USN

Fifth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Control Systems

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. What are the properties of a good control system? **(04 Marks)**

b. Write the comparison between open loop control system and closed loop control system. **(08 Marks)**

c. For the mechanical system shown in Fig Q1(c), write the differential equations of performance. Find write loop equations based on Force – Voltage analogy and write electrical analogous circuit.

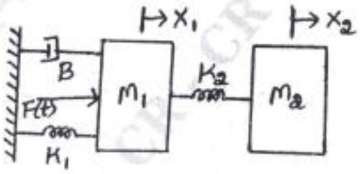


Fig Q1(c) **(08 Marks)**

OR

2 a. Obtain the transfer function of an armature controlled DC servomotor. **(08 Marks)**

b. Write the differential equations governing the mechanical system shown in Fig Q2(b). Draw the torque – current electrical analogous circuit.

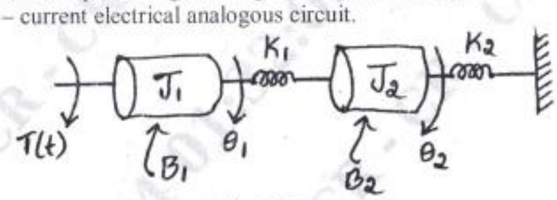


Fig Q2(b) **(08 Marks)**

c. Explain translational motion of mechanical system. **(04 Marks)**

Module-2

3 a. Define the following terms : i) Source node ii) Sink node iii) Forward path iv) Self loop. **(04 Marks)**

b. What is Block diagram? List the properties of Block diagram. **(08 Marks)**

c. Obtain the transfer function for the block diagram, shown in Fig Q3(c), using block diagram reduction technique.

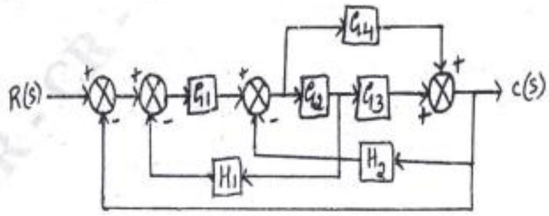


Fig Q3(c) **(08 Marks)**

OR

4 a. Explain the procedure of Block diagram reduction technique. (06 Marks)

b. Construct the signal flow graph and determine the transfer function using Mason's gain formula for Fig Q4(b).

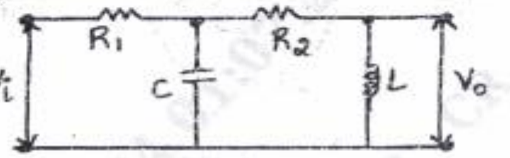


Fig Q4(b) (07 Marks)

c. Find $\frac{C(s)}{R(s)}$ for the signal flow graph shown in Fig Q4(c), using Mason's gain formula.

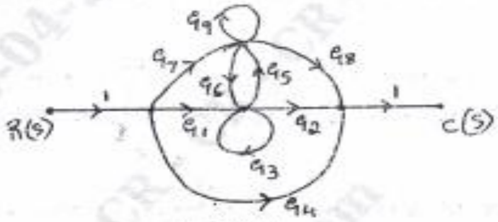


Fig Q4(c) (07 Marks)

Module-3

5 a. Define and derive the expression for i) Rise time ii) Peak overshoot of an underdamped second order control system subjected to step input. (07 Marks)

b. Determine the stability of the following characteristic equations of the system

$s^4 + 6s^3 + 26s^2 + 56s + 80 = 0$

$s^4 + 2s^3 + 4s^2 + 6s + 8 = 0$ (06 Marks)

c. A second order system is given $\frac{C(s)}{R(s)} = \frac{25}{s^2 + 6s + 2s}$. Find the rise time, peak time, setting time and peak overshoot if subjected to unit step input. Also obtain the expression for its output response. (07 Marks)

OR

6 a. Explain Routh-Huruity criterion for determining the stability of the system and mention its limitations. (06 Marks)

b. The open loop transfer function of a unity feedback control system is given by the characteristic equation. Determine the range of values of K for the system stability. What is the value of K which given sustained oscillations? What is the oscillation frequency?

$G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+2s)}$ (07 Marks)

c. A unity feedback system is characterized by an open loop transfer function $G(s) = \frac{K}{s(s+10)}$.

Determine the gain 'K', so that system will have a damping ratio of 0.5. For the value of K determine the settling time, peak overshoot, peak time for a unit step input. (07 Marks)

Module-4

7 a. Define the following terms : i) Angle of asymptotes ii) Asymptote iii) Breakaway points **(06 Marks)**

b. Draw the appropriate root locus diagram for a closed loop system whose loop transfer function is given by $\frac{G(s)}{H(s)} = \frac{K}{s(s+1)(s+2)}$. Comment on the stability. **(14 Marks)**

OR

8 a. Define the following terms :-
i) Gain margin ii) Phase margin iii) Gain crossover frequency **(06 Marks)**

b. A unity feedback control system has $G(s) = \frac{80}{s(s+2)(s+20)}$ **(14 Marks)**

Module-5

9 a. The open loop transfer function of a control system is $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$.
Sketch the Nyquist plot and calculate the value of K. **(14 Marks)**

b. Write short notes on PID controller. **(06 Marks)**

OR

10 a. What is controller? Explain the effect of P, I, PI and PID controller of a second order system. **(12 Marks)**

b. Explain the steps to solve problems by Nyquist criterion. **(08 Marks)**

Solutions

1A)

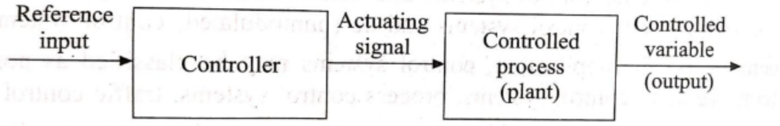
A good control system has many properties, including:

- Strategic control points: The system can deal with deviations based on their severity.
- Simplicity: The system should be easy to understand and implement, even if it doesn't use sophisticated policies.
- Focus on workers: The system should focus on workers, not just the work itself.
- Planning and control: Planning and control are closely linked, and neither can be effective without the other.
- Action: The system should suggest actions to correct deviations between standards and actual results.
- Delegation of authority: The system should grant subordinates the authority to operate within prescribed limits.
- Information flow: The system should ensure that managers receive information promptly.
- Flexibility: The system should be able to continue operating correctly even if there are signal errors or noisy sensors.
- Continuous process: The system should continuously assess progress to ensure that it matches predetermined plans.

1B)

Classification of control systems

1. Open-Loop Control Systems:



Any physical system which does not automatically correct the variation in its output.

- It is not a feedback system
- It operates on a time basis

Example: Washing machine, Electric Toaster, Traffic control.

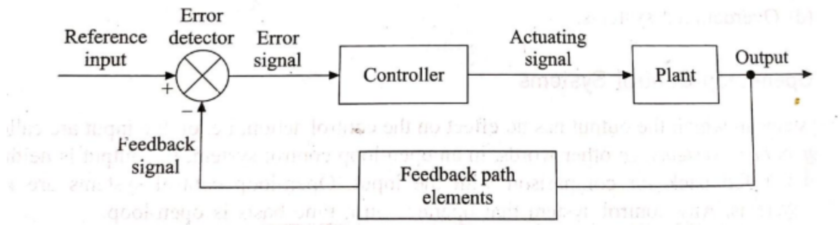
Advantages of Open-loop systems:

- Simple and economical
- Easier to construct
- Generally stable

Disadvantages of open-loop systems:

- Inaccurate and Unreliable
- The changes in the output due to external disturbances are not corrected automatically.

2. Closed-Loop Control Systems:



- Feedback control system.

- A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control.

Example: Traffic control, Room heating system.

Advantages of Closed-loop systems:

- Accurate
- Sensitivity of the systems may be made small to make the system more stable.
- Less affected by noise
- Accurate even in the presence of non-linearities.

