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Internal Assesment Test II – November - 2023

| Sub: | Power System Analysis - II | | | | | | Code: | 18EE71 |
|---|---|---|---|---|---|---|---|---|
| Date: | 5/12/2023 | Duration: | 90 Min | Max Marks: | 50 | Sem: 7 | Section: | A & B |

Note: Answer any **FIVE FULL** Questions & Sketch Neat Figures Wherever Necessary.

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| 1 | What data is required to conduct load flow analysis and discuss the operating constraints considered during the load flow analysis.<br>Data Required for Load Flow Analysis:<br>To conduct a load flow analysis, the following data is required:<br>• System Topology Data:<br>Network topology, including bus and branch data.<br>Bus types (slack, load, generator, etc.).<br>Line and transformer parameters (resistance, reactance, and admittance).<br>• Load Data:<br>Load demand at each bus.<br>Load power factor.<br>• Generator Data:<br>Generator real and reactive power limits.<br>Generator voltage limits.<br>• Transformer Data:<br>Transformer turns ratio and tap settings.<br>• System Control Data:<br>Voltage control devices (e.g., tap changers).<br>Reactive power support devices (e.g., capacitor banks).<br>• Fault Data:<br>Fault location and type.<br>Operating Constraints Considered:<br>During load flow analysis, various operating constraints are considered to ensure a realistic representation of the power system. Some key constraints include:<br>• Voltage Limits:<br>Maintaining voltages within specified limits at all buses.<br>• Reactive Power Limits:<br>Ensuring generators and capacitors operate within specified reactive power limits.<br>• Transmission Line Limits:<br>Avoiding overloading of transmission lines by adhering to thermal limits.<br>• Transformer Tap Limits:<br>Maintaining transformer tap settings within their allowable range.<br>• Generator Limits:<br>Adhering to generator real and reactive power limits.<br>Power Balance:<br>Ensuring that real and reactive power generation equals load demand, considering losses. | [10] | CO2 | L2 |

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| | • Stability Constraints:<br>Checking for stability concerns, especially during contingencies.<br>• Operational Reserve:<br>Allocating sufficient operational reserve to handle uncertainties and contingencies.<br>• Contingency Analysis:<br>Evaluating the impact of possible equipment failures or line outages.<br>By considering these data inputs and constraints, load flow analysis helps ensure a secure and reliable operation of the power system. | | | |
| 2 | Derive the expressions for diagonal elements of Jacobian matrices n NR method of load flow analysis.<br>The diagonal and the off-diagonal elements of $J_1$ are<br>$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} \lvert V_i \rvert \lvert V_j \rvert \lvert Y_{ij} \rvert \sin(\theta_{ij} - \delta_i + \delta_j)$<br>$\frac{\partial P_i}{\partial \delta_j} = -\lvert V_i \rvert \lvert V_j \rvert \lvert Y_{ij} \rvert \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$<br>The diagonal and the off-diagonal elements of $J_2$ are<br>$\frac{\partial P_i}{\partial \lvert V_i \rvert} = 2\lvert V_i \rvert \lvert Y_{ii} \rvert \cos\theta_{ii} + \sum_{j \neq i} \lvert V_j \rvert \lvert Y_{ij} \rvert \cos(\theta_{ij} - \delta_i + \delta_j)$<br>$\frac{\partial P_i}{\partial \lvert V_j \rvert} = \lvert V_i \rvert \lvert Y_{ij} \rvert \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$<br>The diagonal and the off-diagonal elements of $J_3$ are<br>$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} \lvert V_i \rvert \lvert V_j \rvert \lvert Y_{ij} \rvert \cos(\theta_{ij} - \delta_i + \delta_j)$<br>$\frac{\partial Q_i}{\partial \delta_j} = -\lvert V_i \rvert \lvert V_j \rvert \lvert Y_{ij} \rvert \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$<br>The diagonal and the off-diagonal elements of $J_4$ are<br>$\frac{\partial Q_i}{\partial \lvert V_i \rvert} = -2\lvert V_i \rvert \lvert Y_{ii} \rvert \sin\theta_{ii} - \sum_{j \neq i} \lvert V_j \rvert \lvert Y_{ij} \rvert \sin(\theta_{ij} - \delta_i + \delta_j)$<br>$\frac{\partial Q_i}{\partial \lvert V_j \rvert} = -\lvert V_i \rvert \lvert Y_{ij} \rvert \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i$ | [10] | CO3 | L2 |
| 3 | Starting from all the assumptions deduce the Fast decoupled load flow model.<br>Power system transmission lines have a very high $X/R$ ratio. For such a system, real power changes $\Delta P$ are less sensitive to changes in the voltage magnitude and are most sensitive to changes in phase angle $\Delta\delta$. Similarly, reactive power is less sensitive to changes in angle and are mainly dependent on changes in voltage magnitude. Therefore, it is reasonable to set elements $J_2$ and $J_3$ of the Jacobian matrix to zero. Thus, (6.54) becomes<br>$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\lvert V \rvert \end{bmatrix} \qquad (6.68)$ | [10] | CO3 | L2 |

