representation given A= {a, b, c}.	[10]	CO1	L3
	OR			
4	Let A = {1, 2, 3, 4} and R = {(1, 2), (1, 3), (2, 4), (3, 2), (3, 3), (3, 4)} be a relation on A. Verify that $M(R^2) = [M(R)]^2$.	[10]	CO1	L2

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	OR			
2	Let A = {1, 2, 3, 4}, B = {2, 5}, C = {3, 4, 7}. Find $(i)\ A \times B$ $(ii)\ B \times A$ $(iii)\ A \times A$ $(iv)\ (A \times C) \cup (B \times C)$ $(v)\ (A \cup B) \times C$	[10]	CO1	L2
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	PART III			
5	Define the following properties of a relation with an example for each. (i) Reflexive (ii) Symmetric (iii) Anti-symmetric (iv) Transitive (v) partition of a set	[10]	CO1	L1
	OR			
6	Consider A={1, 2, 3,….11, 12}. The relation R on A is defined as $'(x, y) \in R\ iff\ x - y\ is\ a\ multiple\ of\ 5'$. Verify that R is an equivalence relation.	[10]	CO1	L3
	PART IV			
7	Define Equivalence Relations and Partial Order. Let A = {a, b, c, d, e}. Consider the partition $\{\{a, b\},\ \{c, d\},\ \{e\}\}$ of A. Find the equivalence relation inducing this partition.	[10]	CO3	L2
	OR			
8	Let A={1, 2, 3, 4, 5} and R be a relation defined on $A \times A$ by $(x_{1,}\ y_1)R(x_2,\ y_2)\ iff\ x_1 + y_1 = x_2 + y_2$. Determine the partition of $A \times A$ induced by R.	[10]	CO3	L2
	PART V			
9	Let A = {1, 2, 3, 4, 6, 12}. On A, the relation R is defined by 'aRb iff a divides b'. Prove that R is a partial order on A. Draw the Hasse diagram for this relation.	[10]	CO6	L3
	OR			
10	Let A={2, 3, 4, 6, 8, 12, 24} and R denote the partial order of divisibility, ie., xRy iff x divides y. Draw the Hasse diagram and Find the (i) least element (ii) greatest element (iii) minimal elements (iv) maximal elements	[10]	CO6	L2

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	PART IV			
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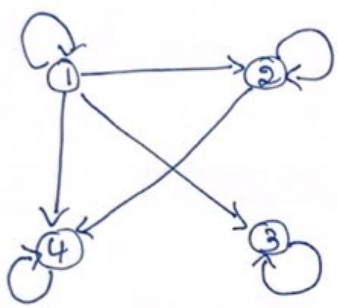
MFCA - 22MCA11

IAT I - March 2024

1. $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

$$M(R) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



vertex	In-deg	Out-deg
1	1	4
2	2	2
3	2	1
4	3	1

3. $M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $M_S = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$R = \{(a,a), (a,c), (b,a), (c,b)\}$

$S = \{(a,c), (b,b), (b,c), (c,a)\}$

i) $R \cup S = \{(a,a), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b)\}$

ii) $R \cap S = \{(a,c)\}$

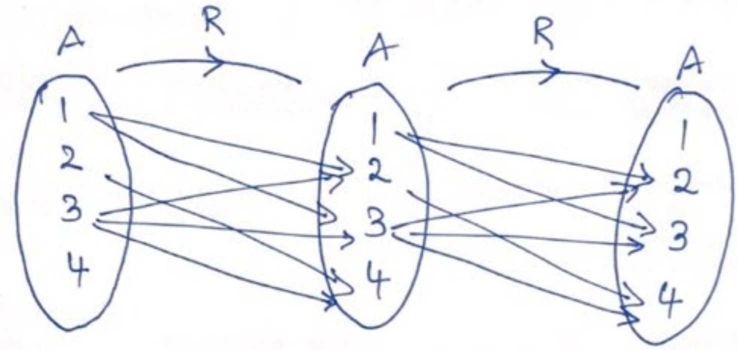
iii) $\overline{R} = (A \times A) - R = \{(a,b), (b,b), (b,c), (c,a), (c,c)\}$

iv) $S^c = \{(c,a), (b,b), (c,b), (a,c)\}$

$M(R \cup S) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $M(R \cap S) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$M(\overline{R}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad M(S^c) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

