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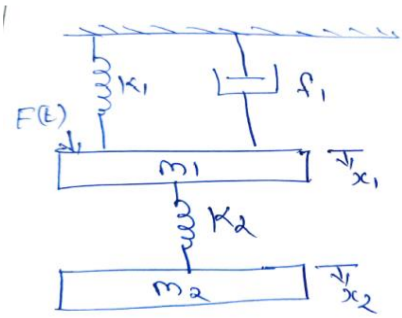
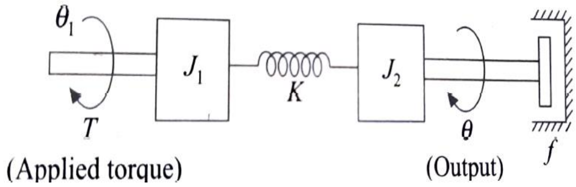
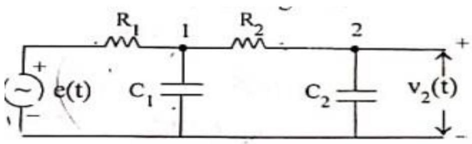
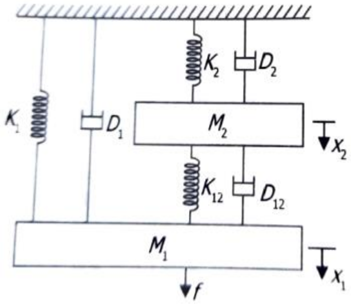
Internal Assesment Test - I

Sub:	Control systems							Code:	21EE52
Date:	18/12/2023	Duration:	90 mins	Max Marks:	50	Sem:	5	Branch:	EEE
Answer Any FIVE FULL Questions									

		Marks	OBE CO	OBE RBT
1	With the help of neat block diagram, define open loop and closed loop control system. Mention the advantages and disadvantages of each system with examples	[10]	CO1	L2
2	Explain about DC servo motor and derive the transfer function of a)Field controlled DC servomotor b)Armature controlled DC servomotor	[10]	CO1	L3
3	Write the force equations of the mechanical system shown in fig and also find the force-voltage(f—v) and force-current(f-i) analogy 	[10]	CO1	L4
4	Find the torque equation of the mechanical system shown in fig and also find the torque-voltage(t-v) and torque-current(t-i) analogy 	[10]	CO1	L4
5(a)	Find the transfer function for the given electrical circuit 	5	CO1	L4
5(b)	Explain about synchros as error detector	5	CO1	L2

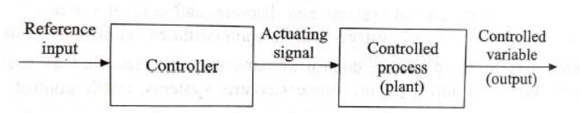
6	Find the force equation and the transfer function [$X_1(s)/F(s)$] of the mechanical system shown in fig and also find the force-voltage(f—v) and force-current(f-i) analogy 	[10]	CO1	L4

Solutions

1)

Classification of control systems

1. Open-Loop Control Systems:



Any physical system which does not automatically correct the variation in its output.

- It is not a feedback system
- It operates on a time basis

Example: Washing machine, Electric Toaster, Traffic control.

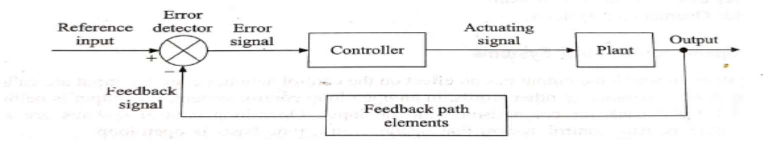
Advantages of Open-loop systems:

- Simple and economical
- Easier to construct
- Generally stable

Disadvantages of open-loop systems:

- Inaccurate and Unreliable
- The changes in the output due to external disturbances are not corrected automatically.

2. Closed-Loop Control Systems:



- Feedback control system.
- A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference a https://cramster-image.s3.amazonaws.com/definitions/DC-959V1.png

Example: Traffic control, Room heating system.

Advantages of Closed-loop systems:

- Accurate
- Sensitivity of the systems may be made small to make the system more stable.
- Less affected by noise
- Accurate even in the presence of non-linearities.

Disadvantages of Closed-loop systems:

- Complex and costlier
- Feedback in closed loop system may lead to oscillatory response.
- Feedback reduces the overall gain of the system.
- Stability is a major problem in closed loop system and more care is needed to design a stable closed loop system.

Comparison

Open-Loop	Closed-Loop
Simple and economical	Complex and costlier
Consume less power	Consume more power
Easier to construct because of less number of components required	Not easy to construct because of more number of components required
Generally stable system	More care is needed to design a stable system
Inaccurate and unreliable	Accurate and more reliable
Changes in the output not corrected automatically	Changes in the output corrected automatically
	Feedback reduces the overall gain of the system.

2)

Servomotors

- The control systems which are used to control the position (i.e) velocity and acceleration are called servomechanisms.
- The motors which are used in automatic control systems are called servomotors.
- Servomotors are used to convert an electrical signal applied to them into an angular displacement of the shaft.
- Depending on the construction, they can operate either in a continuous duty or step duty.

Features of servomotors.

- Linear relationship between speed and electric control signal.
- Steady-state stability
- Wide range of speed control
- Linearity of mechanical characteristics throughout the entire speed range
- Low mechanical and electrical inertia.
- Fast response.

Types of servomotors

- dc servomotors
- ac servomotors

Advantages of dc servomotors:

- Higher output than from an ac motor of the same size.
- Easy achievement of linear characteristics.
- Easier speed control from zero to full speed in both the directions.
- Low electrical and mechanical time constants.
- High torque to inertia ratio that gives them quick response to control signals.

Advantages of ac servomotors:

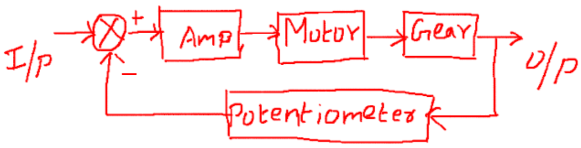
- Lower cost
- Higher efficiency
- Less maintenance

Disadvantages of ac servomotors:

- Nonlinear
- More difficult to control especially for positioning applications.

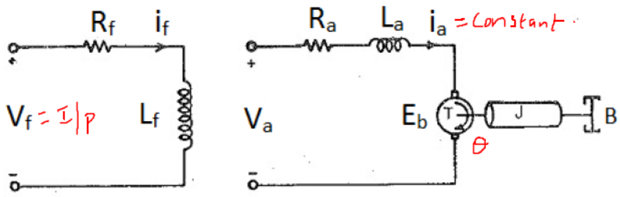
dc Servomotors

- Made up of dc motors which is controlled by potentiometers and gears.



