CMR
INSTITUTE OF
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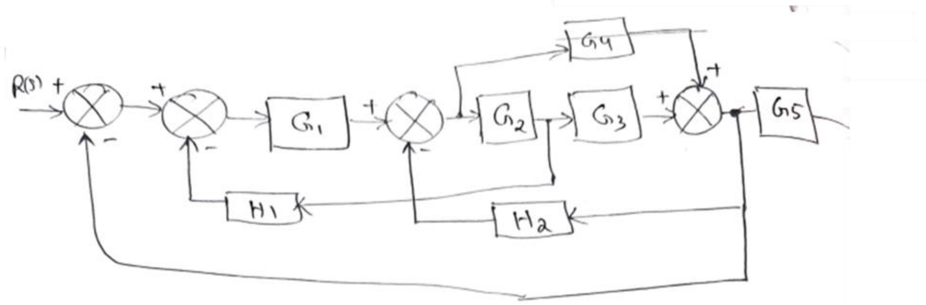
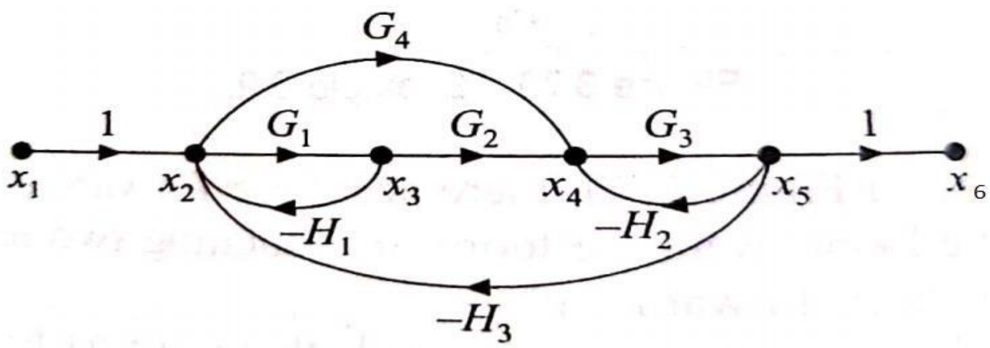
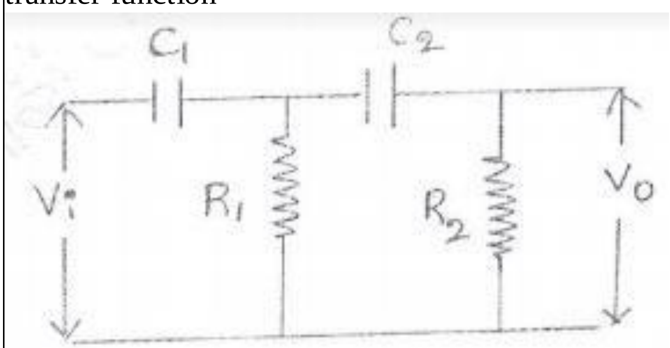
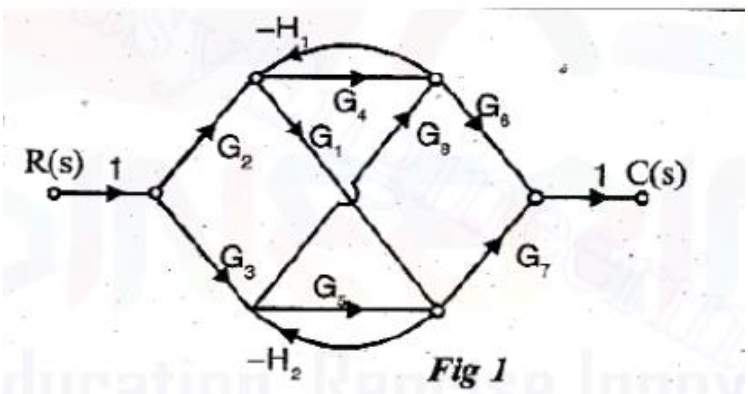
Internal Assesment Test - II

