CMR

INSTITUTE OF

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Internal Assessment Test - III

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|---|---|---|---|---|---|---|---|---|---|
| Sub: | Control systems | | | | | | | Code: | 21EE52 |
| Date: | 14/01/2024 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 5 | Branch: | EEE |
| Answer Any FIVE FULL Questions | | | | | | | | | |

| | | Marks | OBE CO | OBE RBT |
|---|---|---|---|---|
| 1 | Sketch the bode plot for the transfer function and find the gain cross over frequency ,phase cross over frequency, gain margin and phase margin $G(s)=\frac{10}{s(1+0.4s)(1+0.1s)}$ | [10] | CO2 | L3 |
| 2 | Sketch the root locus for the transfer function $G(s)=\frac{K}{s(s^2+4s+13)}$ | [10] | CO2 | L3 |
| 3 | Derive the transfer function of a PD controller along with its steady state error. | [10] | CO2 | L4 |
| 4 | Derive an expression for resonant peak and resonant frequency for a second order system | [10] | CO3 | L2 |

| | | | | |
|---|---|---|---|---|
| 5 | A unity feedback system is characterized by an open-loop transfer function | [10] | CO2 | L3 |

| | | | | |
|---|---|---|---|---|
| | $$G(s) = \frac{K}{s(s+10)}$$ Determine the gain K so that the system will have the damping ration of 0.5.For this value of K, determine the settling time, peak overshoot and time to peak overshoot for a unit step. | | | |
| 6 | For a closed loop system whose open loop transfer function $$G(s)H(s) = \frac{10}{s(s+1)(s+2)}$$ Find the steady state error when the input is r(t) =1+2t+1.5 $t^2$ | [10] | CO2 | L4 |

Solution

1

1) $G(s) = \dfrac{10}{s(1+0.4s)(1+0.1s)}$

Put $s=j\omega$, $G(j\omega) = \dfrac{10}{j\omega(1+0.4j\omega)(1+0.1j\omega)}$

corner frequencies, $\omega_{c1} = \frac{1}{0.4} = 2.5$ rad/sec.

$\omega_{c2} = \frac{1}{0.1} = 10$ rad/sec.