or

$$\Delta P = J_1 \Delta\delta = [\frac{\partial P}{\partial \delta}]\ \Delta\delta \tag{6.69}$$

$$\Delta Q = J_4 \Delta|V| = [\frac{\partial Q}{\partial |V|}]\ \Delta|V| \tag{6.70}$$

(6.69) and (6.70) show that the matrix equation is separated into two decoupled equations requiring considerably less time to solve compared to the time required for the solution of (6.54). Furthermore, considerable simplification can be made to eliminate the need for recomputing $J_1$ and $J_4$ during each iteration. This procedure results in the decoupled power flow equations developed by Stott and Alsac[75–76]. The diagonal elements of $J_1$ described by (6.55) may be written as

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2|Y_{ii}|\sin\theta_{ii}$$

Replacing the first term of the above equation with $-Q_i$, as given by (6.53), results in

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2|Y_{ii}|\sin\theta_{ii}$$
$$= -Q_i - |V_i|^2 B_{ii}$$

Where $B_{ii} = |Y_{ii}|\sin\theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix. $B_{ii}$ is the sum of susceptances of all the elements incident to bus $i$. In a typical power system, the self-susceptance $B_{ii} \gg Q_i$, and we may neglect $Q_i$. Further simplification is obtained by assuming $|V_i|^2 \approx |V_i|$, which yields

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i|B_{ii} \tag{6.71}$$

Under normal operating conditions, $\delta_j - \delta_i$ is quite small. Thus, in (6.56) assuming $\theta_{ii} - \delta_i + \delta_j \approx \theta_{ii}$, the off-diagonal elements of $J_1$ becomes

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j|B_{ij}$$

Further simplification is obtained by assuming $|V_j| \approx 1$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i|B_{ij} \tag{6.72}$$

Similarly, the diagonal elements of $J_4$ described by (6.61) may be written as

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

replacing the second term of the above equation with $-Q_i$, as given by (6.53), results in

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin\theta_{ii} + Q_i$$

Again, since $B_{ii} = Y_{ii}\sin\theta_{ii} \gg Q_i$, $Q_i$ may be neglected and (6.61) reduces to

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i|B_{ii} \tag{6.73}$$

Likewise in (6.62), assuming $\theta_{ij} - \delta_i + \delta_j \approx \theta_{ij}$ yields

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i|B_{ij} \tag{6.74}$$

With these assumptions, equations (6.69) and (6.70) take the following form

$$\frac{\Delta P}{|V_i|} = -B'\ \Delta\delta \tag{6.75}$$

$$\frac{\Delta Q}{|V_i|} = -B''\ \Delta|V| \tag{6.76}$$

Here, $B'$ and $B''$ are the imaginary part of the bus admittance matrix $Y_{bus}$. Since the elements of this matrix are constant, they need to be triangularized and inverted only once at the beginning of the iteration. $B'$ is of order of $(n-1)$. For voltage-controlled buses where $|V_i|$ and $P_i$ are specified and $Q_i$ is not specified, the corresponding row and column of $Y_{bus}$ are eliminated. Thus, $B''$ is of order of $(n-1-m)$, where $m$ is the number of voltage-regulated buses. Therefore, in the fast decoupled power flow algorithm, the successive voltage magnitude and phase angle changes are

$$\Delta\delta = -[B']^{-1}\frac{\Delta P}{|V|} \tag{6.77}$$

$$\Delta|V| = -[B'']^{-1}\frac{\Delta Q}{|V|} \tag{6.78}$$

The fast decoupled power flow solution requires more iterations than the Newton-Raphson method, but requires considerably less time per iteration, and a power flow solution is obtained very rapidly. This technique is very useful in contingency analysis where numerous outages are to be simulated or a power flow solution is required for on-line control.

