CBCS SCHEME

21ME43

USN

Fourth Semester B.E. Degree Examination, June/July 2023

Fluid Mechanics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Explain the following terms:
(i) Total pressure (ii) Centre of pressure
(iii) Gauge pressure (iv) Buoyancy **(08 Marks)**

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

Centre of pressure is defined as the point of application of the total pressure on the surface. There are four cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined. The submerged surfaces may be :

1. Vertical plane surface,
2. Horizontal plane surface,
3. Inclined plane surface, and
4. Curved surface.

▶ 4.2 BUOYANCY

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

b. Derive expression for total pressure force and centre of pressure act on a vertical surface immersed in static fluid. **(08 Marks)**

▶ 3.3 VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in Fig. 3.1.

Let A = Total area of the surface

$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

(*a*) **Total Pressure (F)**. The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force on small strip.

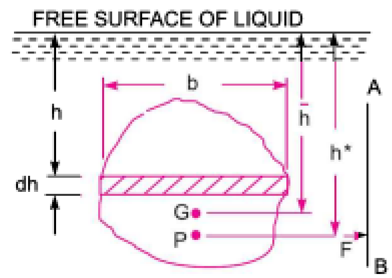

Fig. 3.1

Consider a strip of thickness dh and width b at a depth of h from free surface of liquid as shown in Fig. 3.1

Pressure intensity on the strip, $p = \rho g h$

(See equation 2.5)

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = p \times \text{Area}$

$= \rho g h \times b \times dh$

$\therefore$ Total pressure force on the whole surface,

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$$\therefore \quad F = \rho g A \bar{h} \qquad \ldots(3.1)$$

For water the value of $\rho = 1000\ \text{kg/m}^3$ and $g = 9.81\ \text{m/s}^2$. The force will be in Newton.

(*b*) **Centre of Pressure (h*)**. Centre of pressure is calculated by using the "Principle of Moments", which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from free surface of the liquid as shown in Fig. 3.1. Hence moment of the force F about free surface of the liquid $= F \times h^*$ ...(3.2)

Moment of force dF, acting on a strip about free surface of liquid

$$= dF \times h \qquad \{\because \quad dF = \rho g h \times b \times dh\}$$

$$= \rho g h \times b \times dh \times h$$

Sum of moments of all such forces about free surface of liquid

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2\, dh = \rho g \int h^2\, dA \qquad (\because \quad b dh = dA)$$

But

$$\int h^2 dA = \int b h^2 dh$$

= Moment of Inertia of the surface about free surface of liquid

$$= I_0$$

$\therefore$ Sum of moments about free surface

$$= \rho g I_0 \qquad \text{...(3.3)}$$

Equating (3.2) and (3.3), we get

$$F \times h^* = \rho g I_0$$

But

$$F = \rho g A \bar{h}$$

$\therefore$

$$\rho g A \bar{h} \times h^* = \rho g I_0$$

or

$$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}} \qquad \text{...(3.4)}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \overline{h^2}$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3.4), we get

$$h^* = \frac{I_G + A\overline{h^2}}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h} \qquad \text{...(3.5)}$$

In equation (3.5), $\bar{h}$ is the distance of C.G. of the area of the vertical surface from free surface of the liquid. Hence from equation (3.5), it is clear that :

(*i*) Centre of pressure (*i.e.*, h^*) lies below the centre of gravity of the vertical surface.

(*ii*) The distance of centre of pressure from free surface of liquid is independent of the density of the liquid.

c. Discuss on fluid pressure measuring devices. **(04 Marks)**

▶ 2.5 MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers 2. Mechanical Gauges.

2.5.1 Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2.5.2 Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge, (*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and (*d*) Bellows pressure gauge.

▶ 2.6 SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,`
2. U-tube Manometer, and
3. Single Column Manometer.