4.



$$R^2 = R \circ R = \{(1,4), (1,2), (1,3), (3,4), (3,2), (3,3)\}$$

$$M(R^2) = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[M(R)]^2 = M(R) \times M(R)$$

$$= \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence, $[M(R)]^2 = M(R^2)$

5. A relation R on a set A is said to be

(i) reflexive if $(a,a) \in R, \forall a \in A$.

(ii) symmetric if whenever $(a,b) \in R$ then $(b,a) \in R$

(iii) anti-symmetric if $(a,b) \in R$ & $(b,a) \in R \Rightarrow a=b$.

(iv) transitive if whenever $(a,b) \in R$ & $(b,c) \in R$ $\Rightarrow (a,c) \in R$.

(v) Given a non-empty set A, suppose we have non-empty subsets of A, say $A_1, A_2, \dots A_k$ such that

(i) $A_1 \cup A_2 \cup \dots \cup A_k = A$

(ii) $A_i \cap A_j = \phi$

then $P = \{A_1, A_2\}$ is called partition of the set A.

Eg :- Let $A = \{1,2,3\}$

$R_1 = \{(1,1), (1,2), (2,2), (3,3)\}$ is reflexive.

$R_2 = \{(1,2), (2,1), (2,2)\}$ is symmetric.

$R_3 = \{(1,2), (2,3), (3,3)\}$ is anti-symmetric.

$R_4 = \{(1,1), (1,2), (2,2), (2,2)\}$ is transitive.

$A_1 = \{1,3\}$, $A_2 = \{2\}$

$P = \{A_1, A_2\}$ is partition of A as $A_1 \cup A_2 = A$, $A_1 \cap A_2 = \phi$

7) Let $A = \{a, b, c, d, e\}$

Since a & b belongs to one block, we have aRa, aRb, bRa, bRb.

Since c & d are in same block, we have cRc, cRd, dRc, dRc.

Since e is in one block, we have eRe.

Thus, the equivalence relation inducing the given partition is $R = \{(a,a), (a,b), (b,a), (b,b), (c,c), (c,d), (d,c), (d,d), (e,e)\}$

9) Let $A = \{1, 2, 3, 4, 6, 12\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,12), (2,2), (2,4), (2,6), (2,12), (3,3), (3,6), (3,12), (4,4), (4,12), (6,6), (6,12), (12,12)\}$

Reflexive: WKT a divides a $\forall a \in A$

$\Rightarrow (a,a) \in R \quad \Rightarrow$ R is reflexive.

Anti-symmetric: Let $(a,b) \in R \quad \Rightarrow$ a divides b

But b doesn't divide a $\Rightarrow (b,a) \notin R$ for $a \neq b$

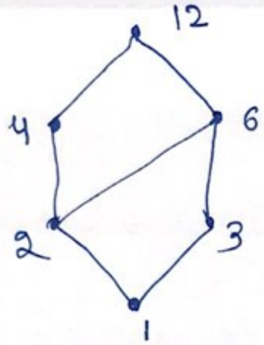
$\therefore$ R is anti-symmetric

Transitive: Let $(a,b) \in R$ & $(b,c) \in R$

$\Rightarrow$ a divides b & b divides c.

$\Rightarrow$ a divides c.

$\Rightarrow (a,c) \in R \quad \therefore$ R is transitive.