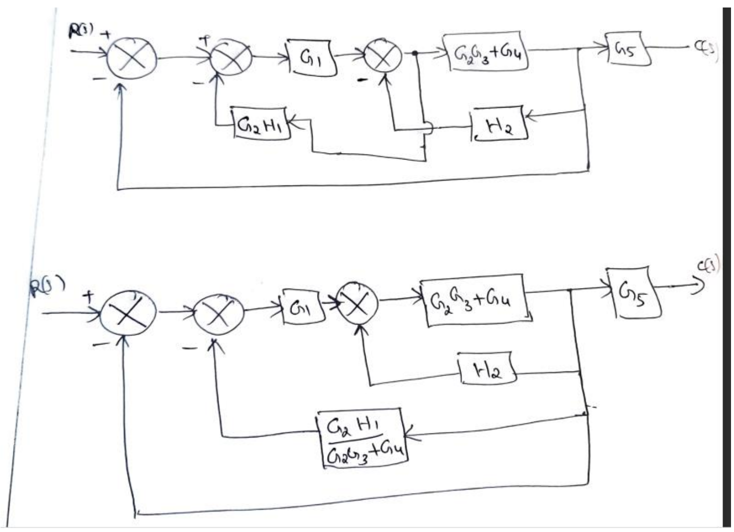
Sub:	Control systems							Code:	21EE52
Date:	01/02/2023	Duration:	90 mins	Max Marks:	50	Sem:	5	Branch:	EEE
Answer Any FIVE FULL Questions									

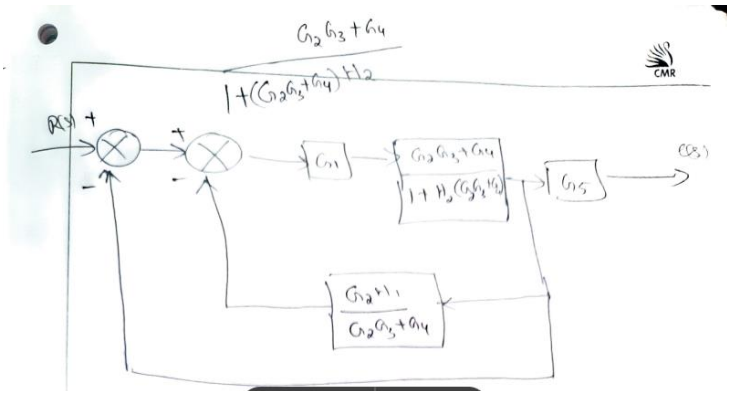
		Marks	OBE	
			CO	RBT
1	Obtain the closed loop transfer function using block diagram reduction technique for the fig. shown 	[10]	CO2	L3
2	Apply the Mason's Gain formula to the signal flow graph shown in fig to find the transfer function x_6 / x_1 	[10]	CO3	L3
3	Draw the signal flow graph for the electrical network shown in fig and find its transfer function 	[10]	CO2	L4

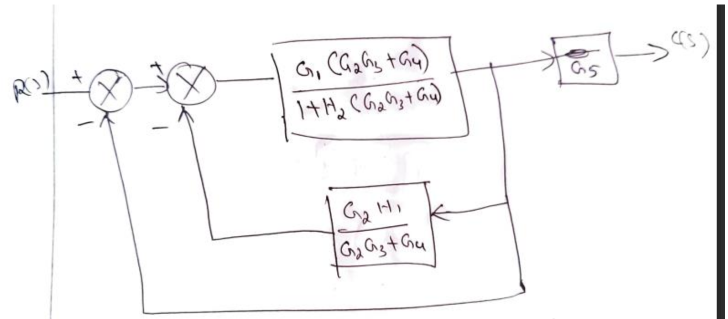
4	Derive the output response of second order system for under damped system for a unit step input	[10]	CO3	L2
5	Find the overall gain of the given SFG using Mason's gain formula  Fig 1	[10]	CO2	L3
6	Find the transfer function represented by the given equations using signal flow graph technique $$x = x_1 + \alpha_0 u$$ $$\frac{dx_1}{dt} = -\alpha_1 x_1 + x_2 + \alpha_2 u$$ $$\frac{dx_2}{dt} = -\alpha_2 x_1 + \alpha_1 u$$	[10]	CO2	L3