2.6.1 Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point *A*, the height of liquid say water is *h* in piezometer tube, then pressure at *A*

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$

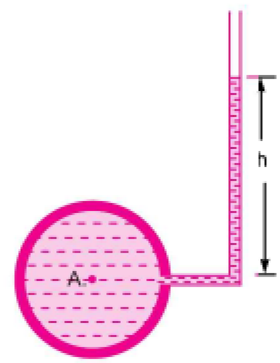

Fig. 2.8 *Piezometer.*

2.6.2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

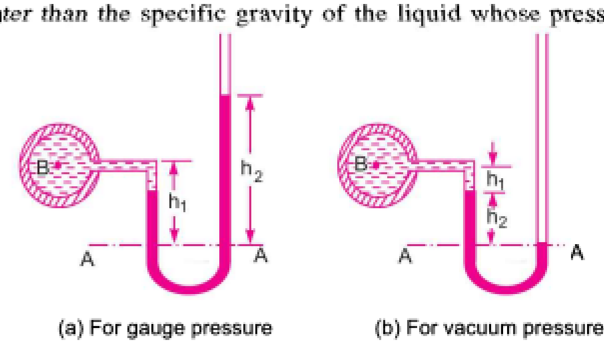

Fig. 2.9 *U-tube Manometer.*

2.6.3 Single Column Manometer. Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.15. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

▶ 2.7 DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

OR

2 a. Explain the Eulerian and Langragian method of fluid flow analysis with suitable example. **(08 Marks)**

▶ 5.2 METHODS OF DESCRIBING FLUID MOTION

The fluid motion is described by two methods. They are —(*i*) Lagrangian Method, and (*ii*) Eulerian Method. In the Lagrangian method, a **single fluid particle** is followed during its motion and its velocity, acceleration, density, etc., are described. In case of Eulerian method, the velocity, acceleration, pressure, density etc., are described **at a point** in flow field. The Eulerian method is commonly used in fluid mechanics.

b. Derive the 3-dimensional flow continuity equation in cartesian coordinates. **(08 Marks)**

▶ 5.6 CONTINUITY EQUATION IN THREE-DIMENSIONS

Consider a fluid element of lengths dx, dy and dz in the direction of x, y and z. Let u, v and w are the inlet velocity components in x, y and z directions respectively. Mass of fluid entering the face $ABCD$ per second

$$= \rho \times \text{Velocity in } x\text{-direction} \times \text{Area of } ABCD$$
$$= \rho \times u \times (dy \times dz)$$

Then mass of fluid leaving the face $EFGH$ per second $= \rho u\, dydz + \frac{\partial}{\partial x}(\rho u\, dydz)\, dx$

∴ Gain of mass in x-direction

$$= \text{Mass through } ABCD - \text{Mass through } EFGH \text{ per second}$$
$$= \rho u\, dydz - \rho u\, dydz - \frac{\partial}{\partial x}(\rho u\, dydz)dx$$
$$= -\frac{\partial}{\partial x}(\rho u\, dydz)\, dx$$
$$= -\frac{\partial}{\partial x}(\rho u)\, dx\, dydz \qquad \{\because dydz \text{ is constant}\}$$

Similarly, the net gain of mass in y-direction

$$= -\frac{\partial}{\partial y}(\rho v)\, dxdydz$$

and in z-direction $$= -\frac{\partial}{\partial z}(\rho w)\, dxdydz$$

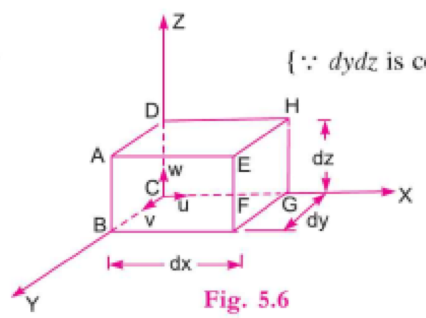

Fig. 5.6

∴ Net gain of masses $= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dxdydz$

Since the mass is neither created nor destroyed in the fluid element, the net increase of mass per unit time in the fluid element must be equal to the rate of increase of mass of fluid in the element. But mass of fluid in the element is ρ. *dx*. *dy*. *dz* and its rate of increase with time is $\frac{\partial}{\partial t}$ (ρ *dx*. *dy*. *dz*) or $\frac{\partial \rho}{\partial t}$. *dx dy dz*.

Equating the two expressions,

or $$-\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right] dxdydz = \frac{\partial \rho}{\partial t}.\ dxdydz$$

or $$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w) = 0$$ [Cancelling *dx.dy.dz* from both sides] ...(5.3*A*)

Equation (5.3*A*) is the continuity equation in cartesian co-ordinates in its most general form. This equation is applicable to :

(*i*) Steady and unsteady flow,

(*ii*) Uniform and non-uniform flow, and

(*iii*) Compressible and incompressible fluids.