Disadvantages of Closed-loop systems:

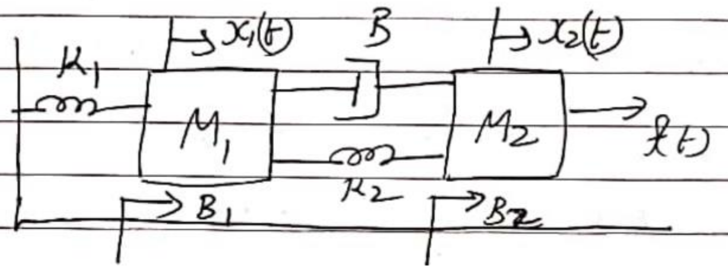
- Complex and costlier
- Feedback in closed loop system may lead to oscillatory response.
- Feedback reduces the overall gain of the system.
- Stability is a major problem in closed loop system and more care is needed to design a stable closed loop system.

Comparison

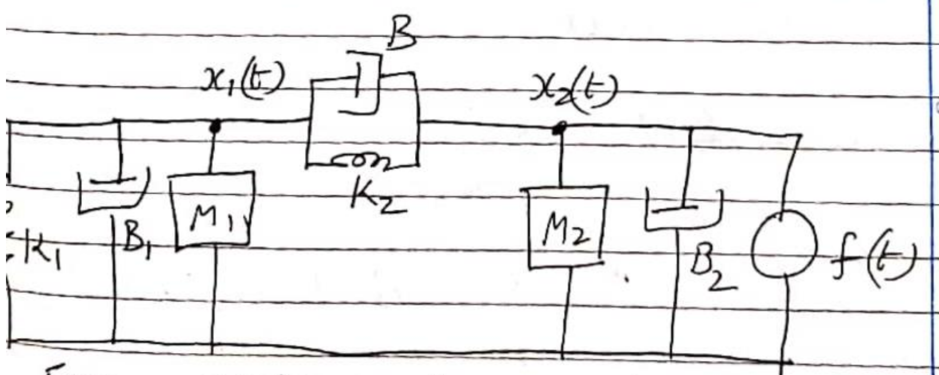
Open-Loop	Closed-Loop
Simple and economical	Complex and costlier
Consume less power	Consume more power
Easier to construct because of less number of components required	Not easy to construct because of more number of components required
Generally stable system	More care is needed to design a stable system
Inaccurate and unreliable	Accurate and more reliable
Changes in the output not corrected automatically	Changes in the output corrected automatically
	Feedback reduces the overall gain of the system.

1C)

Write differential equations [force-displacement equation and force-velocity equation] and find Transfer functions for the given system $\left[\frac{X_2(s)}{F(s)}\right]$.



Mechanical Network



Force-displacement equations.

$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x(t) + B \frac{d(x_1 - x_2)t}{dt} + K_2 [x_1(t) - x_2(t)] = 0 \quad \text{—} (1)$$

$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B \frac{d(x_2 - x_1)}{dt} + K_2 (x_2 - x_1) = f(t) \quad \text{—} (2)$$

Take Laplace Transform (1),

$$M_1 S^2 X_1(S) + B_1 S X_1(S) + K_1 X_1(S) + BS[X_1(S) - X_2(S)] + K_2 [X_1(S) - X_2(S)] = 0.$$

$$X_1(S)[M_1 S^2 + B_1 S + K_1 + BS + K_2] - X_2(S)[BS + K_2] = 0$$

$$X_1(S)[M_1 S^2 + S(B_1 + B) + (K_1 + K_2)] - X_2(S)[BS + K_2] = 0 \quad \text{—} (3)$$

Take Laplace Transform (2),

$$M_2 S^2 X_2(S) + B_2 S X_2(S) + BS[X_2(S) - X_1(S)] + K_2[X_2(S) - X_1(S)] = F(S)$$

$$X_2(S)[M_2 S^2 + B_2 S + BS + K_2] - X_1(S)[BS + K_2] = F(S)$$

$$-X_1(S)[BS + K_2] + X_2(S)[M_2 S^2 + (B_2 + B)S + K_2] = F(S) \quad \text{--- (4)}$$

Apply Kramer's rule in (3), (4),

$$X_2(S) = \frac{\begin{vmatrix} M_1 S^2 + S(B_1 + B) + (K_1 + K_2) & 0 \\ -(BS + K_2) & F(S) \end{vmatrix}}{\begin{vmatrix} M_1 S^2 + S(B_1 + B) + (K_1 + K_2) & -(BS + K_2) \\ -(BS + K_2) & M_2 S^2 + S(B_2 + B) + K_2 \end{vmatrix}}$$

$$X_2(S) = \frac{[M_1 S^2 + S(B_1 + B) + (K_1 + K_2)]\, F(S)}{[M_1 S^2 + S(B_1 + B) + (K_1 + K_2)][M_2 S^2 + S(B_2 + B) + K_2] - (BS + K_2)^2}$$

$$\therefore \frac{X_2(S)}{F(S)} = \frac{M_1 S^2 + S(B_1 + B) + (K_1 + K_2)}{[M_1 S^2 + S(B_1 + B) + (K_1 + K_2)][M_2 S^2 + S(B_2 + B) + K_2]}$$

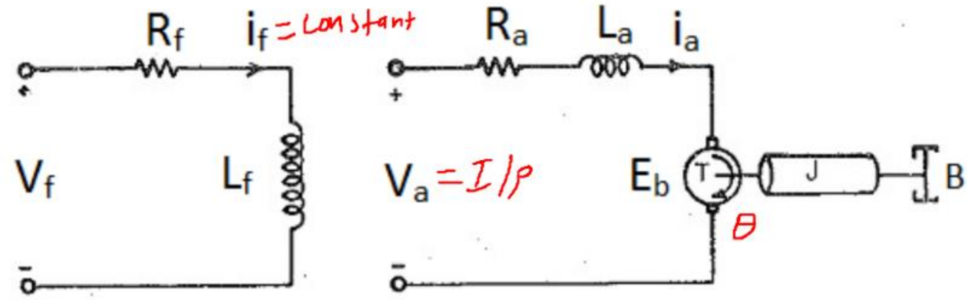
Force-Velocity equation

$$M_1 \frac{dv_1(t)}{dt} + B_1 v_1(t) + K_1 \int v_1(t)\,dt + B\left[v_1(t) - v_2(t)\right] + K_2 \int \left[v_1(t) - v_2(t)\right] dt = 0.$$

$$M_2 \frac{dv_2(t)}{dt} + B_2 v_2(t) + B\left[v_2(t) - v_1(t)\right] + K_2 \int \left[v_2(t) - v_1(t)\right] dt = f(t)$$

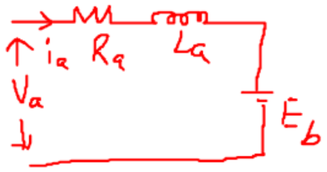
2A)

- Derive the transfer function of armature controlled dc servomotors.



$$T.F \quad \frac{\theta(s)}{V_a(s)}$$

Derivation



Let i_f = constant

$$R_a i_a + L_a \frac{di_a}{dt} + e_b = V_a$$

$$R_a i_a + L_a \frac{di_a}{dt} = V_a - e_b \quad \text{— (1)}$$

$$T \propto i_a \Rightarrow T = K_a i_a \quad \text{— (2)}$$

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T \quad \text{— (3)}$$

$$e_b \propto \frac{d\theta}{dt} \Rightarrow e_b = K_b \frac{d\theta}{dt} \quad \text{— (4)}$$

L.T (1), $R_a I_a(s) + L_a s I_a(s) = V_a(s) - E_b(s)$

$$I_a(s)\left[R_a + L_a s\right] = V_a(s) - E_b(s)$$

$$I_a(s) = \frac{V_a(s) - E_b(s)}{R_a + L_a s} \quad \text{— (5)}$$

L T (2), $T(s) = K_a I_a(s)$ — (6)

L.T (3), $\theta(s)\left[J s^2 + B s\right] = T(s)$ — (7)

L.T (4), $E_b(s) = K_b s\, \theta(s)$ — (8)

From (5), (6), (8)

$$T(s) = K_a \left[\frac{V_a(s) - K_b s\, \theta(s)}{R_a + L_a s}\right] \quad \text{— (9)}$$