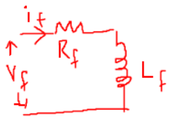
- Transfer function of Field controlled dc Motors.
- Transfer function of Armature controlled dc Motors.

- Derive the transfer function of field controlled dc servomotors



$$\frac{\theta(s)}{V_f(s)} = ?$$

Derivation



Let I_a = Constant.

$$R_f i_f + L_f \frac{di_f}{dt} = V_f \quad \text{— ①}$$

Torque $\propto \phi \times I_a$

$T \propto \phi$

But $\phi \propto I_f$

$\therefore T \propto I_f \Rightarrow T = K_f i_f \quad \text{— ②}$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{— ③}$$

L.T ①, $R_f I_f(s) + L_f s I_f(s) = V_f(s)$

$$I_f(s)\left[R_f + L_f s\right] = V_f(s)$$

$$I_f(s) = \frac{V_f(s)}{R_f + L_f s} \quad \text{— ④}$$

L.T ②, $T(s) = K_f I_f(s)$ — ⑤

L.T ③, $Js^2\theta(s) + Bs\,\theta(s) = T(s)$

$$\theta(s)\left[Js^2 + Bs\right] = T(s) \quad \text{— ⑥}$$

From ④, ⑤, ⑥.

$$\theta(s)\left[Js^2 + Bs\right] = K_f \frac{V_f(s)}{R_f + L_f s}$$

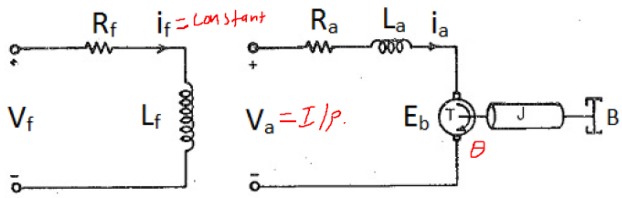
T.F is, $\dfrac{\theta(s)}{V_f(s)} = \dfrac{K_f}{\left[R_f + L_f s\right]\left[Js^2 + Bs\right]}$

Block Diagram representation.

From ④, ⑤, ⑥.

$$\frac{\theta(s)}{V_f(s)} = \frac{K_f}{\left[R_f + L_f s\right]\left[Js^2 + Bs\right]}$$

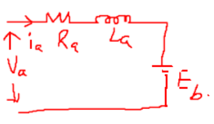
- Derive the transfer function of armature controlled dc servomotors.



T.F $\dfrac{\theta(s)}{V_a(s)}$

Derivation.

Let i_f = constant.



$$R_a i_a + L_a \frac{di_a}{dt} + e_b = V_a.$$

$$R_a i_a + L_a \frac{di_a}{dt} = V_a - e_b \quad \text{— ①}$$

$$T \propto i_a \Rightarrow T = K_a i_a \quad \text{— ②}$$

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad \text{— ③}$$

$$e_b \propto \frac{d\theta}{dt} \Rightarrow e_b = K_b \frac{d\theta}{dt} \quad \text{— ④}$$

L.T ①, $R_a I_a(s) + L_a s I_a(s) = V_a(s) - E_b(s)$

$$I_a(s)\left[R_a + L_a s\right] = V_a(s) - E_b(s)$$

$$I_a(s) = \frac{V_a(s) - E_b(s)}{R_a + L_a s} \quad — (5)$$

L.T (2), $T(s) = K_a I_a(s) \quad — (6)$

L.T (3), $\theta(s)\left[J s^2 + B s\right] = T(s) \quad — (7)$

L.T (4), $E_b(s) = K_b s\,\theta(s) \quad — (8)$

From (5), (6), (8)

$$T(s) = K_a\left[\frac{V_a(s) - K_b s\,\theta(s)}{R_a + L_a s}\right] \quad — (9)$$

(9) in (7),

$$\theta(s)\left[J s^2 + B s\right] = \frac{K_a V_a(s) - K_a K_b s\,\theta(s)}{R_a + L_a s}$$

$$\theta(s)\left\{\left[J s^2 + B s\right]\left[R_a + L_a s\right]\right\} = K_a V_a(s) - K_a K_b s\,\theta(s)$$

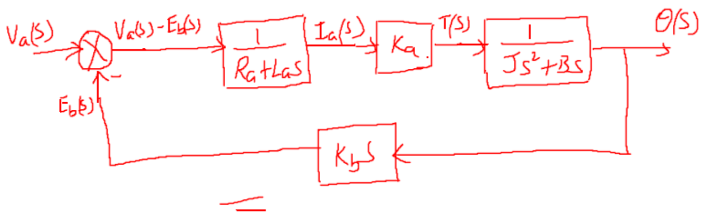
$$\theta(s)\left\{\left(J s^2 + B s\right)\left(R_a + L_a s\right)\right\} + K_a K_b s\,\theta(s) = K_a V_a(s).$$

$$\theta(s)\left[\left(J s^2 + B s\right)\left(R_a + L_a s\right) + K_a K_b s\right] = K_a V_a(s)$$

$$\therefore \frac{\theta(s)}{V_a(s)} = \frac{K_a}{\left(J s^2 + B s\right)\left(R_a + L_a s\right) + K_a K_b s}$$

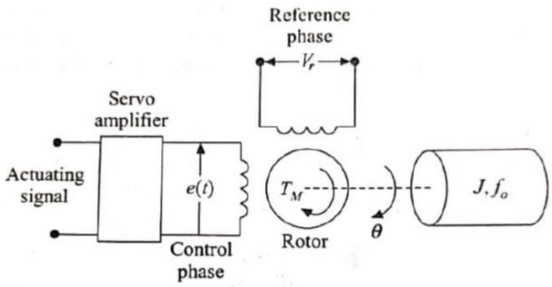
Block Diagram representation.

From (5), (6), (7) (8).



ac servomotor

- An ac servomotor is basically a two-phase induction motor except for certain special design features.