Magnitude plot,

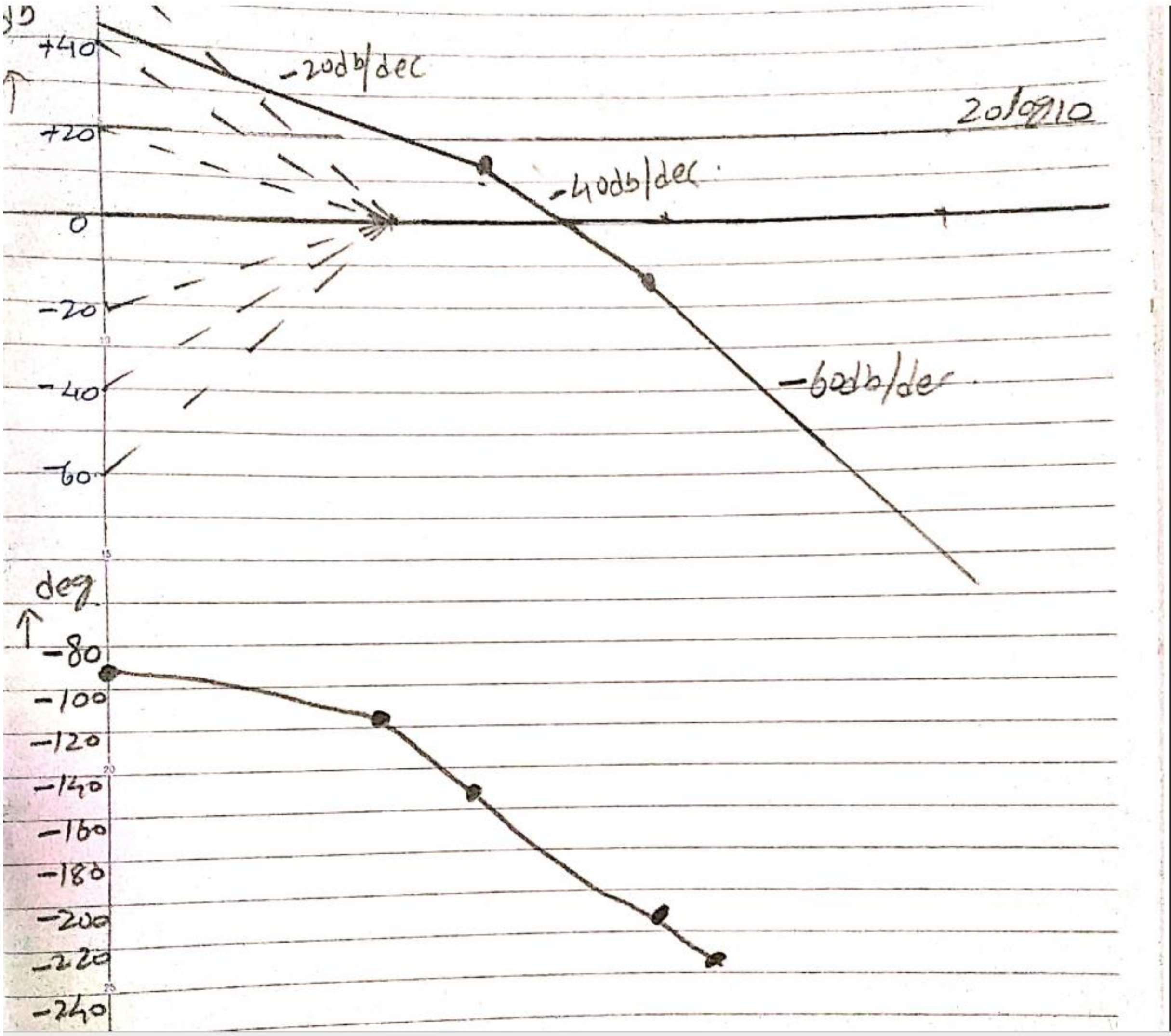
| Basic factors | corner freq rad/sec | slope db/dec | change in slope, db/dec. |
|---|---|---|---|
| 10 | – | 0 | 0 |
| $1/j\omega$ | – | −20 | 0−20 = −20. |
| $\frac{1}{1+0.4j\omega}$ | $\omega_{c1} = 2.5$ | −20 | −20−20 = −40 |
| $\frac{1}{1+0.1j\omega}$ | $\omega_{c2} = 10$ | −20 | −40−20 = −60. |