(9) in (7),

$$\theta(S)\left[JS^2+BS\right] = \frac{K_a V_a(S) - K_a K_b S\,\theta(S)}{R_a + L_a S}$$

$$\theta(S)\left\{\left[JS^2+BS\right]\left[R_a+L_aS\right]\right\} = K_a V_a(S) - K_a K_b S\,\theta(S)$$

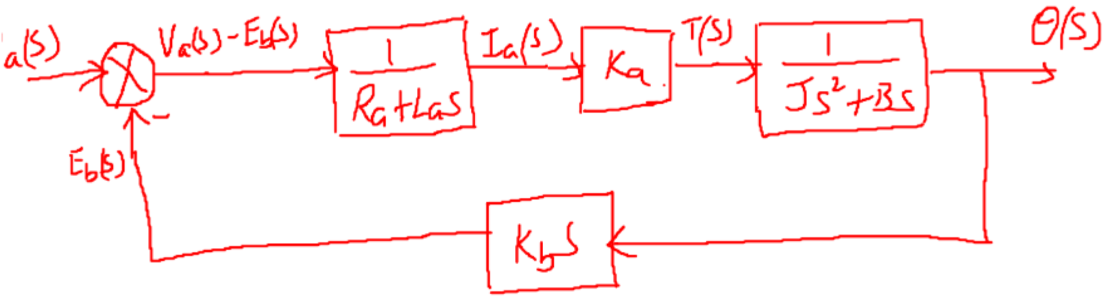
$$\theta(S)\left\{(JS^2+BS)(R_a+L_aS)\right\} + K_a K_b S\,\theta(S) = K_a V_a(S)$$

$$\theta(S)\left[(JS^2+BS)(R_a+L_aS) + K_a K_b S\right] = K_a V_a(S)$$

$$\therefore \frac{\theta(S)}{V_a(S)} = \frac{K_a}{(JS^2+BS)(R_a+L_aS) + K_a K_b S}$$

Block Diagram representation.

From (5), (6), (7), (8)



2C)

Modelling of Mechanical system elements

- Translational Mechanical system

- Rotational Mechanical system

Principle of Newton's Law of Motion

2nd Law is force = mass * acceleration

$$f(t) = m\,a(t)$$

Translational Mechanical System

- Variables are acceleration, velocity and displacement.

- Newton's states that algebraic sum of forces acting on a rigid body in a given direction is equal to the product of mass of body and its acceleration in the same direction.

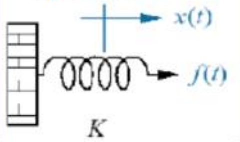
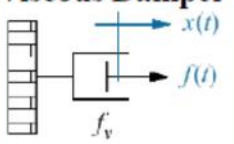
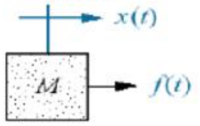
$$\sum Forces = Ma(t) = M\frac{dv(t)}{dt} = M\frac{d^2x(t)}{dt}$$

Translational Mechanical system

Three basic elements are

- Mass, M
- Damper, f or B or D
- Spring, K

Translational elements and correspondin[g] equations of motion

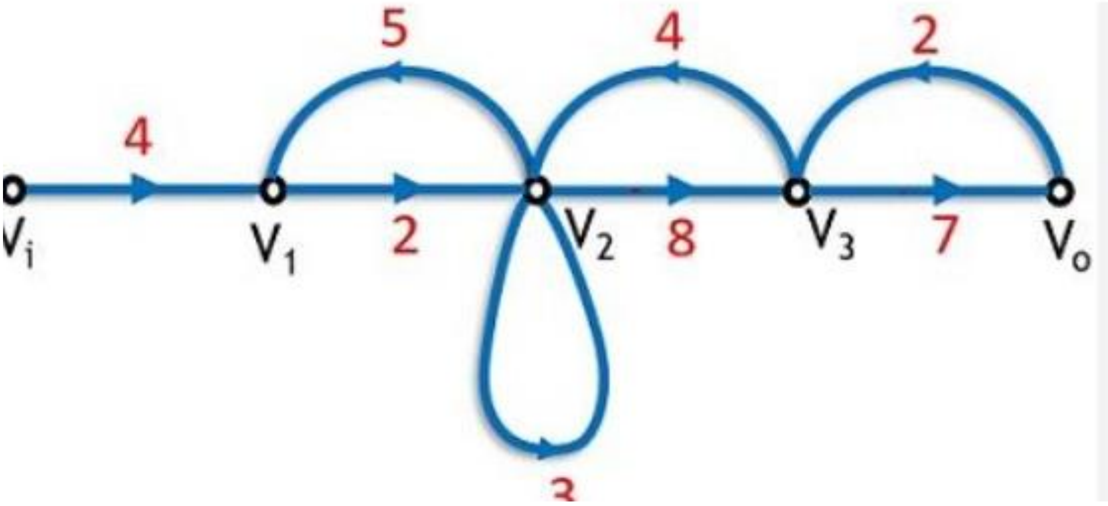
Component	Force-velocity	Force-displacement
Spring 	$f(t)=K\int_0^t v(\tau)d\tau$	$f(t)=Kx(t)$
Viscous Damper 	$f(t)=f_v v(t)$	$f(t)=f_v\frac{dx(t)}{dt}$
Mass 	$f(t)=M\frac{dv(t)}{dt}$	$f(t)=M\frac{d^2x(t)}{dt^2}$

3A)

In a control system, a source node, also known as an input node, is a node that has only outgoing branches and transmits data to a destination node when an event occurs

In a control system's signal flow graph (SFG), a sink node, also known as an output node, is a node that has only incoming branches and no outgoing edges. In contrast, a source node, also known as an input node, has only outgoing branches and no incoming edges.

In a control system, the forward path is the path that connects the input and output nodes, from the error signal to the output. The forward path transfer function is part of the overall transfer function of the control system. The sensitivity of the system to changes in the forward path transfer function can be reduced by increasing the gain of the forward path. A self-loop in a control system is a feedback loop that consists of a single branch and a single node, and the paths in these loops are not defined by any forward path or feedback loop.



3B0

- It is a short hand pictorial representation of the system which depicts
  - Each functional component or sub-system and
  - Flow of signals from one sub-system to another
- Block diagram provides a simple representation of complex systems
- Block diagram enables calculating the overall system transfer function provided the transfer functions of each of the components or sub-systems are known

Block diagrams focus on the input and output of a system, and less on what happens in between. This principle is known as "black box" in engineering, which means that the parts that get the system from input to output are either not known or not important.

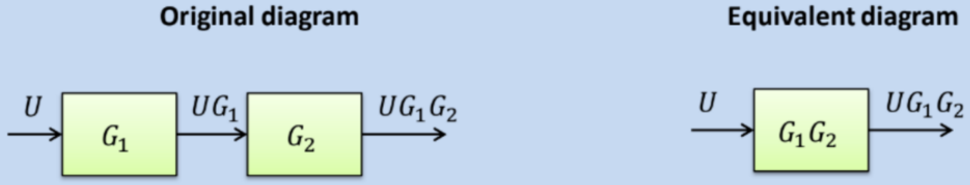
4a)

Block Diagram Reduction

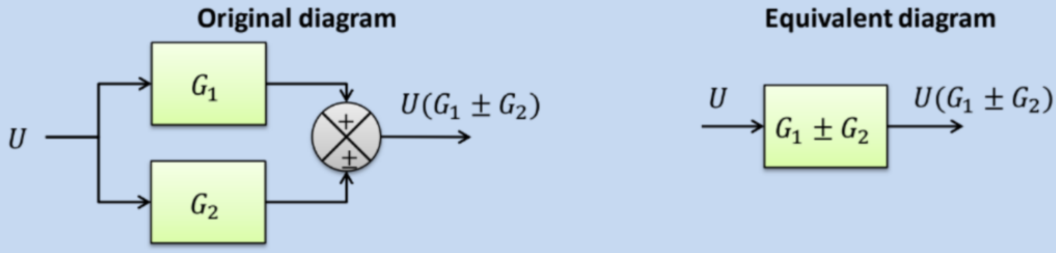
- Block diagram reduction refers to simplification of block diagrams of complex systems through certain rearrangements
- Simplification enables easy calculation of the overall transfer function of the system
- Simplification is done using certain rules called the ‘rules of block diagram algebra’
- All these rules are derived by simply algebraic manipulations of the equations representing the blocks