Mechanical Load,

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = T_M \quad \text{—— (1)}$$

Motor Torque; $T_M = K_1 e_c - K_2\frac{d\theta}{dt}$. —— (2)

(2) in (1),

$$J\frac{d^2\theta}{dt^2} + f\frac{d\theta}{dt} = K_1 e_c - K_2\frac{d\theta}{dt} \quad \text{—— (3)}$$

L.T (3),

$$J s^2\theta(s) + f s\,\theta(s) = K_1 E_c(s) - K_2 s\,\theta(s)$$

$$J s^2\theta(s) + f s\,\theta(s) + K_2 s\,\theta(s) = K_1 E_c(s)$$

$$\theta(s)\left[J s^2 + s[f + K_2]\right] = K_1 E_c(s).$$

$$\therefore \frac{\theta(s)}{E_c(s)} = \frac{K_1}{s[Js + (f+K_2)]} = \frac{K_1}{s(f+K_2)\left(\frac{Js}{f+K_2} + 1\right)}$$

$$= \frac{K_1/(f+K_2)}{s\left[\frac{Js}{f+K_2} + 1\right]} = \frac{K_m}{s[\tau_m s + 1]}$$

where $K_m = \frac{K_1}{f+K_2}$, $\tau_m = \frac{J}{f+K_2}$

↳ Motor Gain ↳ Motor Time

5a)

At node 1, by Kirchoff's current law

$$\frac{v_1}{R_1} + C_1\frac{dv_1}{dt} + \frac{v_1 - v_2}{R_2} = \frac{e}{R_1}$$

On taking Laplace transform with zero initial conditions

$$\frac{V_1(s)}{R_1} + C_1 sV_1(s) + \frac{V_1(s)}{R_2} - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

$$V_1(s)\left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1} \quad (1)$$

At node 2, by Kirchoff's current law

$$\frac{v_2 - v_1}{R_2} + C_2\frac{dv_2}{dt} = 0$$

On taking Laplace transform

$$\frac{V_2(s)}{R_2} - \frac{V_1(s)}{R_2} + C_2 s\, V_2(s) = 0$$

$$\frac{V_1(s)}{R_2} = \frac{V_2(s)}{R_2} + C_2 s\, V_2(s) = \left[\frac{1}{R_2} + sC_2\right] V_2(s)$$

$$\therefore V_1(s) = [1 + sC_2R_2]\, V_2(s) \quad (2)$$

Substitute (2) in (1),

$$(1 + sR_2C_2)\, V_2(s)\left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

$$\left[\frac{(1 + sR_2C_2)\,(R_2 + R_1 + sC_1R_1R_2) - R_1}{R_1R_2}\right] V_2(s) = \frac{E(s)}{R_1}$$

$$\therefore \frac{V_2(s)}{E(s)} = \frac{R_2}{[(1 + sR_2C_2)\,(R_1 + R_2 + sC_1R_1R_2) - R_1]}$$

RESULT

The (node basis) differential equations governing the electrical network are

1. $\frac{v_1}{R_1} + C_1\frac{dv_1}{dt} + \frac{v_1 - v_2}{R_2} = \frac{e}{R_1}$
2. $\frac{v_2 - v_1}{R_2} + C_2\frac{dv_2}{dt} = 0$

The transfer function of the electrical network is

$$\frac{V_2(s)}{E(s)} = \frac{R_2}{[(1 + sR_2C_2)\,(R_1 + R_2 + sC_1R_1R_2) - R_1]}$$

5b)

2.3 SYNCHROS

The term synchro is a generic name for a family of inductive devices which works on the principle of a rotating transformer(Induction motor). The trade names for synchros are Selsyn, Autosyn and Telesyn. Basically they are electro-mechanical devices or electromagnetic transducers which produces an output voltage depending upon angular position of the rotor.

A synchro system is formed by interconnection of the devices called the synchro transmitter and the synchro control transformer. They are also called ***Synchro pair***. The synchro pair measures and compares two angular displacements and its output voltage is approximately linear with angular difference of the axis of both the shafts. They can be used in the following two ways,

1. To control the angular position of load from a remote place/long distance.
2. For automatic correction of changes due to disturbance in the angular position of the load.

SYNCHRO TRANSMITTER

Construction

The constructional features, electrical circuit and a schematic symbol of synchro transmitter are shown in fig 2.4. The two major parts of synchro transmitter are stator and rotor. The stator is identical to the stator of three phase alternator. It is made of laminated silicon steel and slotted on the inner periphery to accommodate a balanced three phase winding. The stator winding is concentric type with the axis of three coils 120° apart. The stator winding is star connected (Y-connection).

The rotor is of dumb bell construction with a single winding. The ends of rotor winding are terminated on two slip rings. A single phase ac excitation voltage is applied to rotor through slip rings.

Working principle

When the rotor is excited by ac voltage, the rotor current flows, and a magnetic field is produced. The rotor magnetic field induces an emf in the stator coils by transformer action. The effective voltage induced in any stator coil depends upon the angular position of the coil's axis with respect to rotor axis

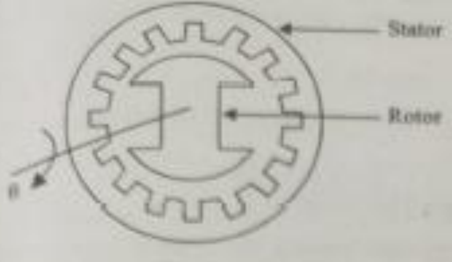


Fig 2.4a : Constructional features.

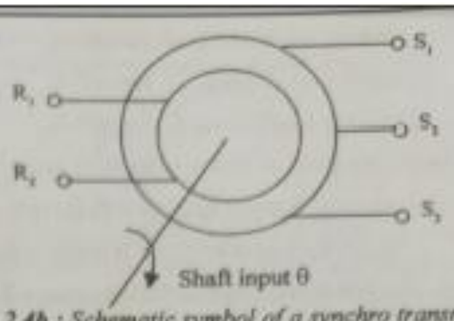


Fig 2.4b : Schematic symbol of a synchro transmitter.

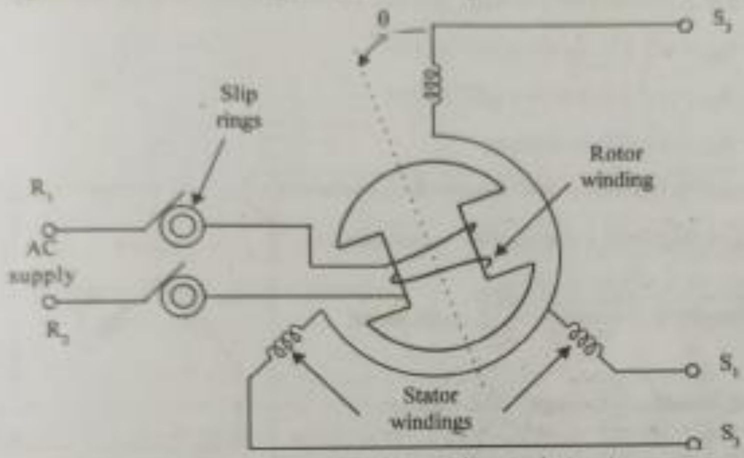


Fig 2.4c : Electrical circuit.

Fig 2.4 : Synchro transmitter.

Let, e_r = Instantaneous value of ac voltage applied to rotor.

e_{s1}, e_{s2}, e_{s3} = Instantaneous value of emf induced in stator coils S1,S2, S3 with respect to neutral respectively.

E_r = Maximum value of rotor excitation voltage.

ω = Angular frequency of rotor excitation voltage.

K_t = Turns ratio of stator and rotor windings.

K_c = Coupling coefficient.