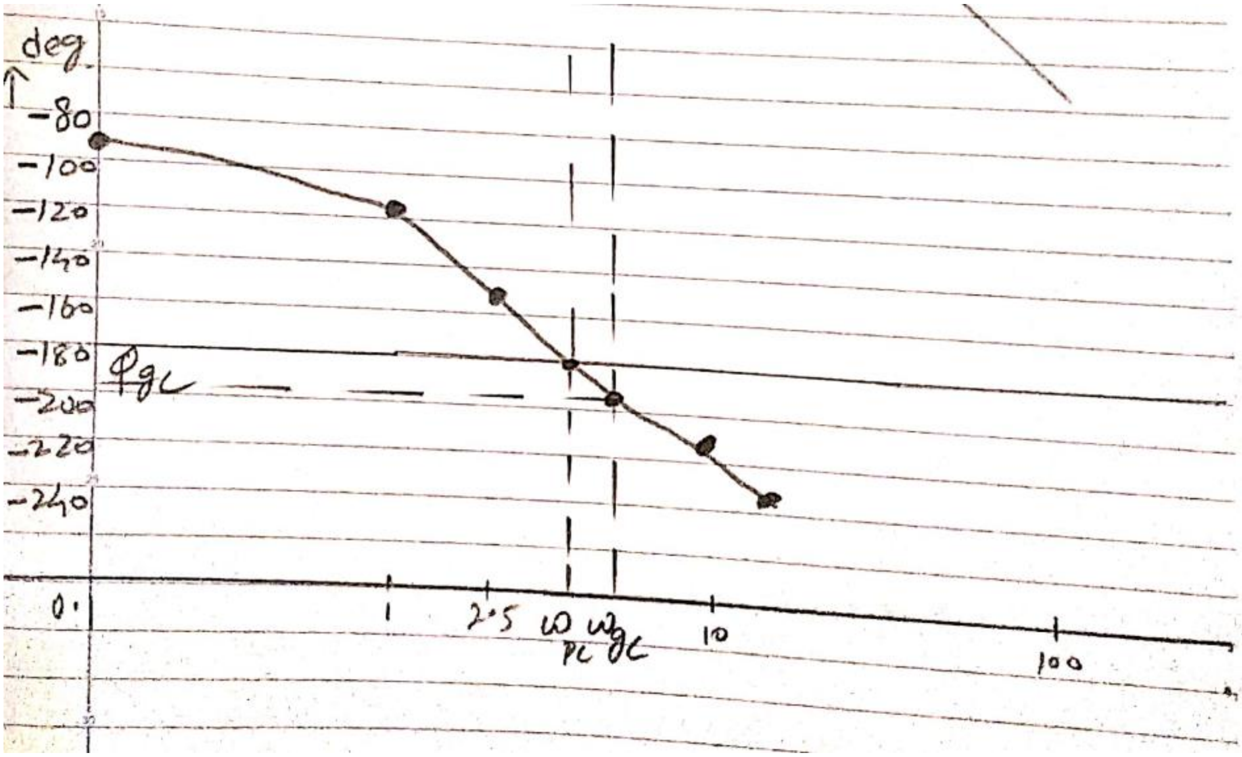
First basic factor $\Rightarrow 20\log K = 20\log 10 = 20$ db.

Phase plot.

$\angle G(j\omega) = -90 - \tan^{-1}0.4\omega - \tan^{-1}0.1\omega$.

| $\omega$ rad/sec | 0.1 | 1 | 2.5 | 10 | 20 |
|---|---|---|---|---|---|
| $\angle G(j\omega)$ (deg) | −92 | −118 | −150 | −210 | −236 |


dB
+40
+20
0
-20
-40
-60
-20db/dec
-40db/dec
-60db/dec
2010910
deg
-80
-100
-120
-140
-160
-180
-200
-220
-240




2)

Solution.

Step 1 Starting and Ending Points.

Starting points are poles.

$S=0, \quad S^2+4S+13=0$

$S=-2\pm j3$.

Ending points are zeros ⇒ No zeros.
⇒ $S=\infty,\infty,\infty$

Step 2 Number of branch = Number of Poles.

= 3.

Step 3 Symmetrical with respect to real-axis.

$= 3.$

Step 3 Symmetrical with respect to real-axis.

Step 4

Angle of Asymptote $\phi_a = \frac{(2q+1)180}{n-m}$

where $q = 0, 1, \ldots, n-m-1.$

$n = 3,\ m = 0,\ n-m-1 = 3-0-1 = 2.$

If $q = 0$, $\phi_a = \frac{180}{3-0} = 60°.$

If $q = 1$, $\phi_a = \frac{3 \times 180}{3} = 180°.$

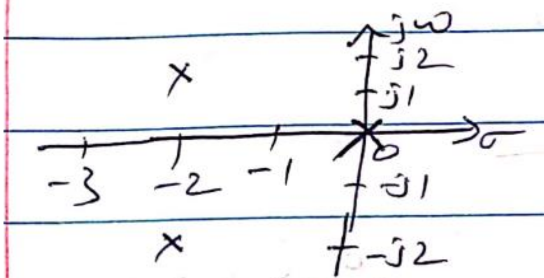
If $q = 2$, $\phi_a = \frac{5 \times 180}{3} = 300 = -60°.$