Solutions

1







$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+H_2(G_2G_3+G_4)}}{1+\left(\dfrac{G_1(G_2G_3+G_4)}{1+H_2(G_2G_3+G_4)}\right)\left(\dfrac{G_2H_1}{G_2G_3+G_4}\right)} =$$

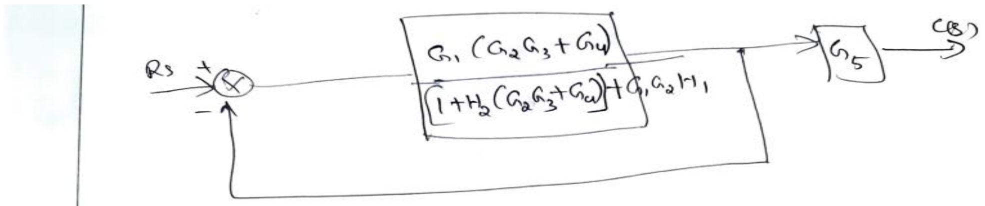
$$\frac{\dfrac{G_1(G_2G_3+G_4)}{[1+H_2(G_2G_3+G_4)]}}{\dfrac{[1+H_2(G_2G_3+G_4)](G_2G_3+G_4)+G_1(G_2G_3+G_4)(G_2H_1)}{[1+H_2(G_2G_3+G_4)](G_2G_3+G_4)}}$$

$$= \frac{G_1(G_2G_3+G_4)}{[1+H_2(G_2G_3+G_4)]} \times \frac{[1+H_2(G_2G_3+G_4)][G_2G_3+G_4]}{[1+H_2(G_2G_3+G_4)][G_2G_3+G_4]+G_1G_2H_1(G_2G_3+G_4)}$$

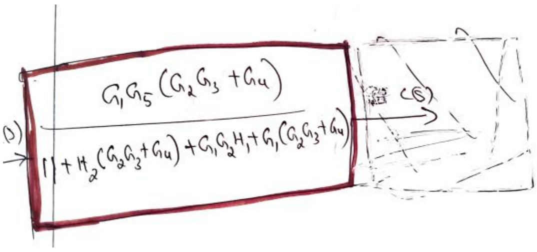
$$= \frac{G_1(G_2G_3+G_4)}{[1+H_2(G_2G_3+G_4)]} \times \frac{[1+H_2(G_2G_3+G_4)][G_2G_3+G_4]}{[1+H_2(G_2G_3+G_4)][G_2G_3+G_4]+G_1G_2H_1(G_2G_3+G_4)}$$

$$= \frac{G_1(G_2G_3+G_4)^2}{[1+H_2(G_2G_3+G_4)][G_2G_3+G_4]+G_1G_2H_1(G_2G_3+G_4)}$$

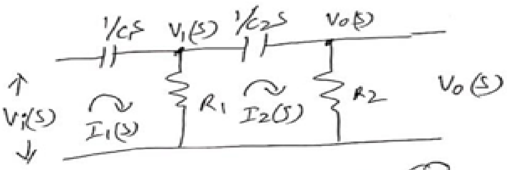
$$= \frac{G_1(G_2G_3+G_4)}{[1+H_2(G_2G_3+G_4)]+G_1G_2H_1}$$



$$\frac{\dfrac{G_1(G_2G_3+G_4)}{[1+H_2(G_2G_3+G_4)]+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{[1+H_2(G_2G_3+G_4)]+G_1G_2H_1}} = \frac{G_1(G_2G_3+G_4)}{[1+H_2(G_2G_3+G_4)]+G_1G_2H_1+G_1(G_2G_3+G_4)}$$



3)



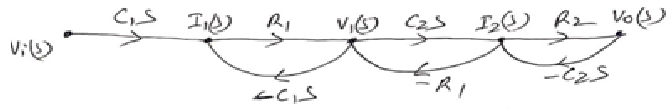
$V_o(s) = I_2(s)R_2$ — (1)

$I_2(s) = [V_1(s) - V_o(s)]C_2s = V_1(s)C_2s - V_o(s)C_2s$ — (2)

$V_1(s) = I_1(s)R_1 - I_2(s)R_1$ — (3)

$I_1(s) = [V_i(s) - V_1(s)]C_1s = V_i(s)C_1s - V_1(s)C_1s$ — (4)

From (1),(2),(3),(4).