θ = Angular displacement of rotor with respect to reference.

Let, the instantaneous value of rotor excitation voltage, $e_r = E_r \sin \omega t$ .....(2.3)

Let the rotor rotates in anticlockwise direction. When the rotor rotates by an angle,θ emfs are induced in stator coils. The frequency of induced emf is same as that of rotor frequency. The magnitude of induced emfs are proportional to the turns ratio and coupling coefficient. The turns ratio, K_t is a constant, but coupling coefficient, K_c is a function of rotor angular position.

$\therefore$ Induced emf in stator coil = $K_t K_c E_r \sin \omega t$ .....(2.4)

Let e_{s2} be reference vector. With reference to fig 2.4, when $\theta = 0$, the flux linkage of coil S_2 is maximum and when $\theta = 90°$, the flux linkage of coil S_2 is zero. Hence the flux linkage of coil S_2 is function of cosθ (i.e., $K_c = K_t \cos\theta$ for coil S_2). The flux linkage of coil s_3 will be maximum after a rotation of 120° in anticlockwise direction and that of S_1 after a rotation of 240°.

$\therefore$ Coupling coefficient, K_x for coil-S_2 $= K_1 \cos\theta$

Coupling coefficient, K_z for coil-S_3 $= K_1 \cos(\theta-120^\circ)$

Coupling coefficient, K_y for coil-S_1 $= K_1 \cos(\theta-240^\circ)$

Hence the emfs of stator coils with respect to neutral can be expressed as follows.

$$e_{S2} = K_r K_1 \cos\theta\, E_r \sin\omega t = K E_r \cos\theta \sin\omega t \qquad ....(2.5)$$

$$e_{S3} = K_r K_1 \cos(\theta-120^\circ)\, E_r \sin\omega t = K E_r \cos(\theta-120^\circ)\sin\omega t \qquad ....(2.6)$$

$$e_{S1} = K_r K_1 \cos(\theta-240^\circ)\, E_r \sin\omega t = K E_r \cos(\theta-240^\circ)\sin\omega t \qquad ....(2.7)$$

With reference to fig 2.5 by kirchoff's voltage law the coil-to-coil emf can be expressed as

$$e_{S1S2} = e_{S1} - e_{S2} = \sqrt{3}\, KE_r \sin(\theta+240^\circ)\sin\omega t \qquad ....(2.8)$$

$$e_{S2S3} = e_{S2} - e_{S3} = \sqrt{3}\, KE_r \sin(\theta+120^\circ)\sin\omega t \qquad ....(2.9)$$

$$e_{S3S1} = e_{S3} - e_{S1} = \sqrt{3}\, KE_r \sin\theta \sin\omega t \qquad ....(2.10)$$

$$\begin{aligned}
e_{S1S2} &= e_{S1} - e_{S2} = K E_r \cos(\theta-240^\circ)\sin\omega t - K E_r \cos\theta \sin\omega t \\
&= K E_r\, [\cos\theta\cos 240^\circ + \sin\theta \sin 240^\circ - \cos\theta]\sin\omega t \\
&= K E_r \left[\cos\theta(-0.5) + \sin\theta\left(-\frac{\sqrt{3}}{2}\right) - \cos\theta\right]\sin\omega t \\
&= \sqrt{3}\, K E_r \left[\sin\theta\left(-\frac{1}{2}\right) + \cos\theta\left(-\frac{\sqrt{3}}{2}\right)\right]\sin\omega t \\
&= \sqrt{3}\, K E_r \left[\sin\theta \cos 240^\circ + \cos\theta \sin 240\right]\sin\omega t \\
&= \sqrt{3}\, KE_r \sin(\theta+240^\circ)\sin\omega t
\end{aligned}$$

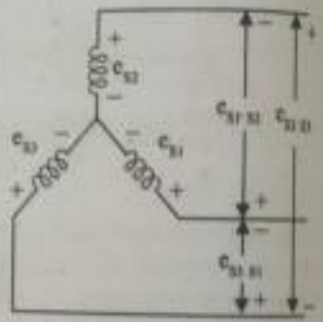


Fig 2.5 : Induced emf in stator coils.

$$\begin{aligned}
e_{S2S3} &= e_{S2} - e_{S3} = K E_r \cos\theta \sin\omega t - K E_r \cos(\theta-120^\circ)\sin\omega t \\
&= K E_r\, [\cos\theta - \cos\theta\cos 120^\circ - \sin\theta \sin 120^\circ]\sin\omega t \\
&= K E_r \left[\cos\theta - \cos\theta(-0.5) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)\right]\sin\omega t = \sqrt{3}\, K E_r \left[\sin\theta\left(-\frac{1}{2}\right) + \cos\theta\left(\frac{\sqrt{3}}{2}\right)\right]\sin\omega t \\
&= \sqrt{3}\, K E_r\, [\sin\theta\cos 120^\circ + \cos\theta \sin 120]\sin\omega t \quad = \sqrt{3}\, KE_r \sin(\theta+120^\circ)\sin\omega t
\end{aligned}$$

$$\begin{aligned}
e_{S3S1} &= e_{S3} - e_{S1} = K E_r \cos(\theta-120^\circ)\sin\omega t - K E_r \cos(\theta-240^\circ)\sin\omega t \\
&= K E_r\, [\cos\theta \cos 120^\circ + \sin\theta \sin 120^\circ - \cos\theta\cos 240^\circ - \sin\theta \sin 240^\circ]\sin\omega t \\
&= K E_r \left[\cos\theta(-0.5) + \sin\theta\left(\frac{\sqrt{3}}{2}\right) - \cos\theta(-0.5) - \sin\theta\left(-\frac{\sqrt{3}}{2}\right)\right]\sin\omega t = \sqrt{3}\, K E_r \sin\theta \sin\omega t
\end{aligned}$$

When $\theta = 0$, from equation 2.5 we can say that maximum emf is induced in coil S_2. But from equation (2.10) it is observed that the coil-to-coil voltage e_{S3S1} is zero. This position of the rotor is defined as the electrical zero of the transmitter. The electrical zero position is used as reference for specifying the angular position of the rotor.

The input to the synchro transmitter is the angular position of its rotor shaft and the output is a set of three stator coil-to-coil voltages. By measuring and identifying ...

it is possible to identify the angular position of the rotor. [A device called ***synchro/digital converter*** is available to measure the stator voltages and to calculate the angular measure and then display the direction and angle of rotation of the rotor].

SYNCHRO CONTROL TRANSFORMER

Construction

The constructional features of synchro control transformer is similar to that of synchro transmitter, except the shape of rotor. The rotor of the control transformer is made cylindrical so that the air gap is practically uniform. This feature of the control transformer minimizes the changes in the rotor impedance with the rotation of the shaft. The constructional features, electrical circuit and a schematic symbol of control transformer are shown in fig 2.6.