Intersection Point of Asymptote,

$$\sigma = \frac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{n-m}$$

$$= \frac{(0-2-2)-0}{3-0} = \frac{-4}{3} = -1.33$$

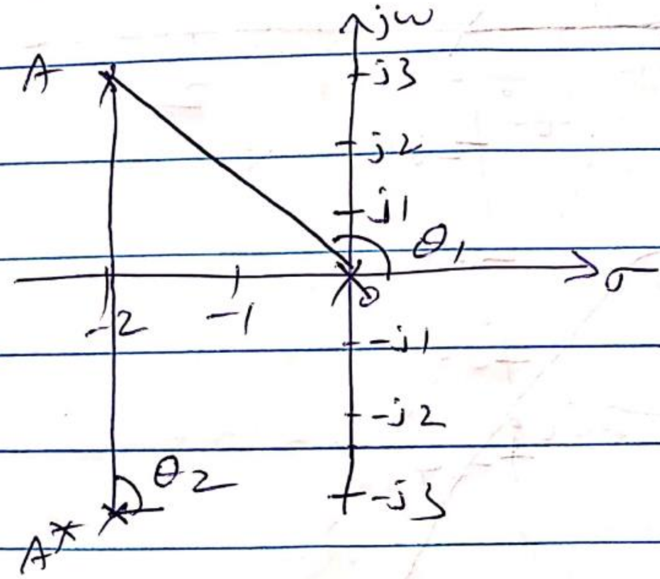
Step 5 Root locus on real axis




$-\infty$ to $0 \Rightarrow$ Odd roots $\Rightarrow$ Yes.

$0$ to $\infty \Rightarrow$ Even $\Rightarrow$ NO.

## Step:-6 Angle of Departure.




$\theta_1 = 180 - \tan^{-1}\frac{3}{2}$

$= 117.4^\circ$

$\theta_2 = 90^\circ$

Angle of Departure from A $\left.\right\} = 180 - (\theta_1 + \theta_2) + (0)$

$= 180 - 117.4 - 90 = -27.4^\circ$

Angle of Departure from A* $\left.\right\} = 27.4^\circ$

Step:-7 Intersection of root locus with Imaginary axis.

$$\frac{C(s)}{R(s)} = \frac{K}{s(s^2+4s+13)+K}$$

$$s(s^2+4s+13)+K=0.$$

$$s^3+4s^2+13s+K=0.$$

Routh table.

| | | |
|---|---|---|
| $s^3$ | $1$ | $13$ |
| $s^2$ | $4$ | $K$ |
| $s^1$ | $\frac{52-K}{4}$ | |
| $s^0$ | $K$ | |

$s^0 \Rightarrow K > 0.$

$s^1 \Rightarrow K < 52$

For stable $0 < K < 52.$

Critical value of $K = 52.$

$4s^2 + K = 0.$

$4s^2 + 52 = 0 \Rightarrow s^2 + 13 = 0.$

$s^2 = -13$

$s = \pm j3.6$

Step:- 8. Breakaway point.