Forward Path Gain:-

$M_1 = R_1R_2C_1C_2s^2, \quad \Delta_1 = 1.$

Individual loop Gains

$L_{11} = -R_1C_1s.$

$L_{12} = -R_1C_2s$

$L_{13} = -R_2C_2s$

Two Non-Touching loop Gains

$L_{21} = R_1R_2C_1C_2s^2$

$\Delta = 1 - [L_{11}+L_{12}+L_{13}] + L_{21}$

$= 1 + R_1C_1s + R_1C_2s + R_2C_2s + R_1R_2C_1C_2s^2.$

$$\frac{V_o(s)}{V_i(s)} = \frac{M_1\Delta_1}{\Delta}$$

$$= \frac{R_1R_2C_1C_2s^2}{1+R_1C_1s+R_1C_2s+R_2C_2s+R_1R_2C_1C_2s^2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_1R_2C_1C_2s^2}{R_1R_2C_1C_2s^2+(R_1C_1+R_1C_2+R_2C_2)s+1}$$

2)

Forward Path Gain.

$M_1 = G_1 G_2 G_3$, $\Delta_1 = 1$.

$M_2 = G_3 G_4$, $\Delta_2 = 1$.

Individual loop Gain.

$L_{11} = -G_1 H_1$

$L_{12} = -G_3 H_2$.

$L_{13} = -G_1 G_2 G_3 H_3$.

$L_{14} = -G_3 G_4 H_3$

Two Non-Touching loop Gain

$L_{21} = G_1 G_3 H_1 H_2$

$\Delta = 1 - [L_{11} + L_{12} + L_{13} + L_{14}] + L_{21}$

$= 1 + G_1 H_1 + G_3 H_2 + G_1 G_2 G_3 H_3 + G_3 G_4 H_3 + G_1 G_3 H_1 H_2$.

$$\frac{C(s)}{R(s)} = \frac{M_1 \Delta_1 + M_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 + G_3 G_4}{1 + G_1 H_1 + G_3 H_2 + G_1 G_2 G_3 H_3 + G_3 G_4 H_3 + G_1 G_3 H_1 H_2}.$$

4

Most control systems are designed as under damped systems to have fast response

Step Response of an under damped system $0 < \xi < 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad R(s) = 1/s$$

$$C(s) = \frac{1}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$C(s) = \frac{1}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$C(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Solving for A, B and C we can find that A=1 B= -1 and

$C = -2\xi\omega_n$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$C(s)=\frac{1}{s}-\frac{s+2\zeta\omega_n}{s^2+2\zeta\omega_n s+\zeta^2\omega_n^2+\omega_n^2-\zeta^2\omega_n^2}$$

$$C(s)=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)} \qquad C(s)=\frac{1}{s}-\frac{s+2\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

- Where $\omega_d=\omega_n\sqrt{1-\xi^2}$, is the frequency of transient oscillations and is called ***damped natural frequency***.

$$C(s)=\frac{1}{s}-\frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}-\frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