Rules of Block Diagram reduction

1. Combining blocks in cascade

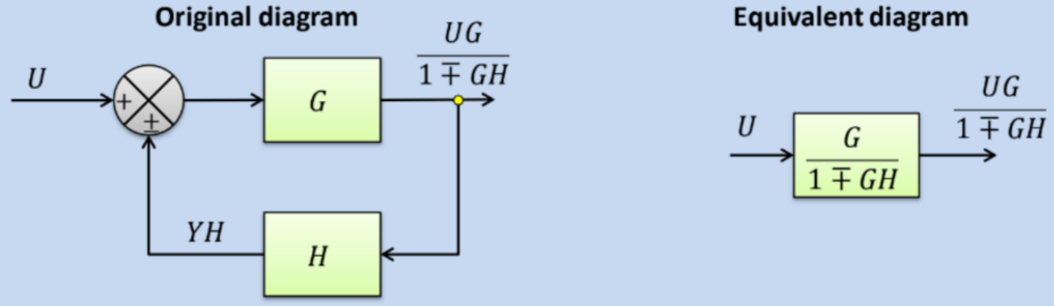


2. Combining blocks in parallel



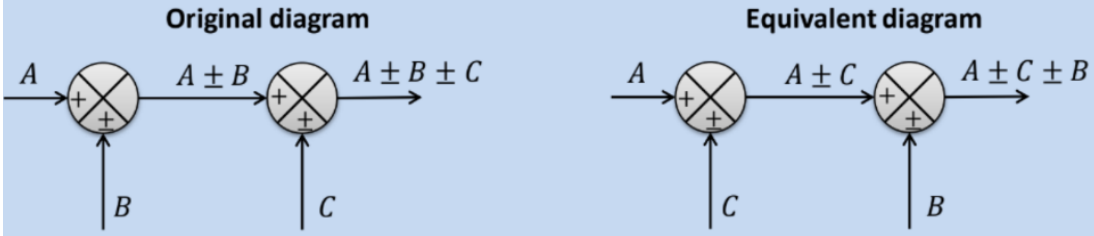
Rules of Block Diagram reduction

3. Eliminating a feedback loop



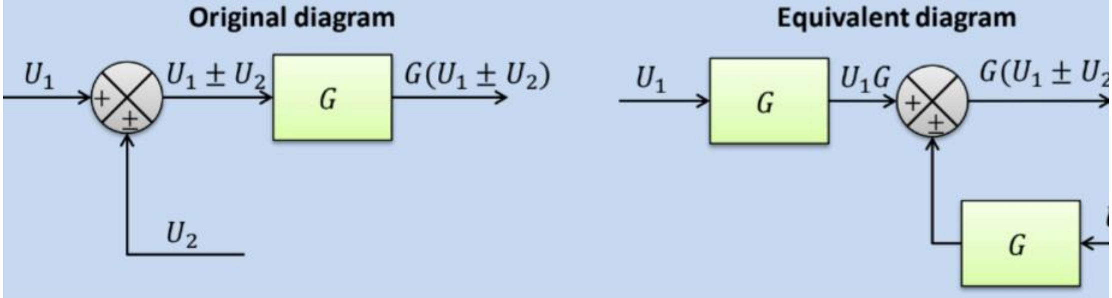
Rules of Block Diagram reduction

4. Interchanging the summing point

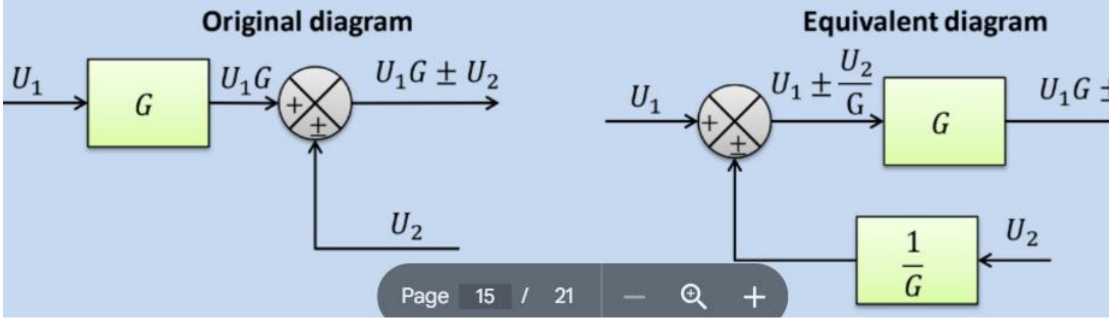


Rules of Block Diagram reduction

5. Moving a summing point after a block

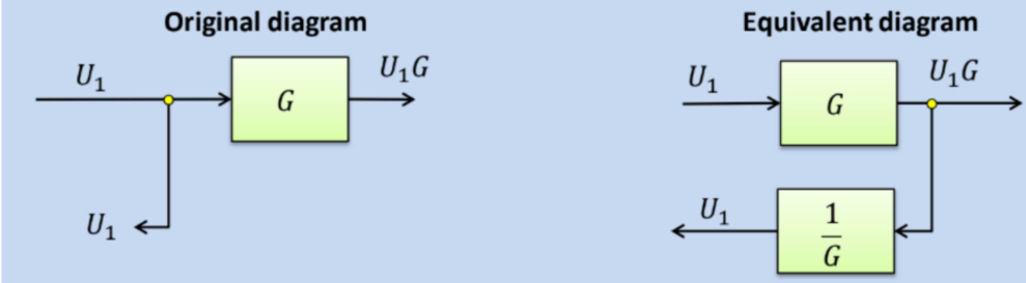


6. Moving a summing point ahead of a block

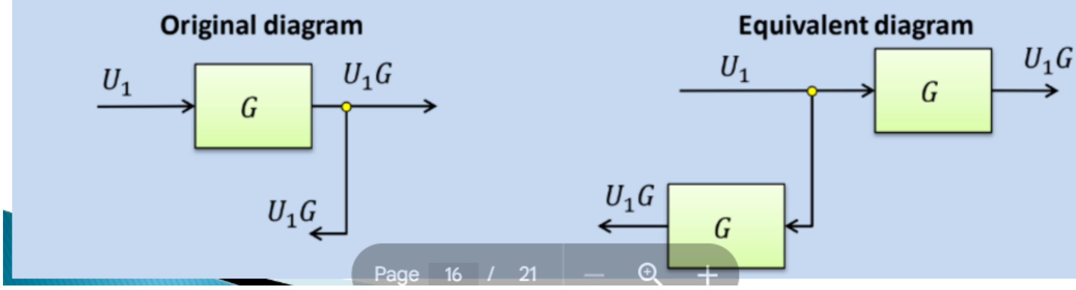


Rules of Block Diagram reduction

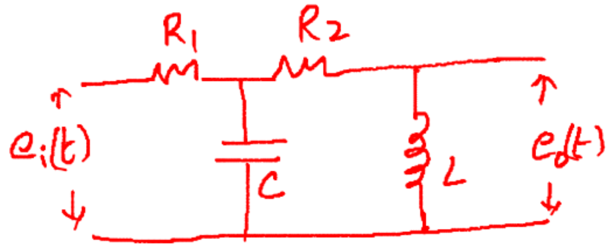
7. Moving a take-off point after a block



8. Moving a take-off point ahead of a block



Draw signal flow graph for the given electrical network and find its transfer function.