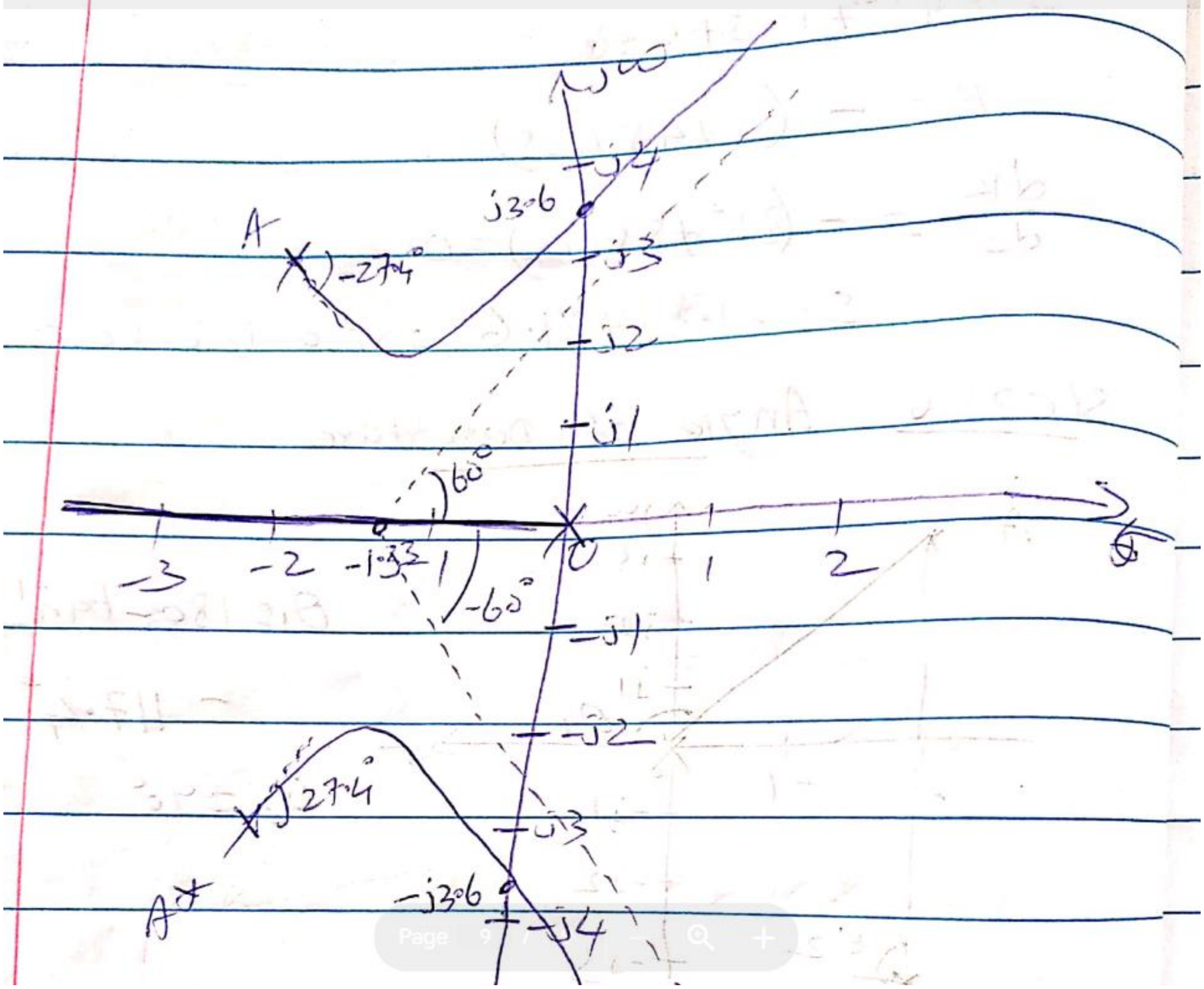
$s^3 + 4s^2 + 13s + K = 0.$

$K = -(s^3 + 4s^2 + 13s)$

$\frac{dK}{ds} = -(3s^2 + 8s + 13) = 0.$

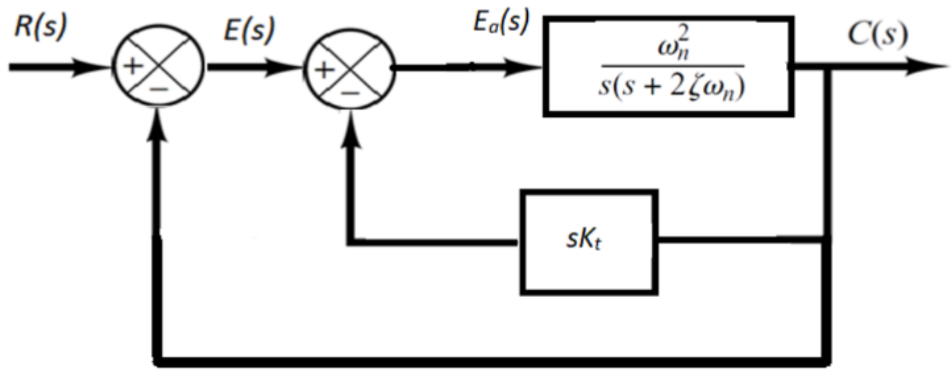
$s = -1.33 \pm j1.6 \Rightarrow$ No Breakaway points