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Discuss the flow chart and equation how the load flow analysis is carried out using Gauss seidel load flow analysis.

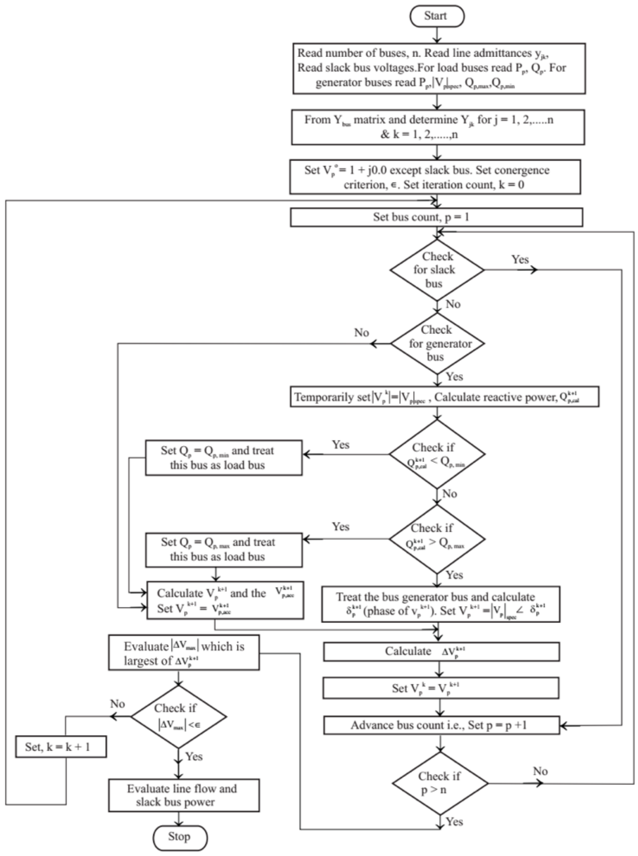


***Fig 2.2 :** Flowchart for load flow solution bu Gauss-Seidel method*

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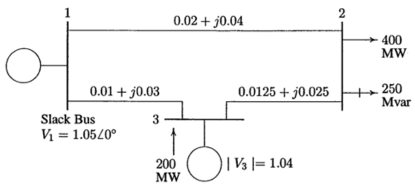
Figure shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus I is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptance's are neglected. Obtain the power flow solution by the **Gauss-Seidel method** including line flows.




Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.04 + j0.0$, $V_2$ and $V_3$ are computed from (6.28).

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0 + j2.5}{1.0 - j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.04 + j0)}{(26 - j52)} \\ &= 0.97462 - j0.042307 \end{aligned}$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$\begin{aligned} Q_3^{(1)} &= -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\ &= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - \\ &\quad (16 - j32)(0.97462 - j0.042307)]\} \\ &= 1.16 \end{aligned}$$

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The value of $Q_3^{(1)}$ is used as $Q_3^{sch}$ for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$V_{c3}^{(1)} = \frac{\frac{P_3^{sch}-jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13}+y_{23}}$$
$$= \frac{\frac{2.0-j1.16}{1.04-j0} + (10-j30)(1.05+j0) + (16-j32)(0.97462-j0.042307)}{(26-j62)}$$
$$= 1.03783 - j0.005170$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = -0.005170$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$$

Thus

$$V_3^{(1)} = 1.039987 - j0.005170$$

For the second iteration, we have

$$V_2^{(2)} = \frac{\frac{P_2^{sch}-jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12}+y_{23}}$$
$$= \frac{\frac{-4.0+j2.5}{.97462+j.042307} + (10-j20)(1.05) + (16-j32)(1.039987+j0.005170)}{(26-j52)}$$
$$= 0.971057 - j0.043432$$

$$Q_3^{(2)} = -\Im\{V_3^{*(1)}[V_3^{(1)}(y_{13}+y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\}$$
$$= -\Im\{(1.039987+j0.005170)[(1.039987-j0.005170)(26-j62) -$$
$$(10-j30)(1.05+j0) - (16-j32)(0.971057-j0.043432)]\}$$
$$= 1.38796$$