For steady flow, $\frac{\partial \rho}{\partial t} = 0$ and hence equation (5.3*A*) becomes as

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w) = 0 \quad ...(5.3B)$$

If the fluid is incompressible, then ρ is constant and the above equation becomes as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0 \quad ...(5.4)$$

Equation (5.4) is the continuity equation in three-dimensions. For a two-dimensional flow, the component $w = 0$ and hence continuity equation becomes as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y} = 0. \quad ...(5.5)$$

c. Calculate the velocity of fluid flow at a point (2, 3) if its 2-D flow stream function is given by ψ = 2xy. **(04 Marks)**

Problem 5.14 *The stream function for a two-dimensional flow is given by $\psi = 2xy$, calculate the velocity at the point P (2, 3). Find the velocity potential function ϕ.*

Solution. Given : $\psi = 2xy$

The velocity components u and v in terms of ψ are

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(2xy) = -2x$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y.$$

At the point P (2, 3), we get $u = -2 \times 2 = -4$ units/sec

$v = 2 \times 3 = 6$ units/sec

$\therefore$ Resultant velocity at $P = \sqrt{u^2 + v^2} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 7.21$ units/sec.

Velocity Potential Function ϕ

We know $\frac{\partial \phi}{\partial x} = -u = -(-2x) = 2x$ ...(*i*)

$\frac{\partial \phi}{\partial y} = -v = -2y$ ...(*ii*)

Integrating equation (*i*), we get

$$\int d\phi = \int 2x dx$$

or $\phi = \frac{2x^2}{2} + C = x^2 + C$ ...(*iii*)

where C is a constant which is independent of x but can be a function of y.

Differentiating equation (*iii*) w.r.t. 'y', we get $\frac{\partial \phi}{\partial y} = \frac{\partial C}{\partial y}$

But from (*ii*), $\frac{\partial \phi}{\partial y} = -2y$

$\therefore$ $\frac{\partial C}{\partial y} = -2y$

Integrating this equation, we get $C = \int -2y \, dy = -\frac{2y^2}{2} = -y^2$

Module-2

3 a. Derive the Euler's equation of fluid motion and hence deduce Bernoulli's equation.

(10 Marks)

▶ 6.3 EULER'S EQUATION OF MOTION

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in *s*-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section *dA* and length *ds*. The forces acting on the cylindrical element are:

1. Pressure force *pdA* in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of *s* must be equal to the mass of fluid element × acceleration in the direction *s*.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s \qquad \text{...(6.2)}$$

where a_s is the acceleration in the direction of *s*.

Now $a_s = \frac{dv}{dt}$, where *v* is a function of *s* and *t*.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v\partial v}{\partial s}$$

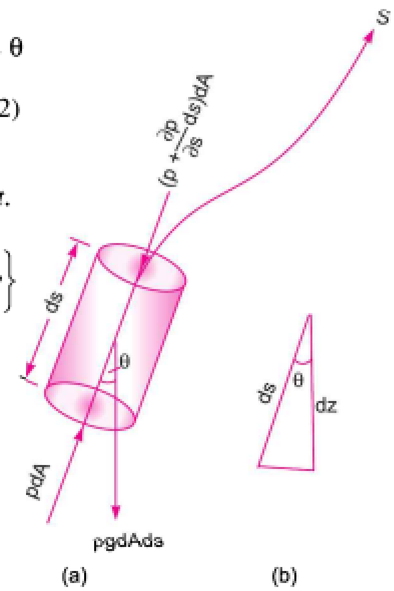

Fig. 6.1 *Forces on a fluid element.*

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g\, dA ds \cos\theta = \rho dA ds \times \frac{\partial v}{\partial s}$$

Dividing by $\rho ds dA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (*b*), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + gdz + vdv = 0$$

or $$\frac{dp}{\rho} + gdz + vdv = 0 \qquad \text{...(6.3)}$$

Equation (6.3) is known as Euler's equation of motion.

▶ 6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{...(6.4)}$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

▶ 6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero (*ii*) The flow is steady

(*iii*) The flow is incompressible (*iv*) The flow is irrotational.

OR

4 a. Derive expression for discharge through a triangular notch. (10 Marks)

▶ 8.4 DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the V- notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig. 8.3.