Sketch root locus:

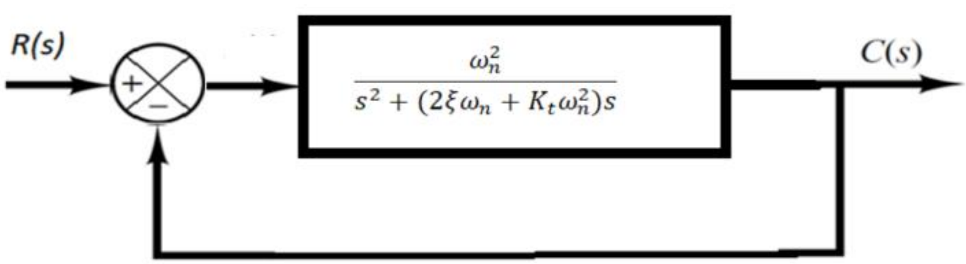


In derivative feedback control, the actuating signal is the difference of proportional error signal and the derivative of the output signal.

$$e_a(t) = e(t) - K_t \frac{dc(t)}{dt} \qquad E_a(s) = E(s) - K_t s C(s)$$




The simplified block diagram is given by.




$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + (2\xi\omega_n + K_t\omega_n^2)s + \omega_n^2}$$

The characteristic equation is

$$s^2 + (2\xi\omega_n + K_t\omega_n^2)s + \omega_n^2 = 0$$

Therefore the effective damping ratio

$$2\xi'\omega_n = 2\xi\omega_n + K_t\omega_n^2$$

$$\xi' = \frac{2\xi\omega_n + K_t\omega_n^2}{2\omega_n} \qquad \xi' = \xi + \frac{K_t\,\omega_n}{2}$$

The damping ratio increases, so the maximum overshoot is reduced.

We know that

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+\dfrac{\omega_n^2}{s^2+(2\xi\omega_n+K_t\omega_n^2)s}}$$

$$\frac{E(s)}{R(s)} = \frac{s^2+(2\xi\omega_n+K_t\omega_n^2)s}{s^2+(2\xi\omega_n+K_t\omega_n^2)s+\omega_n^2}$$

$$\frac{E(s)}{R(s)} = \frac{s(s+2\xi\omega_n+K_t\omega_n^2)}{s^2+(2\xi\omega_n+K_t\omega_n^2)s+\omega_n^2}$$

For ramp input $R(s) = \frac{1}{s^2}$

$$E(s) = \frac{1}{s^2}\,\frac{s(s+2\xi\omega_n+K_t\omega_n^2)}{s^2+(2\xi\omega_n+K_t\omega_n^2)s+\omega_n^2}$$

Steady state error

$$e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s\,\frac{1}{s^2}\,\frac{s(s+2\xi\omega_n + K_t\omega_n^2)}{s^2+(2\xi\omega_n + K_t\omega_n^2)s+\omega_n^2}$$

$$= \frac{2\xi\omega_n + K_t\omega_n^2}{\omega_n^2} = \frac{2\xi}{\omega_n} + K_t$$

So with derivative feedback control the steady state error increases

4)

## ***Correlation between Time and frequency response***

CLTF (general expression), $M(s) = \dfrac{Y(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

Sinusoidal steady state, $s=j\omega$. Thus,

$$\boxed{M(j\omega) = \frac{Y(j\omega)}{R(j\omega)} = \frac{G(j\omega)}{1+G(j\omega)H(j\omega)}}$$

Magnitude of M(jω) is

$$\boxed{|M(j\omega)| = \left|\frac{G(j\omega)}{1+G(j\omega)H(j\omega)}\right|}$$