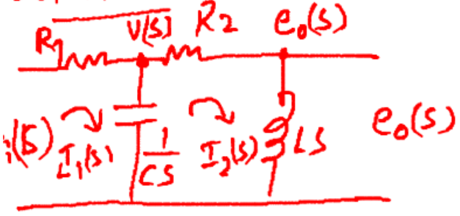
$\frac{L(O/P)}{L(I/P)}$

$\frac{E_o(s)}{E_i(s)} = ?$

$\int \frac{1}{c} = \frac{1}{cs}$

$L \frac{di}{dt} = Ls$

Solution:



$e_o(s) = I_2(s) LS$ — ①

$I_2(s) = \frac{V(s) - e_o(s)}{R_2}$

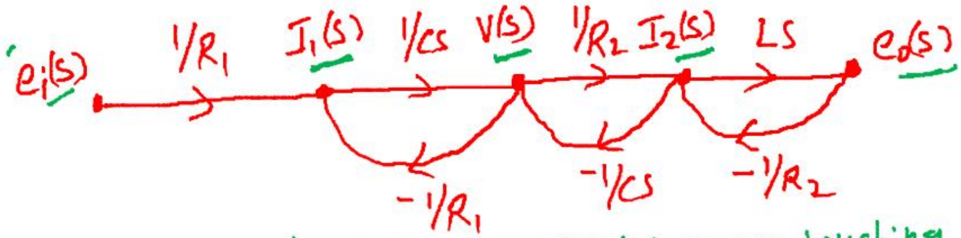
$= \frac{V(s)}{R_2} - \frac{e_o(s)}{R_2}$ — ②

$V(s) = I_1(s) \frac{1}{CS} - I_2(s) \frac{1}{CS}$ — ③

$I_1(s) = \frac{e_i(s) - V(s)}{R_1} = \frac{e_i(s)}{R_1} - \frac{V(s)}{R_1}$ — ④

$R \xrightarrow{G} C = RG$

Draw SFG for ①, ②, ③, ④.



$M_1 = \frac{1}{R_1} \times \frac{1}{CS} \times \frac{1}{R_2} \times LS = \frac{L}{R_1 R_2 C}$

$\Delta_1 = 1$

Individual Loops

$L_{11} = -\frac{1}{R_1 CS}$

$L_{12} = -\frac{1}{R_2 CS}$

$L_{13} = -\frac{LS}{R_2}$

Two non-touching loops.

$L_{21} = \frac{LS}{R_1 R_2 C S} = \frac{L}{R_1 R_2 C}$

$\Delta = 1 - [L_{11} + L_{12} + L_{13}] + [L_{21}]$

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5A)

Response of undamped second order system for unit step Input.

(output response, $c(t) = ?$)

Solution

undamped $\Rightarrow \delta = 0$.

unit step Input $\Rightarrow R(s) = \frac{1}{s}$.

second order system $\Rightarrow \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$.

Let $\delta = 0$, $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$

$$c(s) = R(s)\,\frac{\omega_n^2}{s^2 + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + \omega_n^2)}$$

By PFE,

$$C(s) = \frac{A}{s} + \frac{Bs + C}{s^2 + \omega_n^2}$$

$$\frac{\omega_n^2}{s(s^2 + \omega_n^2)} = \frac{A}{s} + \frac{Bs + C}{s^2 + \omega_n^2}$$

$$\omega_n^2 = \frac{A\cancel{s}(s^2 + \omega_n^2)}{\cancel{s}} + \frac{(Bs + C)\,s\,\cancel{(s^2 + \omega_n^2)}}{\cancel{s^2 + \omega_n^2}}$$

$$\omega_n^2 = As^2 + A\omega_n^2 + Bs^2 + Cs.$$

$s^2 \Rightarrow 0 = A + B$

$s \Rightarrow 0 = C$

$s^0 \Rightarrow \omega_n^2 = A\omega_n^2$

$A = 1$,

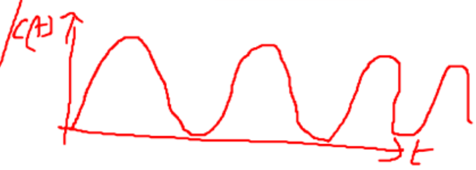
$$\frac{A}{s} + \frac{Bs}{s^2 + \omega_n^2}$$

$B = -1$, $C = 0$.

$$C(s) = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}$$

$$c(t) = L^{-1}[C(s)]$$

$$c(t) = 1 - \cos\omega_n t$$



$$L[\sin at] = \frac{a}{s^2 + a^2}$$

$$L(\cos at) = \frac{s}{s^2 + a^2}$$

5C)

i) $\frac{C(s)}{R(s)} = \frac{8}{s^2+3s+8}$

standard form of second order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$$

By comparing,

$$s^2+2\delta\omega_n s+\omega_n^2 = s^2+3s+8.$$

$\omega_n^2 = 8.$ | $2\delta\omega_n = 3$

$\omega_n = \sqrt{8} = 2.83$ | $\delta = \frac{3}{2\times\omega_n} = 0.53$

$\because \delta < 1$, It is underdamped system.

6A)

Routh Stability Criteria

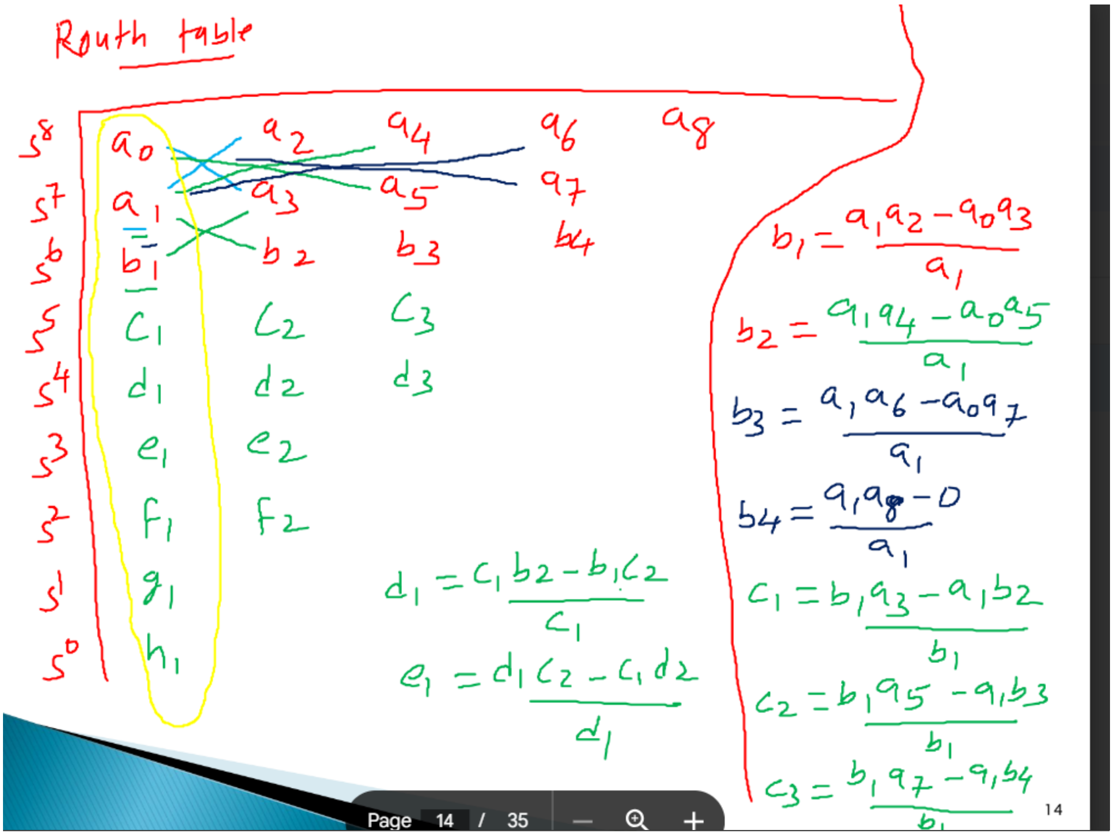
(or)

Routh Hurwitz criteria.

Let $q(s) = a_0 s^8 + a_1 s^7 + a_2 s^6 + a_3 s^5 + a_4 s^4 + a_5 s^3 + a_6 s^2 + a_7 s + a_8 = 0$

Routh table

s^8	———— 1st, 3rd, 5th . . . .
s^7	———— 2nd, 4th, 6th . . . .
s^6	
s^5	
s^4	
s^3	
s	



See the first column of Routh table

1) If no sign change → Stable system.