$$V_{c3}^{(2)} = \frac{\frac{P_3^{sch}-jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13}+y_{23}}$$
$$= \frac{\frac{2.0-j1.38796}{1.039987+j0.00517} + (10-j30)(1.05) + (16-j32)(.971057-j.043432)}{(26-j62)}$$
$$= 1.03908 - j0.00730$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = -0.00730$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2} = 1.039974$$

or

$$V_3^{(2)} = 1.039974 - j0.00730$$

The process is continued and a solution is converged with an accuracy of $5 \times 10^{-5}$ pu in seven iterations as given below.

$V_2^{(3)} = 0.97073 - j0.04479 \quad Q_3^{(3)} = 1.42904 \quad V_3^{(3)} = 1.03996 - j0.00833$
$V_2^{(4)} = 0.97065 - j0.04533 \quad Q_3^{(4)} = 1.44833 \quad V_3^{(4)} = 1.03996 - j0.00873$
$V_2^{(5)} = 0.97062 - j0.04555 \quad Q_3^{(5)} = 1.45621 \quad V_3^{(5)} = 1.03996 - j0.00893$
$V_2^{(6)} = 0.97061 - j0.04565 \quad Q_3^{(6)} = 1.45947 \quad V_3^{(6)} = 1.03996 - j0.00900$
$V_2^{(7)} = 0.97061 - j0.04569 \quad Q_3^{(7)} = 1.46082 \quad V_3^{(7)} = 1.03996 - j0.00903$

The final solution is

$$V_2 = 0.97168\angle{-2.6948^\circ} \quad \text{pu}$$

$$S_3 = 2.0 + j1.4617 \quad \text{pu}$$
$$V_3 = 1.04\angle{-.498^\circ} \quad \text{pu}$$
$$S_1 = 2.1842 + j1.4085 \quad \text{pu}$$

Line flows and line losses are computed as in Example 6.7, and the results expressed in MW and Mvar are

$S_{12} = 179.36 + j118.734 \quad S_{21} = -170.97 - j101.947 \quad S_{L12} = 8.39 + j16.79$
$S_{13} = 39.06 + j22.118 \quad S_{31} = -38.88 - j\,21.569 \quad S_{L13} = 0.18 + j0.548$
$S_{23} = -229.03 - j148.05 \quad S_{32} = 238.88 + j167.746 \quad S_{L23} = 9.85 + j19.69$

The power flow diagram is shown in Figure 6.13, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

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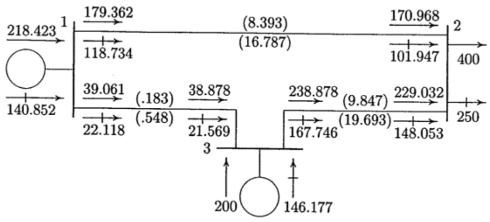


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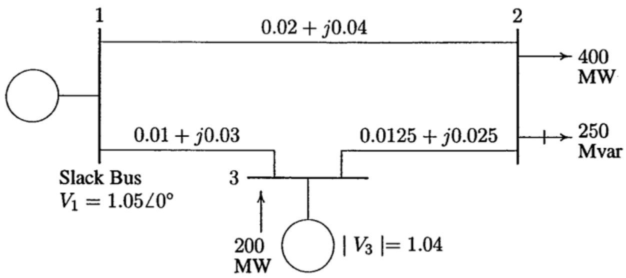
Figure shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus I is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptance's are neglected. Obtain the power flow solution by the **Newton Raphson's method** including line losses.




Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$, and $y_{23} = 16 - j32$. This results in the bus admittance matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Converting the bus admittance matrix to polar form with angles in radian yields

$$Y_{bus} = \begin{bmatrix} 53.85165\angle{-1.9029} & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle{-1.1071} & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle{-1.1737} \end{bmatrix}$$

From (6.52) and (6.53), the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are

$$\begin{aligned} P_2 &= |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + \\ &\quad |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ P_3 &= |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \\ &\quad \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33} \\ Q_2 &= -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - \\ &\quad |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \end{aligned}$$

Elements of the Jacobian matrix are obtained by taking partial derivatives of the above equations with respect to $\delta_2$, $\delta_3$ and $|V_2|$.