From Fig. 8.3 (b), we have

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

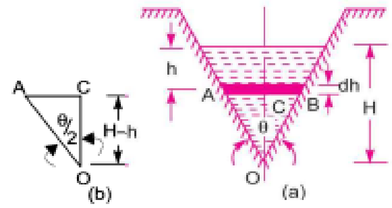

Fig. 8.3 *The triangular notch.*

$$\therefore \quad AC = (H-h)\tan\frac{\theta}{2}$$

Width of strip $= AB = 2AC = 2\,(H-h)\tan\dfrac{\theta}{2}$

$\therefore$ Area of strip $= 2\,(H-h)\tan\dfrac{\theta}{2} \times dh$

The theoretical velocity of water through strip $= \sqrt{2gh}$

$\therefore$ Discharge, through the strip,

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H-h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$\therefore$ Total discharge,

$$Q = \int_0^H 2C_d\,(H-h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15}C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \quad \text{...(8.2)}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge, $$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2} \quad \text{...(8.3)}$$

$$= 1.417\ H^{5/2}.$$

b. A horizontal venturimeter of 20 cm inlet diameter and 10 cm throat diameter is used to measure an oil flow. The discharge of oil through venturimeter is 60 lit/s. Calculate the reading of oil-mercury differential manometer. Take $C_d = 0.98$ and specific gravity = 0.8.

(10 Marks)

Solution. Given : $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} 20^2 = 314.16\ \text{cm}^2$

$d_2 = 10$ cm

$$\therefore \qquad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3\text{/s}$$

Using the equation (6.8), $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$$\therefore \qquad h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$$\therefore \qquad 289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$$\therefore \qquad x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

∴ Reading of oil-mercury differential manometer = **18.12 cm. Ans.**

Module-3

5 a. Derive Hagen Poiseulle equation for laminar flow through a circular pipe. **(10 Marks)**

▶ 9.2 FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure for a given length is to be determined. The flow through the circular pipe will be viscous or laminar, if the Reynolds number (R_e*) is less than 2000. The expression for Reynold number is given by

$$R_e = \frac{\rho V D}{\mu}$$

where ρ = Density of fluid flowing through pipe

V = Average velocity of fluid

D = Diameter of pipe and

μ = Viscosity of fluid.

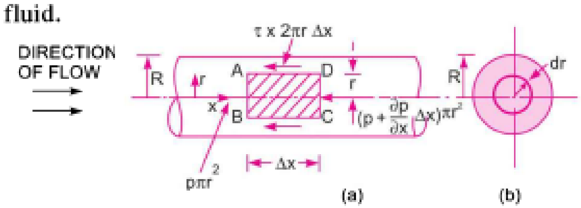

Fig. 9.1 *Viscous flow through a pipe.*

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in Fig. 9.1 (*a*). Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius $(r + dr)$. Let the length of fluid element be Δx. If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. Then the forces acting on the fluid element are :

1. The pressure force, $p \times \pi r^2$ on face AB.
2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ on face CD.
3. The shear force, $\tau \times 2\pi r\Delta x$ on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero *i.e.*,

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x} . r - 2\tau = 0$$

$\therefore$
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} \qquad ...(9.1)$$

The shear stress τ across a section varies with 'r' as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress distribution across a section is linear as shown in Fig. 9.2 (*a*).

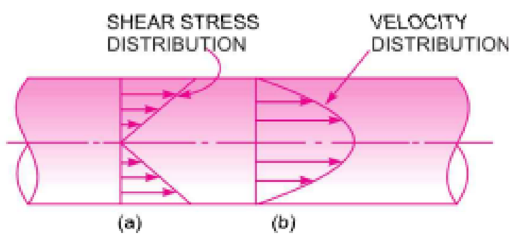

Fig. 9.2 *Shear stress and velocity distribution across a section.*

(*i*) **Velocity Distribution.** To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{du}{dy}$ is substituted in equation (9.1).

But in the relation $\tau = \mu \frac{du}{dy}$, y is measured from the pipe wall. Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$\therefore$
$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting this value in (9.1), we get

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{or} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$$

Integrating this above equation w.r.t. 'r', we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C \qquad ...(9.2)$$

where C is the constant of integration and its value is obtained from the boundary condition that at $r = R$, $u = 0$.

$$\therefore \qquad 0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C$$

$$\therefore \qquad C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

Substituting this value of C in equation (9.2), we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \qquad ...(9.3)$$

In equation (9.3), values of μ, $\frac{\partial p}{\partial x}$ and R are constant, which means the velocity, u varies with the square of r. Thus equation (9.3) is a equation of parabola. This shows that the velocity distribution across the section of a pipe is parabolic. This velocity distribution is shown in Fig. 9.2 (*b*).