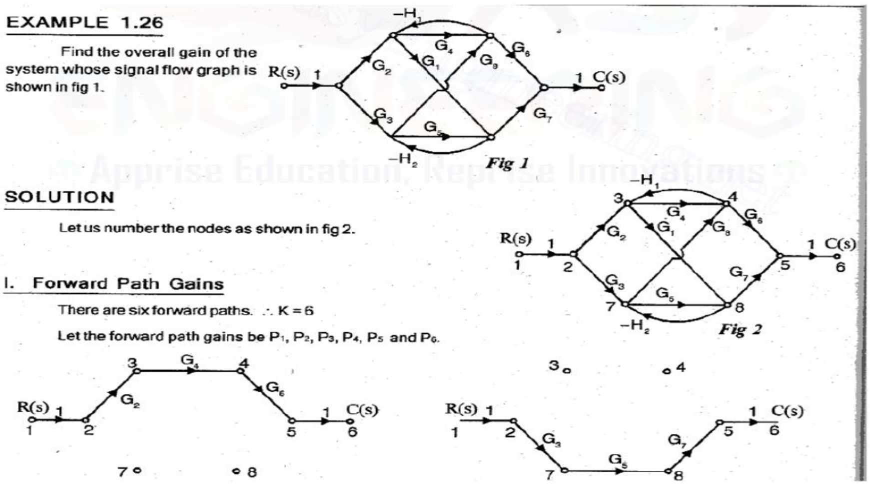
$$=\frac{1}{s}-\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}-\frac{\xi\omega_n}{\omega_d}\cdot\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$=\frac{1}{s}-\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}-\frac{\xi}{\sqrt{1-\xi^2}}\cdot\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

Taking the inverse Laplace transform of the above equation

$$c(t)=1-e^{-\xi\omega_n t}\cos\omega_d t-\frac{\xi e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\omega_d t$$

5



EXAMPLE 1.26

Find the overall gain of the system whose signal flow graph is shown in fig 1.

SOLUTION

Let us number the nodes as shown in fig 2.

I. Forward Path Gains

There are six forward paths. ∴ K = 6

Let the forward path gains be P_1, P_2, P_3, P_4, P_5 and P_6.

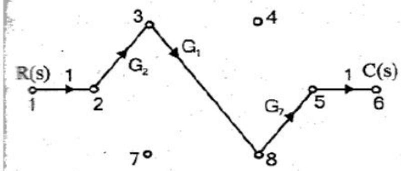


Fig 5 : Forward path-3

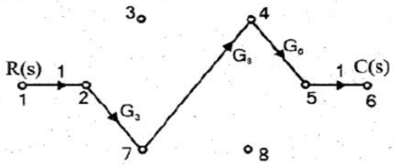


Fig 6 : Forward path-4

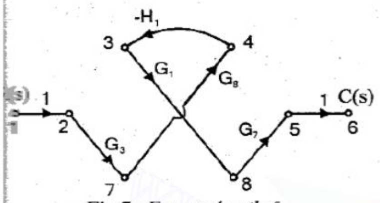


Fig 7 : Forward path-5

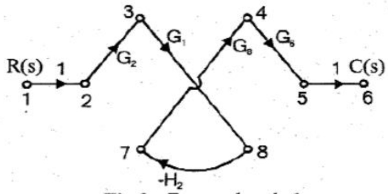


Fig 8 : Forward path-6

Gain of forward path-1, $P_1 = G_2 G_4 G_6$

Gain of forward path-2, $P_2 = G_3 G_5 G_7$

Gain of forward path-3, $P_3 = G_1 G_2 G_7$

Gain of forward path-4, $P_4 = G_3 G_8 G_6$

Gain of forward path-5, $P_5 = -G_1G_3G_7G_8H_1$

Gain of forward path-6, $P_6 = -G_1G_2G_6G_8H_2$

Gain of forward path-6, $P_6 = -G_1G_2G_6G_8H_2$

Individual Loop Gain

There are three individual loops.

Let individual loop gains be P_{11}, P_{21} and P_{31}.

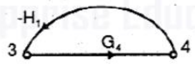


Fig 9 : Loop-1

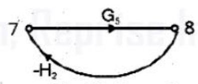


Fig 10 : Loop-2

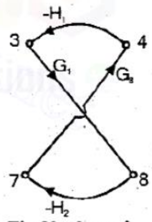


Fig 11 : Loop-3

Loop gain of individual loop-1, $P_{11} = -G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_5H_2$

Loop gain of individual loop-3, $P_{31} = G_1G_8H_1H_2$

Gain Products of Two Non-touching Loops

There is only one combination of two non-touching loops. Let gain product of two non-touching loops be P_{12}.