12
4
6
2
3
1

2. $A = \{1, 2, 3, 4\}$

$B = \{2, 5\}$

$C = \{3, 4, 7\}$

(i) $A \times B = \{(1,2), (1,5), (2,2), (2,5), (3,2), (3,5), (4,2), (4,5)\}$ — 1M

(ii) $B \times A = \{(2,1), (2,2), (2,3), (2,4), (5,1), (5,2), (5,3), (5,4)\}$ — 1M

(iii) $A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$ — 1M

$(A \times C) = \{(1,3), (1,4), (1,7), (2,3), (2,4), (2,7), (3,3), (3,4), (3,7), (4,3), (4,4), (4,7)\}$ — 1M

$B \times C = \{(2,3), (2,4), (2,7), (5,3), (5,4), (5,7)\}$ — 1M

(iv) $(A \times C) \cup (B \times C) = \{(1,3), (1,4), (1,7), (2,3), (2,4), (2,7), (3,3), (3,4), (3,7), (4,3), (4,4), (4,7), (5,3), (5,4), (5,7)\}$ — 2M

(v) $A \cup B = \{1, 2, 3, 4, 5\}$ — 1M

$(A \cup B) \times C = \{(1,3), (1,4), (1,7), (2,3), (2,4), (2,7), (3,3), (3,4), (3,7), (4,3), (4,4), (4,7), (5,3), (5,4), (5,7)\}$ — 2M

5. $R = \{(1,1), (1,6), (1,11), (2,2), (2,7), \ldots \ldots (12,12)\}$ — 1M

<u>Reflexive:</u> Wkt $x - x = 0$ is a multiple of 5.

$\therefore (x, x) \in R, \ \forall\ x \in A$.

$\therefore$ R is reflexive. — 3M

<u>Symmetric:</u> If $(x, y) \in R$

$\Rightarrow x - y$ is a multiple of 5.

$\Rightarrow y - x$ — " —

$\Rightarrow (y, x) \in R$

$\therefore$ R is symmetric. — 3M

<u>Transitive:</u> If $(x, y) \in R$, $(y, z) \in R$

$\Rightarrow x - y$ is a multiple of 5, $\quad y - z$ is a multiple of 5

$\Rightarrow x - y = 5k_1 \qquad y - z = 5k_2$

$\Rightarrow x - \cancel{y} + \cancel{y} - z = 5(k_1 + k_2)$ which is a multiple of 5

$\Rightarrow (x, z) \in R \quad \therefore$ R is transitive.

Hence R is an equivalence relation. — 3M

8. $[(1,1)] = \{(1,1)\} = A_1$

$[(1,2)] = \{(1,2),(2,1)\} = A_2$

$[(1,3)] = \{(1,3),(3,1),(2,2)\} = A_3$

$[(1,4)] = \{(1,4),(4,1),(2,3),(3,2)\} = A_4$

$[(1,5)] = \{(1,5),(5,1),(2,4),(4,2),(3,3)\} = A_5$

$[(2,5)] = \{(2,5),(5,2),(3,4),(4,3)\} = A_6$

$[(3,5)] = \{(3,5),(5,3),(4,4)\} = A_7$

$[(4,5)] = \{(4,5),(5,4)\} = A_8$

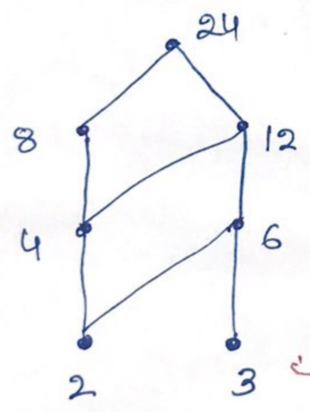
$[(5,5)] = \{(5,5)\} = A_9$

Partition of $A \times A$ induced by R is given by

$P = \{A_1, A_2, \ldots A_9\}$

10 Let $A = \{2, 3, 4, 6, 8, 12, 24\}$

$R = \{(2,2), (2,4), (2,6), (2,8), (2,12), (2,24), (3,3),$
$(3,6), (3,12), (3,24), (4,4), (4,8), (4,12), (4,24),$
$(6,6), (6,12), (6,24), (8,8), (8,24), (12,12), (12,24),$
$(24,24)\}$ 3



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(iv) Maximal elements : 24

(iii) Minimal elements : 2 & 3

(ii) Greatest element : 24

(i) Least element : Doesn't exist

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