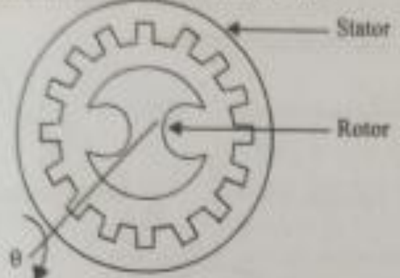


Fig 2.6a : Constructional features.

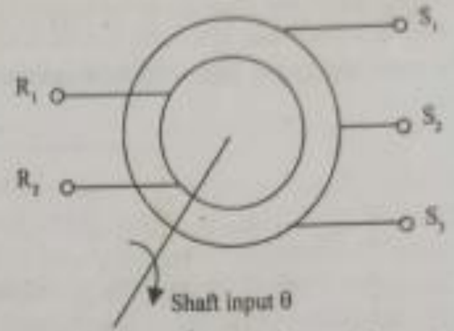


Fig 2.6b : Schematic symbol of a synchro control transformer.

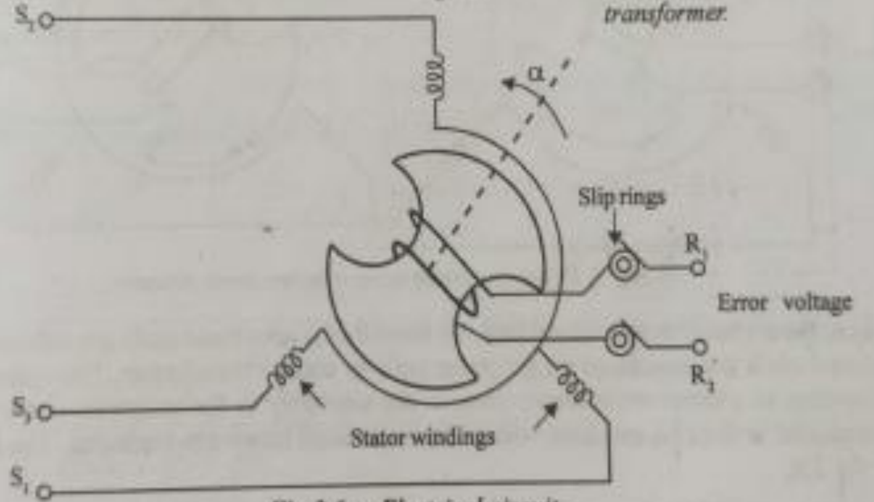


Fig 2.6c : Electrical circuit.

Fig 2.6 : Synchro control transformer.

Working

The generated emf of the synchro transmitter is applied as input to the stator coils of control transformer. The rotor shaft is connected to the load whose position has to be maintained at the desired value. Depending on the current position of the rotor and the applied emf on the stator, an emf is induced on the rotor winding. This emf can be measured and used to drive a motor so that the position of the load is corrected.

SYNCHRO AS ERROR DETECTOR

The synchro error detector is formed by interconnection of a synchro transmitter and synchro control transformer. In this arrangement, the stator leads of the transmitter are directly connected to the stator leads of the control transformer. The angular position of the transmitter-rotor is the reference input (or the input corresponding to the desired output) and the rotor is ... supply with frequency, ω.

The control transformer rotor is connected to a servo motor and to the shaft of the load, whose position is the desired output. The induced emf (error voltage) available across the rotor slip rings of control transformer is measured by a signal conditioning circuit. The output of signal conditioning circuit is used to drive motor so that desired load position is achieved. A simple schematic diagram of synchro as error detector is shown in fig 2.7.

Initially the shafts of transmitter and control transformer are assumed to be in aligned position. In this position the transmitter rotor will be in electrical zero position and the control transformer rotor will be in null position and the angular separation of both rotor axis in aligned position is 90°. The null position of a control transformer in a servo system is defined as position of its rotor for which the output voltage on the rotor winding is zero with the transmitter in its electrical zero position.

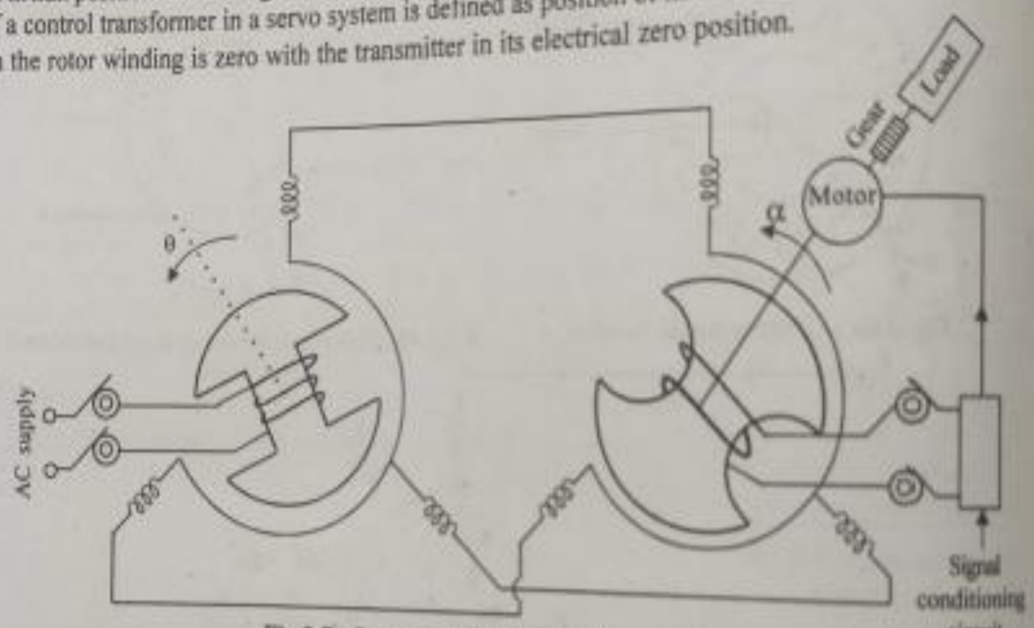


Fig 2.7 : Servo system using synchro error detector.

When, the transmitter rotor is excited, the rotor flux is set-up and emfs are induced in stator coils. These induced emfs are impressed on the stator coils of control transformer. The currents in the stator coils set up flux in control transformer. Due to the similarity in the magnetic construction, the flux patterns produced in the two synchros will be the same if all losses are neglected. The flux patterns are shown in fig 2.8.