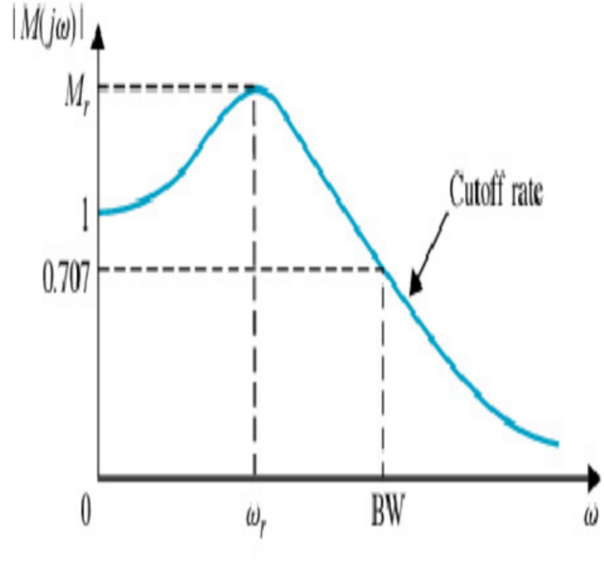
Phase of M(jω) is

$$\boxed{\angle M(j\omega) = \angle G(j\omega) - \angle[1+G(j\omega)H(j\omega)]}$$

# Frequency Domain Specifications

## 1. Resonant Peak, ($M_r$)

**The resonant peak $M_r$ is the maximum value of** M(jω). In general $M_r$ gives indication of the relative stability. Normally, a large $M_r$ corresponds to a large maximum overshoot of the step response.




## 2. Resonant Frequency, |M(jω)|

The resonant frequency ωr is the frequency at which the peak resonance Mr occurs.

## 3. Bandwidth, BW

The bandwidth BW is the frequency at which M( jω) drops to 70.7 % of, or 3 dB down from, its zero-frequency value. Generally, the bandwidth of a control system gives an indication of the transient response properties in time domain. A large BW corresponds to faster rise time, whereas small BW refers to slow and sluggish time response.

## 4. Cutoff Rate

Cutoff rate is the slope of |M(jω)| at high frequencies.

- 2nd order system: $M_r$, $j\omega$ and BW are uniquely related to $\zeta$ and $\omega_n$

$$M(j\omega) = \frac{Y(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{1}{1 + \mathrm{j}2(\omega/\omega_{\mathrm{n}})\zeta - (\omega/\omega_n)^2}$$

- By letting **u= ω/ ω$_\mathbf{n}$**,

$$M(j\omega) = \frac{1}{1 + j2u\zeta - u^2}$$

- Magnitude of M(ju),

$$|M(ju)| = \frac{1}{\left[(1-u^2)^2 + (2\zeta u)^2\right]^{1/2}}$$

- $\omega_r$ can be found by $$\boxed{\frac{d|M(ju)|}{du} = 4u(u^2 - 1 + 2\zeta^2) = 0}$$

$u_r = \sqrt{1-2\zeta^2}$ , $\omega_r = \omega_n \sqrt{1-2\zeta^2}$ , $\zeta < 0.707$

- Since frequency is a real quantity, the equation is meaningful only for $2\zeta^2 < 1$, or $\zeta < 0.707$

- For all values of $\zeta > 0.707$, the resonant frequency is $\omega_r = 0$, and $M_r = 1$

$$\boxed{M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}, \qquad \zeta < 0.707}$$

- BW of prototype second order system is given by

$$BW = \omega_n \sqrt{\left(1-2\zeta^2\right) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

A summary of the relationships between the time-domain response and frequency domain characteristics of a second order system, is as follows:

1. The resonant peak Mr depends on $\xi$ only. When $\xi$ *= 0, Mr is infinite. When* $\xi$ is negative, the system is unstable, and the value of Mr ceases to have any meaning. As $\xi$ increases, Mr decreases.

2. For $\xi \geq 0.707$, Mr =1 and $\omega_r = 0$. In comparison with the unit-step time response, the maximum overshoot also depends on $\xi$.

3. Bandwidth is directly proportional to $\omega_n$. For the unit-step response, rise time increases as $\omega_n$ decreases. Therefore, BW and rise time are inversely proportional to each other.

4. Bandwidth and Mr are proportional to each other for $0 \leq \xi \leq 0.707$.