2) Any sign change → unstable system

3) If any row is fully zero → Marginally system

6B)

(eg) Determine the range of K for stability of unity feedback system whose open loop Transfer function is $G(s) = \frac{K}{s(s+1)(s+2)}$

Solution closed loop T.F $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$

characteristic equation is,

$$1+G(s)H(s) = 1 + \frac{K}{s(s+1)(s+2)} = 0.$$

$$s(s+1)(s+2) + K = 0.$$

$$s^3 + 3s^2 + 2s + K = 0.$$

Routh table,

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	
s^0	K	

For stable system,

s^0 row $\Rightarrow K > 0$.

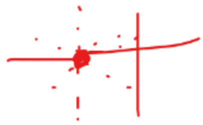
s^1 row $\Rightarrow \frac{6-K}{3} > 0 \Rightarrow K < 6$

∴ Range of K for stable system is

$$0 < K < 6.$$

Step 4: Behaviour at infinity

- When the number of finite poles, n is greater than the number of finite zeros, m of $G(s)H(s)$, $(n - m)$ branches of the root loci will approach infinity.
- The properties of the root loci near infinity are described by the
  - Angle of asymptotes and
  - Intersect of the asymptotes with real axis.



Step 4: Behaviour at infinity (cont.)

<u>Angles of Asymptotes</u>

$(s) = \frac{(s+1)}{s(s+3)}$ ⇒ 1, ⇒ 2.

$G(s) = \frac{(s+1)(s+2)}{s(s+3)}$ ⇒ 2, ⇒ 2.

$$\phi_a = \frac{\pm(2q + 1) \times 180^\circ}{n - m}, n \neq m$$

where $q = 0,1,2, \ldots, n - m - 1$, n and m are numbers of finite poles and zeros

where n = number of finite poles.

m = number of finite zeros.

Step 4: Behaviour at infinity (cont.)

Intersect of the asymptotes with real axis

- The intersect of the asymptotes lies on the real axis at:

$$\sigma_a = \frac{\text{Sum } Finite\ poles - \text{Sum of } Finite\ zeros}{n - m}$$

NOTE:

1. *Finite poles* ***and*** *Finite zeros* ***means real part***

2. This step only applies if you have infinite poles and/or zeros.

Step 4: Behaviour at infinity (cont.)

Example:

$$KG(s)H(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

Zeros ⇒ $S = -3$ } 1

Poles ⇒ $S = 0, -1, -2, -4$

4

Numbers of finite poles, $n = 4$

Numbers of finite zeros, $m = 1$

$q = 0, 1, \ldots, n - m - 1 = 0, 1, 2$

$n - m - 1 = 4 - 1 - 1 = 2.$

The angles of asymptotes are:

$$\phi_a = \frac{\pm(2q+1)\times 180^\circ}{n-m}, n \neq m, q = 0, 1, \ldots, n - m - 1$$

$$= \frac{\pm(2q+1)\times 180^\circ}{4-1}$$

$$= 180^\circ, \pm 60^\circ$$

$q = 0$, $\phi = \frac{180}{4-1} = 60^\circ$.

$q = 1$, $\phi = \frac{3\times 180}{3} = 180^\circ$.

$q = 2$, $\phi = \frac{5\times 180}{3} = 300^\circ = -60^\circ$

Step 4: Behaviour at infinity (cont.)

Example: $$KG(s)H(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

The intersect of the asymptotes on the real axis are at:

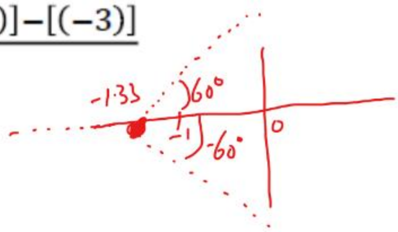
zeros ⇒ -3

Poles ⇒ 0, -1, -2, -4

$$\sigma_a = \frac{\sum Finite\ poles - \sum Finite\ zeros}{n-m}$$

$$= \frac{[0+(-1)+(-2)+(-4)]-[(-3)]}{4-1}$$

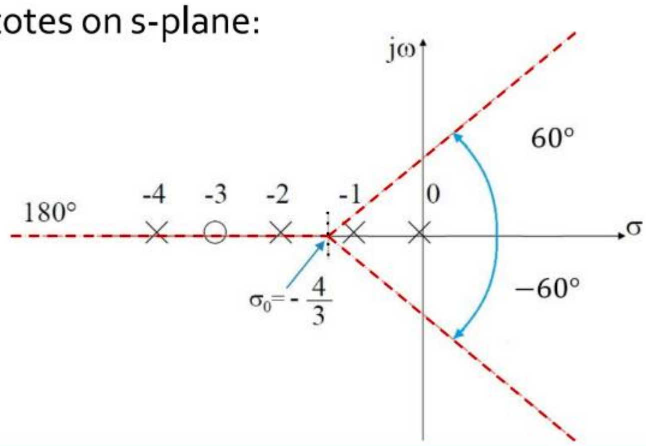
$$= -\frac{4}{3} = -1.33$$



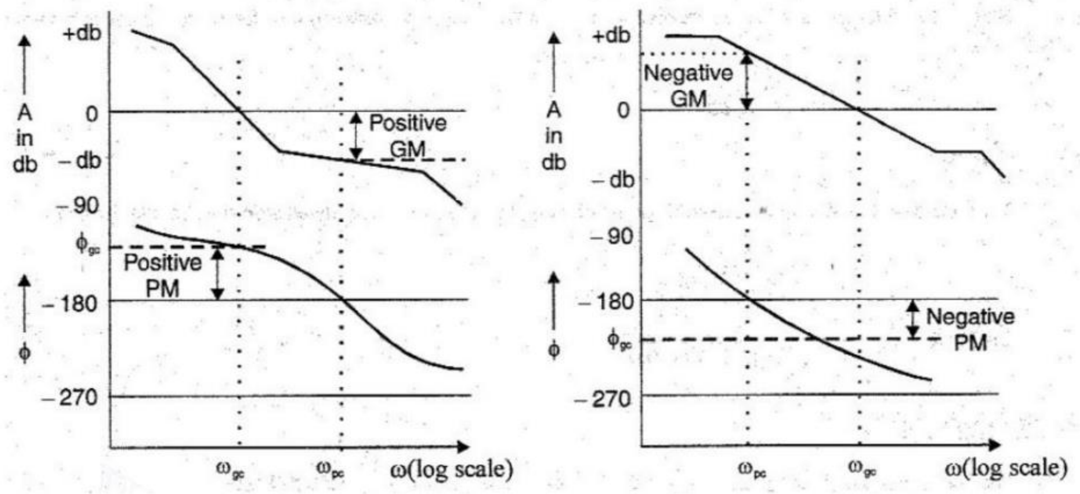
Step 4: Behaviour at infinity (cont.)

Example: $$KG(s)H(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+4)}$$

Asymptotes on s-plane:



Determination of Gain Margin and Phase Margin



phase margin, $\gamma = 180° + \phi_{gc}$

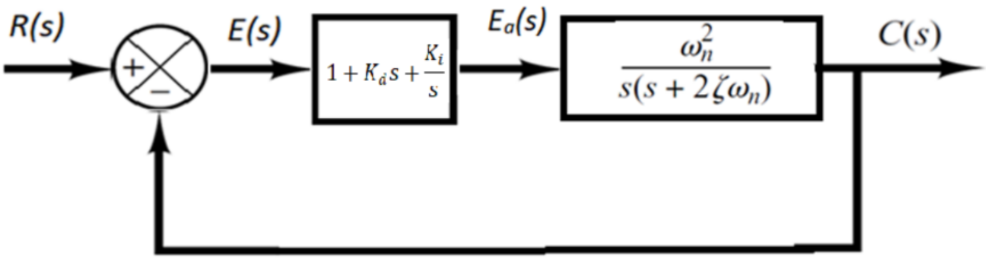
9A

Proportional plus Integral plus Derivative control (PID control)

The actuating signal consists of proportional error signal Added with integral and derivative of error control