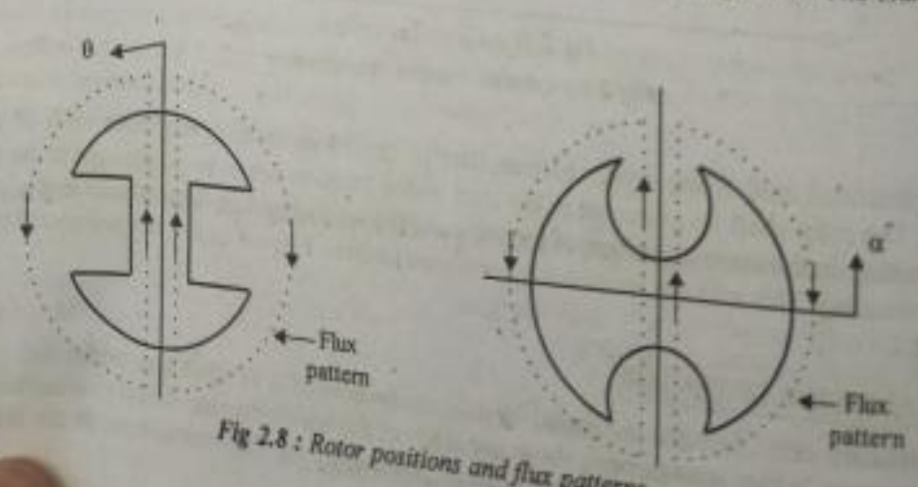


Fig 2.8 : Rotor positions and flux patterns.

Let the rotor of the transmitter rotate through an angle θ from its electrical zero position. Now the rotor of the control transformer will rotate in the same direction through an angle α from its null position. The net angular separation of the two rotors is equal to $(90 - \theta + \alpha)$ and the voltage induced in the control transformer rotor is proportional to the cosine of this angle. The error voltage is amplified and used to drive a servo motor. The motor drives the shaft of the synchro control transformer until it comes to a new aligned position at which the error voltage is zero.

$$\therefore \left.\begin{array}{l}\text{Voltage across slip rings of control}\\ \text{transformer (modulated error voltage)}\end{array}\right\} e_m = K'E_r \cos(90 - \theta + \alpha)\sin\omega t$$

$$= K' E_r \cos(90 - (\theta - \alpha)) \sin \omega t$$

$$= K'E_r \sin(\theta - \alpha) \sin \omega t \qquad .....(2.11)$$

where K' is a proportional constant

Let, $\phi(t) = \theta - \alpha$ .....(2.12)

For small values of $\phi(t)$, $\sin(\theta - \alpha) = \sin\phi(t) \approx \phi(t)$

$$\therefore e_m = K'E_r\phi(t) \sin\omega t \qquad .....(2.13)$$

From the equation (2.13) we can say that the output voltage of the synchro error detector is a modulated signal with carrier frequency, ω (which is same as supply frequency of the transmitter rotor). The magnitude of the modulated carrier wave is proportional to $\phi(t)$ and the phase reversals of the modulated wave depend on the sign of $\phi(t)$. The signal conditioning circuit demodulates the voltage available across slip rings and develops a demodulated and amplified error voltage to drive the motor.

The demodulated error voltage, $e = K_s\phi(t)$ .....(2.14)

where K_s = Sensitivity of synchro error detector in Volts/deg.

On taking Laplace transform of equation (2.14) We get,

$$E(s) = K_s\,\phi(s) \qquad \therefore \frac{E(s)}{\phi(s)} = K_s \qquad .....(2.15)$$

The equation (2.15) is the transfer function of the Synchro error detector.

Note : *If the motor employed is an ac servomotor then the signal conditioning circuit will not include a demodulator.*

voltage

3 Force equation :-

$$F(t) = m_1 \frac{d^2 x_1(t)}{dt^2} + K_1 x_1(t) + f_1 \frac{dx_1(t)}{dt} + K_2\left(x_1(t) - x_2(t)\right) \quad — (1)$$

$$0 = m_2 \frac{d^2 x_2(t)}{dt^2} + K_2\left(x_2(t) - x_1(t)\right) \quad — (2)$$

Force Voltage Analogy :-

$$\boxed{V(t) = C_1 \frac{d^2 q_1(t)}{dt^2} + \frac{1}{R_1}\frac{dq_1(t)}{dt} + K_2}$$

$$V(t) = L_1 \frac{d^2 q_1(t)}{dt^2} + R_1 \frac{dq_1(t)}{dt} + \frac{1}{C_1} q_1(t) + \frac{1}{C_2}\left(q_1(t) - q\right.$$

$i = \frac{q}{t}$

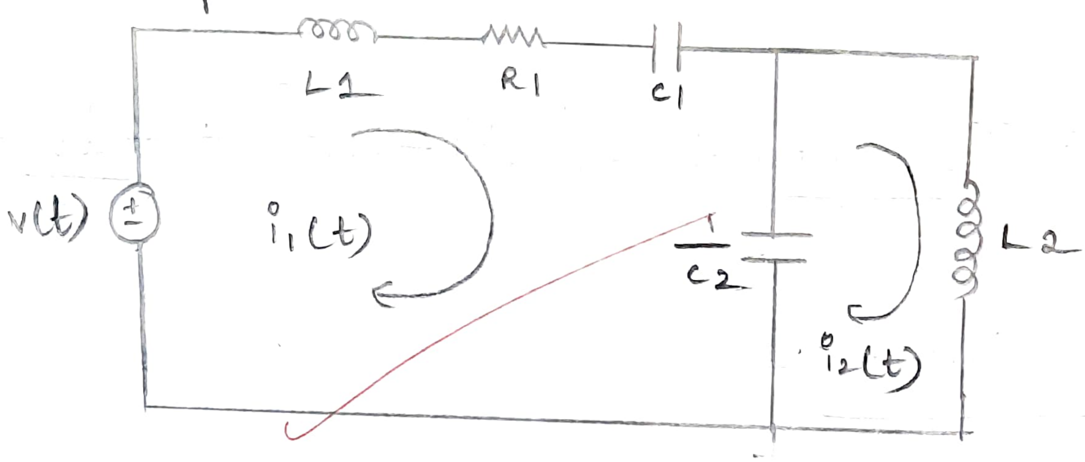
$$V(t) = L_1 \frac{d i_1(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1}\int i_1(t)\,dt + \frac{1}{C_2}\int \left(i_1(t\right.$$

$$0 = \frac{L_2\, \cancel{m_2}\, d^2 q_2(t)}{dt^2} + \frac{1}{C_2}\left(q_2(t) - q_1(t)\right)$$

$$0 = L_2 \frac{d i_2(t)}{dt} + \frac{1}{C_2}\int \left(i_2(t) - i_1(t)\right) dt \quad — (4)$$

From eq ③ and eq ④

Circuit diagram :- (F − V)



$$v = \frac{\phi}{t}$$

Force – Current analogy :-

$$i(t) = C_1 \frac{d^2 \phi_1(t)}{dt^2} + \frac{1}{R_1}\frac{d\phi_1(t)}{dt} + \frac{1}{L_1}\phi_1(t) + \frac{1}{L_2}\left(\phi_1(t) - \phi_2(t)\right)$$