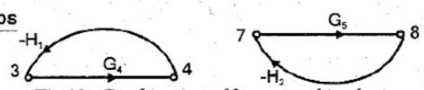


Fig 12 : Combination of 2 non-touching loops

Gain product of first combination of two non - touching loops $\Big\}$ $P_{12} = P_{11}P_{21} = (-G_4H_1)(-G_5H_2) = G_4G_5H_1H_2$

Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + P_{12} = 1 - (-G_4H_1 - G_5H_2 + G_1G_8H_1H_2) + G_4G_5H_1H_2$$

$$= 1 + G_4H_1 + G_5H_2 - G_1G_8H_1H_2 + G_4G_5H_1H_2$$

$$X(s) = X_1(s) + \alpha_0 U(s)$$
$$sX_1(s) = -\alpha_1 X_1(s) + X_2(s) + \alpha_2 U(s)$$
$$sX_2(s) = -\alpha_2 X_1(s) + \alpha_1 U(s)$$
$$X(s) = X_1(s) + \alpha_0 U(s)$$

i.e.

$$X_1(s) = \frac{-\alpha_1}{s} X_1(s) + \frac{1}{s} X_2(s) + \frac{\alpha_2}{s} U(s)$$

$$X_2(s) = \frac{-\alpha_2}{s} X_1(s) + \frac{\alpha_1}{s} U(s)$$

Based on these equations, the signal flow graph is drawn as shown in Figure 3.70.

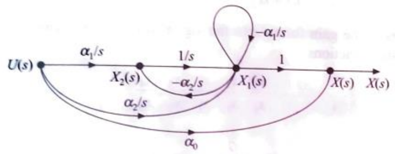


Figure 3.70 Example 3.9.

loops are not touching the third forward path.

The forward paths and the gains associated with them are as follows:

Forward path $U(s)$-$X_2(s)$-$X_1(s)$-$X(s)$ $\quad M_1 = \frac{\alpha_1}{s} \cdot \frac{1}{s} \cdot 1 = \frac{\alpha_1}{s^2}$

Forward path $U(s)$-$X_1(s)$-$X(s)$ $\quad M_2 = \frac{\alpha_2}{s} \cdot 1 = \frac{\alpha_2}{s}$

Forward path $U(s)$-$X(s)$ $\quad M_3 = \alpha_0$

The loops and the gains associated with them are as follows:

Loop $X_2(s)$-$X_1(s)$-$X_2(s)$ $\quad L_1 = \left(\frac{-\alpha_2}{s}\right)\left(\frac{1}{s}\right) = \frac{-\alpha_2}{s^2}$

Loop $X_1(s)$-$X_1(s)$ $\quad L_2 = \frac{-\alpha_1}{s}$

Both the loops are touching the first and second forward paths and no loop is touching the third forward path

$\therefore$

$$\Delta_1 = 1,\ \Delta_2 = 1,\ \Delta_3 = 1 - \left(-\frac{\alpha_2}{s^2} - \frac{\alpha_1}{s}\right) = 1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2} = \Delta$$

The determinant of the signal flow graph is D = 1 − (L_1 + L_2)

$$\therefore \qquad \Delta = 1 - \left(-\frac{\alpha_2}{s^2} - \frac{\alpha_1}{s} \right) = 1 + \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}$$

Using Mason's gain formula, the transfer function is

$$\frac{X(s)}{U(s)} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta} = \frac{\dfrac{\alpha_1}{s^2} + \dfrac{\alpha_2}{s}}{1 + \dfrac{\alpha_1}{s} + \dfrac{\alpha_2}{s^2}} + \alpha_0 = \alpha_0 + \frac{\alpha_1 + \alpha_2 s}{s^2 + \alpha_1 s + \alpha_2}$$

$$\frac{X(s)}{U(s)} = \frac{\alpha_0(s^2 + \alpha_1 s + \alpha_2) + \alpha_1 + \alpha_2 s}{s^2 + \alpha_1 s + \alpha_2}$$