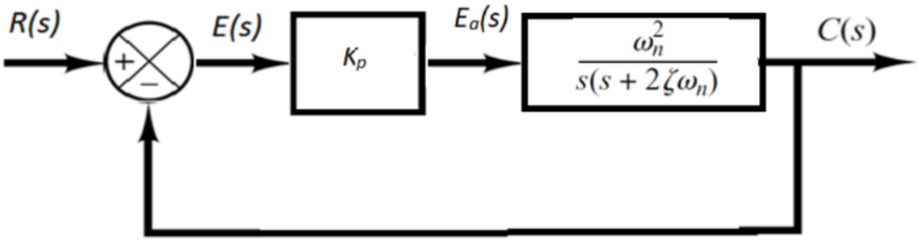
$$e_a(t) = e(t) + K_d \frac{de(t)}{dt} + K_i \int e(t)dt$$

$$E_a(s) = E(s) + K_d s E(s) + K_i \frac{E(s)}{s} \qquad E_a(s) = \left[1 + K_d s + \frac{K_i}{s}\right] E(s)$$



10A

PROPORTIONAL CONTROLLER



The actuating signal is given by

$$e_a(t) = K_p\, e(t)$$

Laplace transform of the above equation gives

$$E_a(s) = K_p E(s)$$

$$E(s) = R(s) - C(s)$$

$$E(s) = R(s) - E(s)\frac{K_p\omega_n^2}{s(s + 2\xi\omega_n)}$$

$$E(s)\left[1 + \frac{K_p\omega_n^2}{s(s + 2\xi\omega_n)}\right] = R(s)$$

$$E(s)\left[\frac{s^2 + 2\xi\omega_n s + K_p\omega_n^2}{s(s + 2\xi\omega_n)}\right] = R(s)$$

$$\frac{E(s)}{R(s)} = \frac{s(s + 2\xi\omega_n)}{s^2 + 2\xi\omega_n s + K_p\omega_n^2}$$

For ramp input

$$E(s) = \frac{1}{s^2}\frac{s(s + 2\xi\omega_n)}{s^2 + 2\xi\omega_n s + K_p\omega_n^2}$$

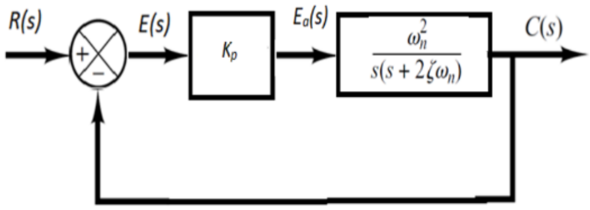
Steady state error

$$e_{ss} = \lim_{s\to 0} sE(s) = \frac{2\xi}{K_p\omega_n}$$

Note: for a system with out controller, Steady state error for ramp input is $\frac{2\xi}{\omega_n}$

As K_p increases steady state error decreases

The closed loop transfer function is given by



$$\frac{C(s)}{R(s)} = \frac{K_p\omega_n^2}{s^2 + 2\xi\omega_n s + K_p\omega_n^2}$$

The characteristic equation is

$$s^2 + 2\xi\omega_n s + K_p\omega_n^2 = 0$$

The roots of the characteristic equation are

$$-\xi\omega_n \pm \omega_n\sqrt{K_p - \xi^2}$$

$$\omega_d = \omega_n \sqrt{K_p - \xi^2}$$

As K_p increases ω_d increases,

Hence

the output will oscillate with higher frequencies
the rise time decreases,
maximum overshoot increases

The proportional controller reduces steady state error, makes the system faster but system response is oscillatory

Proportional plus Derivative controller (PD controller)

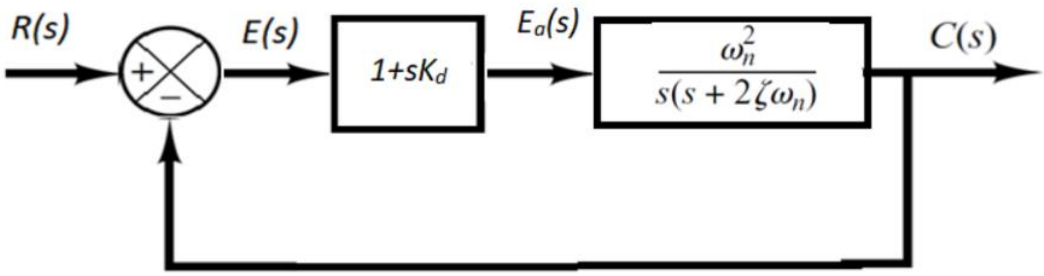
The actuating signal consists of proportional error signal and derivative of the error signal.

$$e_a(t) = e(t) + K_d \frac{de(t)}{dt}$$

Laplace transform of the above equation gives

$$E_a(s) = E(s) + sK_d E(s)$$

$$E_a(s) = [1 + sK_d]E(s)$$



$$\frac{C(s)}{R(s)} = \frac{(1+sK_d)\omega_n^2/s(s+2\xi\omega_n)}{1+(1+sK_d)\omega_n^2/s(s+2\xi\omega_n)}$$

$$\frac{C(s)}{R(s)} = \frac{(1+sK_d)\omega_n^2}{s^2+(2\xi\omega_n+K_d\omega_n^2)s+\omega_n^2}$$

The characteristic equation given by.

$$s^2+(2\xi\omega_n+K_d\omega_n^2)s+\omega_n^2 = 0$$

Comparing characteristic equation with standard equation

$$s^2+(2\xi'\omega_n)s+\omega_n^2 = 0$$

$$2\xi'\omega_n = 2\xi\omega_n + K_d\omega_n^2$$

Therefore the effective damping ratio

$$\xi' = \frac{2\xi\omega_n + K_d\omega_n^2}{2\omega_n} \qquad \xi' = \xi + \frac{K_d\,\omega_n}{2}$$

The damping ratio increases, the maximum overshoot reduces

The error function is given by

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+\frac{\omega_n^2(1+sK_d)}{s(s+2\xi\omega_n)}}$$

$$\frac{E(s)}{R(s)} = \frac{s(s+2\xi\omega_n)}{s^2+(2\xi\omega_n+K_d\omega_n^2)s+\omega_n^2}$$

10A

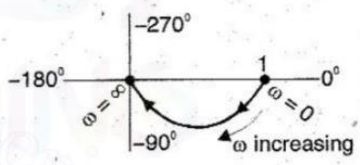
Typical sketches of Polar Plot (Nyquist plot)

Type : 0, Order : 1 $\qquad G(s) = \dfrac{1}{1+sT}$

$$G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2 T^2}\ \angle \tan^{-1}\omega T} = \frac{1}{\sqrt{1+\omega^2 T^2}}\ \angle -\tan^{-1}\omega T$$

As $\omega \to 0$, $\quad G(j\omega) \to 1\angle 0^\circ$

As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -90^\circ$

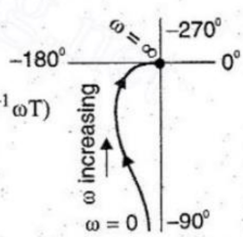


Type : 1, Order : 2 $\qquad G(s) = \dfrac{1}{s(1+sT)}$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)} = \frac{1}{\omega\ \angle 90^\circ\ \sqrt{1+\omega^2 T^2}\ \angle \tan^{-1}\omega T} = \frac{1}{\omega\sqrt{1+\omega^2 T^2}}\ \angle(-90^\circ - \tan^{-1}\omega T)$$

As $\omega \to 0$, $\quad G(j\omega) \to \infty\angle -90^\circ$

As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -180^\circ$



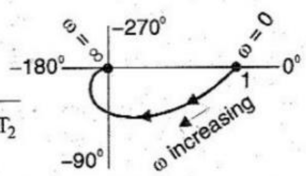
Type : 0, Order : 2 $\qquad G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)}$

$$G(j\omega) = \frac{1}{(1+j\omega T_1)\ (1+j\omega T_2)} = \frac{1}{\sqrt{1+\omega^2 T_1^2}\ \angle \tan^{-1}\omega T_1\ \sqrt{1+\omega^2 T_2^2}\ \angle \tan^{-1}\omega T_2}$$

$$= \frac{1}{\sqrt{\left(1+\omega^2 T_1^2\right)\left(1+\omega^2 T_2^2\right)}}\ \angle(-\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2)$$