(t))

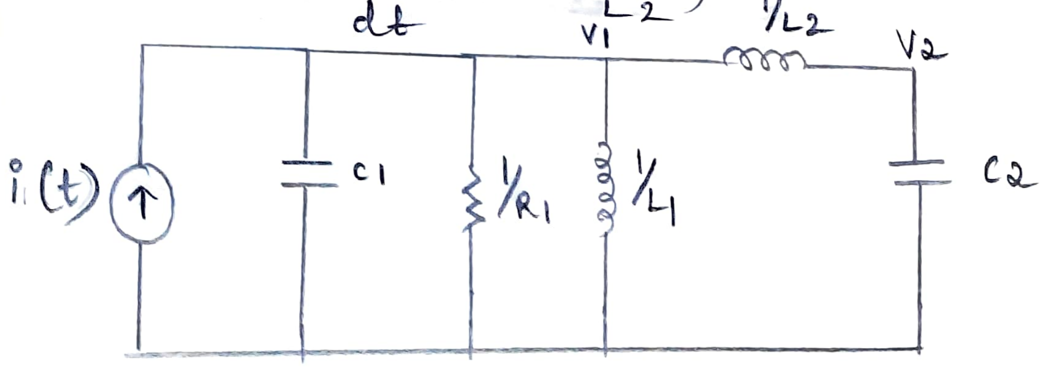
$$= C_1 \frac{dv_1(t)}{dt} + \frac{1}{R_1} v_1(t) + \frac{1}{L_1}\int v_1(t) + \frac{1}{L_2}\int (v_1(t) - v_2(t))\,dt \quad —①$$

$-i_2(t)\,dt$

—③

$$0 = \frac{C_2 d^2\phi_2(t)}{dt^2} + \frac{1}{L_2}\left(\phi_2(t) - \phi_1(t)\right)$$

$$0 = C_2 \frac{dv_2(t)}{dt} + \frac{1}{L_2}\int (v_2(t) - v_1(t))\,dt \quad —②$$



(4) **Find Torque equation :-**

$$T(t) = J_1 \frac{d^2 \theta_1(t)}{dt} + K(\theta_1(t) - \theta_2(t)) \quad \text{--- (1)}$$

$$0 = J_2 \frac{d^2 \theta(t)}{dt} + K(\theta(t) - \theta_1(t)) + f\frac{d\theta}{dt} \quad \text{--- (2)}$$

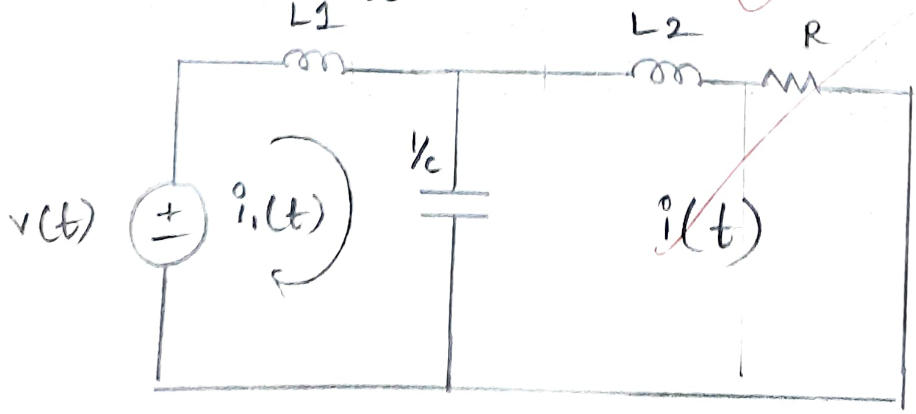
T – V Analogy :-

$$V(t) = L_1 \frac{d^2 q_1(t)}{dt} + \frac{K1}{C}(q_1(t) - q(t))$$

$$0 = L_2 \frac{d^2 q(t)}{dt} + \frac{K1}{C}(q(t) - q_1(t)) + f\frac{dq}{dt}$$

$$V(t) = L_1 \frac{di_1(t)}{dt} + \frac{K1}{C}\int (i_1(t) - i(t))\,dt + \quad \text{--- (1)}$$

$$0 = L_2 \frac{di(t)}{dt} + \frac{1}{C}\int (i(t) - i_1(t))\,dt + R\,i(t) \quad \text{--- (2)}$$



T– Current Analogy

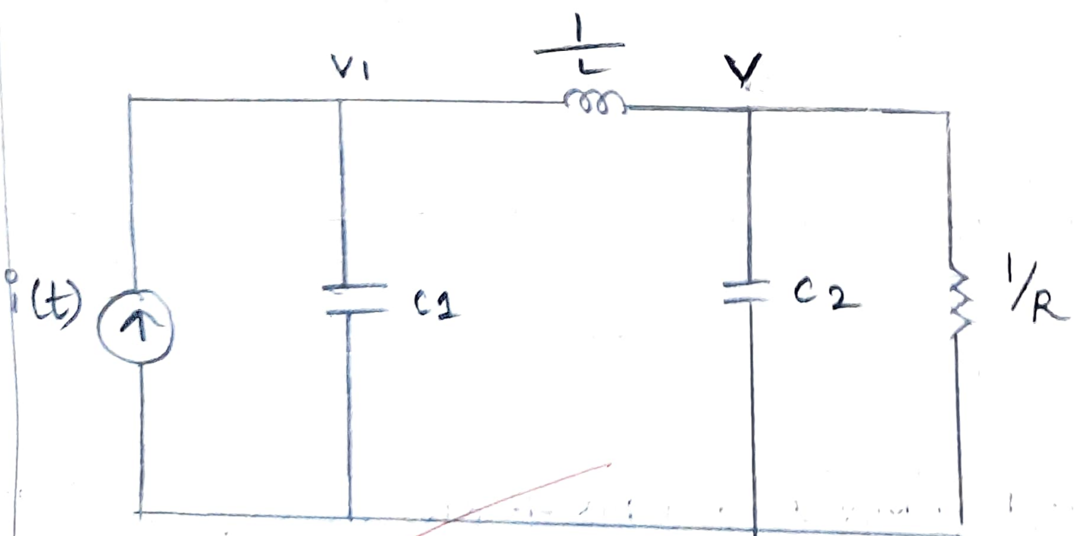
$$i(t) = C_1 \frac{d^2\phi_1(t)}{dt} + \frac{1}{L}\left(\phi_1(t) - \phi(t)\right) \quad \text{— (1)}$$

$$i(t) = C_1 \frac{d v_1(t)}{dt} + \frac{1}{L}\left(\int (v_1(t) - v(t))\right) dt \quad \text{— (2)}$$

$$0 = C_2 \frac{d^2\phi(t)}{dt} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\left(\phi(t) - \phi_1(t)\right) \quad \text{— (3)}$$

$$0 = C_2 \frac{d v(t)}{dt} + \frac{1}{R} v(t) + \frac{1}{L}\left(\int v(t) - v_1(t)\right) dt \quad \text{— (4)}$$

from (2) and (4)