$$\begin{aligned} \frac{\partial P_2}{\partial \delta_2} &= |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \\ &\quad \sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial \delta_3} &= -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial |V_2|} &= |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + \\ &\quad |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_3}{\partial \delta_2} &= -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) \\ \frac{\partial P_3}{\partial \delta_3} &= |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \\ &\quad \sin(\theta_{32} - \delta_3 + \delta_2) \\ \frac{\partial P_3}{\partial |V_2|} &= |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) \end{aligned}$$

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$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

The slack bus voltage is $V_1 = 1.05\angle 0$ pu, and the bus 3 voltage magnitude is $|V_3| = 1.04$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$

$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\begin{aligned} \Delta\delta_2^{(0)} &= -0.045263 & \delta_2^{(1)} &= 0 + (-0.045263) = -0.045263 \\ \Delta\delta_3^{(0)} &= -0.007718 & \delta_3^{(1)} &= 0 + (-0.007718) = -0.007718 \\ \Delta|V_2^{(0)}| &= -0.026548 & |V_2^{(1)}| &= 1 + (-0.026548) = 0.97345 \end{aligned}$$

Voltage phase angles are in radians. For the second iteration, we have

$$\begin{bmatrix} -0.099218 \\ 0.021715 \\ -0.050914 \end{bmatrix} = \begin{bmatrix} 51.724675 & -31.765618 & 21.302567 \\ -32.981642 & 65.656383 & -15.379086 \\ -28.538577 & 17.402838 & 48.103589 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$

and

$$\begin{aligned} \Delta\delta_2^{(1)} &= -0.001795 & \delta_2^{(2)} &= -0.045263 + (-0.001795) = -0.04706 \\ \Delta\delta_3^{(1)} &= -0.000985 & \delta_3^{(2)} &= -0.007718 + (-0.000985) = -0.00870 \\ \Delta|V_2^{(1)}| &= -0.001767 & |V_2^{(2)}| &= 0.973451 + (-0.001767) = 0.971684 \end{aligned}$$

For the third iteration, we have

$$\begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} 51.596701 & -31.693866 & 21.147447 \\ -32.933865 & 65.597585 & -15.351628 \\ -28.548205 & 17.396932 & 47.954870 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(2)} \\ \Delta\delta_3^{(2)} \\ \Delta|V_2^{(2)}| \end{bmatrix}$$

and

$$\begin{aligned} \Delta\delta_2^{(2)} &= -0.000038 & \delta_2^{(3)} &= -0.047058 + (-0.0000038) = -0.04706 \\ \Delta\delta_3^{(2)} &= -0.0000024 & \delta_3^{(3)} &= -0.008703 + (-0.0000024) = 0.008705 \\ \Delta|V_2^{(2)}| &= -0.0000044 & |V_2^{(3)}| &= 0.971684 + (-0.0000044) = 0.97168 \end{aligned}$$

The solution converges in 3 iterations with a maximum power mismatch of $2.5 \times 10^{-4}$ with $V_2 = 0.97168\angle -2.696°$ and $V_3 = 1.04\angle -0.4988°$. From (6.52) and (6.53), the expressions for reactive power at bus 3 and the slack bus real and reactive powers are

$$Q_3 = -|V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2|Y_{33}|\sin\theta_{33}$$

$$P_1 = |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3| |Y_{13}|\cos(\theta_{13} - \delta_1 + \delta_3)$$

$$Q_1 = -|V_1|^2|Y_{11}|\sin\theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3| |Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3)$$

Upon substitution, we have

$$Q_3 = 1.4617 \quad \text{pu}$$

$$P_1 = 2.1842 \quad \text{pu}$$

$$Q_1 = 1.4085 \quad \text{pu}$$

Finally, the line flows are calculated in the same manner as the line flow calculations in the Gauss-Seidel method described in Example 6.7, and the power flow diagram is as shown in Figure 6.13.

A program named **lfnewton** is developed for power flow solution by the Newton-Raphson method for practical power systems. This program must be preceded by the **lfybus** program. **busout** and **lineflow** programs can be used to print the load flow solution and the line flow results. The format is the same as the Gauss-Seidel. The following is a brief description of the **lfnewton** program.

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****** **ALL THE BEST** ******

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