As $\omega \to 0$, $\quad G(j\omega) \to 1\angle 0^\circ$

As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -180^\circ$



Type : 0, Order : 3 $\qquad G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$

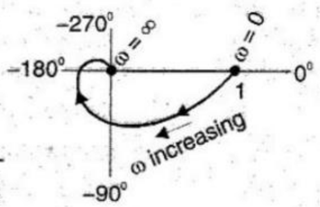
$$G(j\omega) = \frac{1}{(1+j\omega T_1)\ (1+j\omega T_2)\ (1+j\omega T_3)}$$

$$= \frac{1}{\sqrt{1+\omega^2 T_1^2}\ \angle \tan^{-1}\omega T_1\ \sqrt{1+\omega^2 T_2^2}\ \angle \tan^{-1}\omega T_2\ \sqrt{1+\omega^2 T_3^2}\ \angle \tan^{-1}\omega T_3}$$

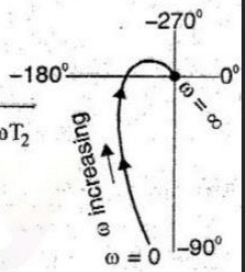
$$= \frac{1}{\sqrt{\left(1+\omega^2 T_1^2\right)\left(1+\omega^2 T_2^2\right)\left(1+\omega^2 T_3^2\right)}}\ \angle(-\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3)$$

As $\omega \to 0$, $\quad G(j\omega) \to 1\angle 0^\circ$

As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -270^\circ$



Type : 1, Order : 3 $G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$



$$G(j\omega) = \frac{1}{j\omega(1+j\omega T_1)\ (1+j\omega T_2)} = \frac{1}{\omega\angle 90^\circ \sqrt{1+\omega^2T_1^2}\angle \tan^{-1}\omega T_1 \sqrt{1+\omega^2T_2^2}\ \angle\tan^{-1}\omega T_2}$$

$$= \frac{1}{\omega\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)}}\angle(-90^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2)$$

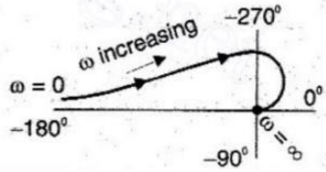
As $\omega \to 0$, $\quad G(j\omega) \to \infty\angle -90^\circ$

As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -270^\circ$

Type : 2, Order : 4 $G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)}$

$$G(j\omega) = \frac{1}{(j\omega)^2\ (1+j\omega T_1)\ (1+j\omega T_2)} = \frac{1}{\omega^2\angle -180^\circ \sqrt{1+\omega^2T_1^2}\angle \tan^{-1}\omega T_1 \sqrt{1+\omega^2T_2^2}\angle\tan^{-1}\omega T_2}$$

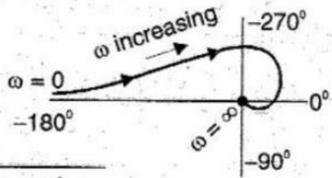
$$= \frac{1}{\omega^2\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)}}\angle(-180^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2)$$



As $\omega \to 0$, $\quad G(j\omega) \to \infty\angle -180^\circ$

As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -360^\circ$

Type : 2, Order : 5 $G(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$



$$G(j\omega) = \frac{1}{(j\omega)^2\left(1+j\omega T_1\right)\left(1+j\omega T_2\right)\left(1+j\omega T_3\right)}$$

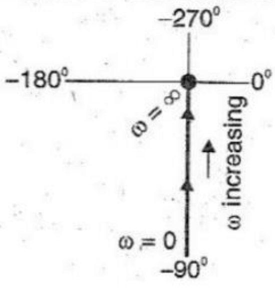
$$= \frac{1}{\omega^2\angle -180^\circ \sqrt{1+\omega^2T_1^2}\angle \tan^{-1}\omega T_1 \sqrt{1+\omega^2T_2^2}\angle\tan^{-1}\omega T_2 \sqrt{1+\omega^2T_3^2}\angle\tan^{-1}\omega T_3}$$

$$= \frac{1}{\omega^2\sqrt{\left(1+\omega^2T_1^2\right)\left(1+\omega^2T_2^2\right)\left(1+\omega^2T_3^2\right)}}\angle(-180^\circ - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 - \tan^{-1}\omega T_3)$$

As $\omega \to 0$, $\quad G(j\omega) \to \infty\angle -180^\circ$

As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -450^\circ = 0\angle -90^\circ$

Type : 1, Order : 1 $G(s) = \frac{1}{s}$



$$G(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega\angle 90^\circ} = \frac{1}{\omega}\angle -90^\circ$$

As $\omega \to 0$, $\quad G(j\omega) \to \infty\angle -90^\circ$

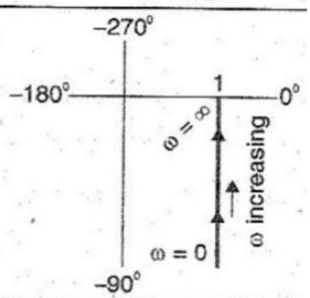
As $\omega \to \infty$, $\quad G(j\omega) \to 0\angle -90^\circ$

$$G(s) = \frac{1+sT}{sT}$$

$$G(j\omega) = \frac{1+j\omega T}{j\omega T} = \frac{1}{j\omega T} + 1 = \frac{1}{\omega T \angle 90^\circ} + 1 = \frac{1}{\omega T}\angle -90^\circ + 1$$

As $\omega \to 0$, $G(j\omega) \to \infty\angle -90^\circ + 1$

As $\omega \to \infty$, $G(j\omega) \to 0\angle -90^\circ + 1$

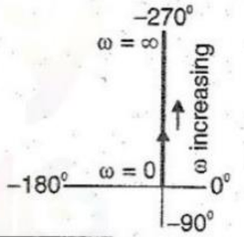


$G(s) = s$

$G(j\omega) = j\omega = \omega\angle 90^\circ$

As $\omega \to 0$, $G(j\omega) \to 0\angle 90^\circ$

As $\omega \to \infty$, $G(j\omega) \to \infty\angle 90^\circ$

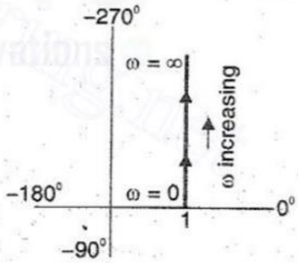


$G(s) = 1 + sT$

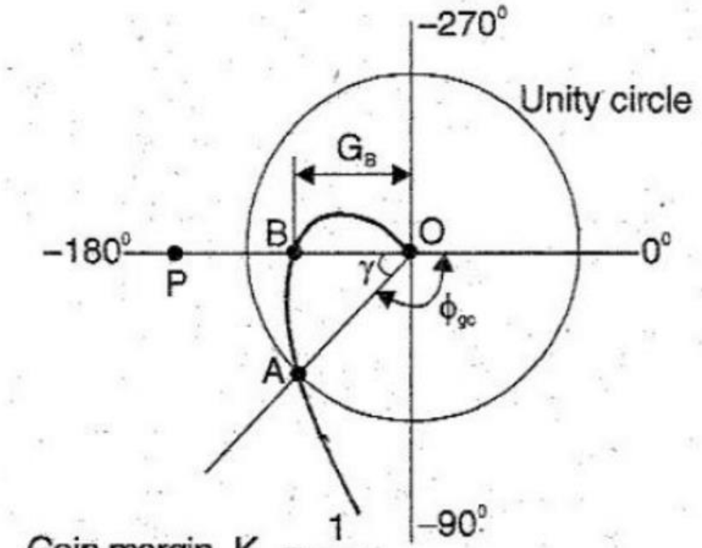
$G(j\omega) = 1 + j\omega T = 1 + \omega T\angle 90^\circ$

As $\omega \to 0$, $G(j\omega) \to 1 + 0\angle 90^\circ$

As $\omega \to \infty$, $G(j\omega) \to 1 + \infty\angle 90^\circ$



Determination of Gain Margin and Phase Margin



Gain margin, $K_g = \frac{1}{G_B}$

Phase margin, $\gamma = 180^\circ + \phi_{gc}$