CMR

(6) $\frac{X_1(s)}{F(s)}$ f - V and f - I

$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + K_1 x_1(t) + D_1 \frac{dx_1(t)}{dt} + K_{12}(x_1(t) - x_2(t))$$

$$+ D_{12}\left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right] \quad —(1)$$

$$0 = \frac{m_2 d^2 x_2(t)}{dt^2} + K_2 x_2(t) + D_2 \frac{dx_2(t)}{dt}$$

$$+ K_{12}(x_2(t) - x_1(t)) + D_{12}\left[\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\right]$$

$$—(2)$$

L.S eq 1 :-

$$F(s) = M_1 s^2 X_1(s) + K_1 X_1(s) + D_1 s X_1(s) + K_{12}(X_1(s) - X_2(s))$$

$$+ D_{12}\left[s X_1(s) - s X_2(s)\right]$$

$$F(s) = X_1[s]\left[M_1 s^2 + K_1 + D_1 s + K_{12} + D_{12} s\right] - X_2(s)\left[K_{12} + D_{12}\right.$$

eq eq (2) L.T

$$0 = M_2 s^2 X_2(s) + K_2 X_2(s) + D_2 s X_2(s) + K_{12}(X_2(s) - X_1(s))$$

$$+ D_{12}\left[s X_2(s) - s X_1(s)\right]$$

$$0 = X_1(s)\left[-D_{12}s - K_{12}\right] + X_2(s)\left[M_2s^2 + K_2 + D_2s + K_{12} + D_{12}s\right]$$

$$\begin{bmatrix} M_1s^2 + K_1 + D_1s + K_{12} + D_{12}s & -\left[K_{12} + D_{12}s\right] \\ -\left(D_{12}s + K_{12}\right) & M_2s^2 + K_2 + D_2s + K_1s + D_{12}s \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}$$

$$X_1(s) = \frac{\begin{vmatrix} F(s) & -\left[K_{12} + D_1s\right] \\ 0 & M_2s^2 + K_2D_2s + K_1s + D_{12}s \end{vmatrix}}{\begin{bmatrix} M_1s^2 + K_1 + D_1s + K_{12} + D_{12} & -K_{12} + D_{12}s \\ -\left(D_{12}s + K_{12}\right) & M_2s^2 + K_2 + D_2s + K_1s + D_{12}s \end{bmatrix}}$$

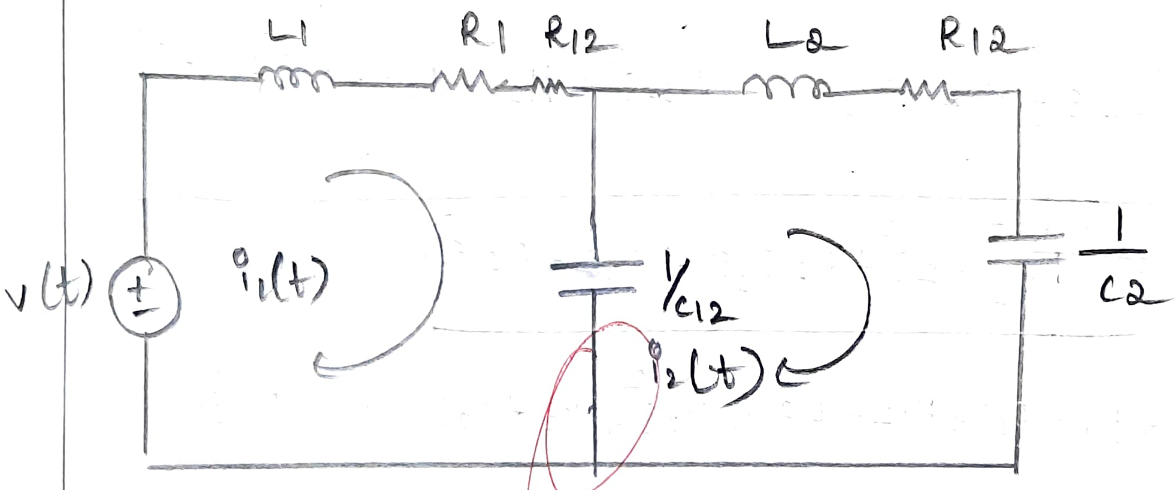
$$\boxed{\frac{X_1(s)}{F(s)} = \frac{\cancel{F(s)}\left[M_2s^2 + K_2D_2s + K_1s + D_{12}s\right)}{\left(M_1s^2 + K_1 + D_1s + K_{12} + D_{12}\right)\left(M_2s^2 + K_2 + D_2s + K_1s + D_{12}s\right) - \left(D_{12}s + K_{12}\right)^2}} \quad (7)$$

f – V Analogy

L	R	$\frac{1}{C}$
C	$\frac{1}{R}$	$\cancel{\frac{1}{C}}$ L

$$v(t) = \overset{L_1}{\cancel{M_1}}\frac{di_1(t)}{dt} + R_1 i_1(t) + \underset{R_{12}}{\overset{\cancel{R_{12}}\ \frac{1}{\cancel{C}_{12}}}{\cancel{D_{12}}}}\left[i_1(t) - i_2(t)\right] + \frac{1}{C_1}\int i_1(t) + \frac{1}{C_{12}}\int\left(i_1(t) - i_2(t)\right)dt \quad \text{—(1)}$$

$$0 = \overset{L_2}{\cancel{M_2}}\frac{di_2(t)}{dt} + \overset{R_{12}}{\cancel{D_{12}}}\left[i_2(t) - i_1(t)\right] + \frac{1}{C_2}\int i_2(t) + \frac{1}{C_{12}}\int\left(i_2(t) - i_1(t)\right)dt \quad \text{—(2)}$$



R $\frac{1}{C}$

L C

C   R   L

f - I :-

$$i(t) = \frac{C_1\, dv_1(t)}{dt} + \frac{1}{R}\, v_1(t) + \frac{1}{R_{12}}\left[v_1(t) - v_2(t)\right] + R$$

$$+ \frac{1}{L_1}\int v_1(t) + \frac{1}{L_{12}}\int v_1(t) - v_2(t)\, dt \quad \text{— (1)}$$

$$0 = \frac{C_2\, dv_2(t)}{dt} + \frac{1}{R_{12}}\left[v_2(t) - v_1(t)\right] + \frac{1}{L_2}\int v_2(t)$$

$$+ \frac{1}{L_{12}}\int v_2(t) - v_1(t)\, dt + R$$