(*ii*) **Ratio of Maximum Velocity to Average Velocity.** The velocity is maximum, when $r = 0$ in equation (9.3). Thus maximum velocity, U_{max} is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \qquad ...(9.4)$$

The average velocity, $\bar{u}$, is obtained by dividing the discharge of the fluid across the section by the area of the pipe (πR^2). The discharge (Q) across the section is obtained by considering the flow through a circular ring element of radius r and thickness dr as shown in Fig. 9.1 (*b*). The fluid flowing per second through this elementary ring

$$dQ = \text{velocity at a radius } r \times \text{area of ring element}$$

$$= u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times 2\pi r\, dr$$

$$\therefore \qquad Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\, r dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4} = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

∴ Average velocity, $$\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}{\pi R^2}$$

or $$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \qquad ...(9.5)$$

Dividing equation (9.4) by equation (9.5),

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2} = 2.0$$

∴ Ratio of maximum velocity to average velocity = 2.0.

(*iii*) **Drop of Pressure for a given Length (L) of a pipe**

From equation (9.5), we have

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \quad \text{or} \quad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

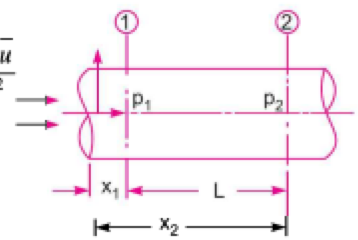

Fig. 9.3

Integrating the above equation w.r.t. x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\, dx$$

∴ $$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or } (p_1 - p_2) = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2} L \qquad \{\because \ x_2 - x_1 = L \text{ from Fig. 9.3}\}$$

$$= \frac{8\mu\bar{u}L}{(D/2)^2} \qquad \left\{\because \ R = \frac{D}{2}\right\}$$

or $$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2},$$ where $p_1 - p_2$ is the drop of pressure.

∴ Loss of pressure head $$= \frac{p_1 - p_2}{\rho g}$$

∴ $$\frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2} \qquad ...(9.6)$$

Equation (9.6) is called **Hagen Poiseuille Formula.**

b. A crude oil flowing through a horizontal circular pipe of 10 cm diameter and 100 cm length. Assume laminar flow and calculate pressure drop if 100 kg oil collected in a tank in 30 seconds. Take viscosity = 0.97 N-S/m^2 and specific gravity = 0.9. **(10 Marks)**

Solution. Given : $\mu = 0.97 \text{ poise} = \frac{0.97}{10} = 0.097 \text{ Ns/m}^2$

Relative density $= 0.9$

$\therefore$ ρ_0, or density, $= 0.9 \times 1000 = 900 \text{ kg/m}^3$

Dia. of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

$L = 10 \text{ m}$

Mass of oil collected, $M = 100 \text{ kg}$

Time, $t = 30$ seconds

Calculate difference of pressure or $(p_1 - p_2)$.

The difference of pressure $(p_1 - p_2)$ for viscous or laminar flow is given by

$$p_1 - p_2 = \frac{32\mu \bar{u} L}{D^2}, \text{ where } \bar{u} = \text{average velocity} = \frac{Q}{\text{Area}}$$

Now, mass of oil/sec $= \frac{100}{30}$ kg/s

$= \rho_0 \times Q = 900 \times Q$ $\quad (\because \rho_0 = 900)$

$$\therefore \quad \frac{100}{30} = 900 \times Q$$

$$\therefore \quad Q = \frac{100}{30} \times \frac{1}{900} = 0.0037 \text{ m}^3\text{/s}$$

$$\therefore \quad \bar{u} = \frac{Q}{\text{Area}} = \frac{.0037}{\frac{\pi}{4} D^2} = \frac{.0037}{\frac{\pi}{4}(.1)^2} = 0.471 \text{ m/s.}$$

For laminar or viscous flow, the Reynolds number (R_e) is less than 2000. Let us calculate the Reynolds number for this problem.

Reynolds number, $R_e^* = \frac{\rho V D}{\mu}$

where $\rho = \rho_0 = 900$, $V = \bar{u} = 0.471$, $D = 0.1$ m, $\mu = 0.097$

$$\therefore \quad R_e = 900 \times \frac{.471 \times 0.1}{0.097} = 436.91$$

As Reynolds number is less than 2000, the flow is laminar.

$$\therefore \quad p_1 - p_2 = \frac{32\mu \bar{u} L}{D^2} = \frac{32 \times 0.097 \times .471 \times 10}{(.1)^2} \text{ N/m}^2$$

$= 1462.28 \text{ N/m}^2 = 1462.28 \times 10^{-4} \text{ N/cm}^2 =$ **0.1462 N/cm². Ans.**

OR

6 a. Discuss the energy losses that occur in pipe flow. **(10 Marks)**

▶ 11.2 LOSS OF ENERGY IN PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses
   This is due to friction and it is calculated by the following formulae :
   (*a*) Darcy-Weisbach Formula
   (*b*) Chezy's Formula

2. Minor Energy Losses
   This is due to
   (*a*) Sudden expansion of pipe
   (*b*) Sudden contraction of pipe
   (*c*) Bend in pipe
   (*d*) Pipe fittings etc.
   (*e*) An obstruction in pipe.

▶ 11.4 MINOR ENERGY (HEAD) LOSSES

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance of a pipe,
4. Loss of head at the exit of a pipe,
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

b. Derive Darcy-Weisbach equation for determining loss of head due to friction. **(10 Marks)**

▶ 11.3 LOSS OF ENERGY (OR HEAD) DUE TO FRICTION

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 \, . \, f \, . \, L \, . \, V^2}{d \times 2g} \qquad ...(11.1)$$

where h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynolds number

$$= \frac{16}{R_e} \text{ for } R_e < 2000 \text{ (viscous flow)}$$

$$= \frac{0.079}{R_e^{1/4}} \text{ for } R_e \text{ varying from 4000 to } 10^6$$

L = length of pipe,

V = mean velocity of flow,

d = diameter of pipe.

(*b*) **Chezy's Formula for loss of head due to friction in pipes.** Refer to chapter 10 article 10.3.1 in which expression for loss of head due to friction in pipes is derived. Equation (*iii*) of article 10.3.1, is

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad \text{...(11.2)}$$

where h_f = loss of head due to friction, P = wetted perimeter of pipe,
A = area of cross-section of pipe, L = length of pipe,
and V = mean velocity of flow.

Module-4

7 a. Explain the following terms:
(i) Boundary layer thickness (ii) Streamline body (iii) Bluff body
(iv) Lift (v) Drag
(10 Marks)

13.2.4 Boundary Layer Thickness (δ). It is defined as the distance from the boundary of the solid body measured in the y-direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free stream velocity (U) of the fluid. It is denoted by the symbol δ. For laminar and turbulent zone it is denoted as :

1. δ_{lam} = Thickness of laminar boundary layer,
2. δ_{tur} = Thickness of turbulent boundary layer, and
3. δ' = Thickness of laminar sub-layer.

14.3.3 Stream-lined Body. A stream-lined body is defined as that body whose surface coincides with the stream-lines, when the body is placed in a flow. In that case the separation of flow will take place only at the trailing edge (or rearmost part of the body). Though the boundary layer will start at the leading edge, will become turbulent from laminar, yet it does not separate upto the rearmost part of the body in the case of stream-lined body. Thus behind a stream-lined body, wake formation zone will be very small and consequently the pressure drag will be very small. Then the total drag on the stream-lined body will be due to friction (shear) only. A body may be stream-lined :

1. at low velocities but may not be so at higher velocities.
2. when placed in a particular position in the flow but may not be so when placed in another position.

14.3.4 Bluff Body. A bluff body is defined as that body whose surface does not coincide with the streamlines, when placed in a flow. Then the flow is separated from the surface of the body much ahead of its trailing edge with the result of a very large wake formation zone. Then the drag due to pressure will be very large as compared to the drag due to friction on the body. Thus the bodies of such a shape in which the pressure drag is very large as compared to friction drag are called bluff bodies.

14.2.1 Drag. The component of the total force (F_R) in the direction of motion is called 'drag'. This component is denoted by F_D. Thus drag is the force exerted by the fluid in the direction of motion.

14.2.2 Lift. The component of the total force (F_R) in the direction perpendicular to the direction of motion is known as 'lift'. This is denoted by F_L. Thus lift is the force exerted by the fluid in the direction perpendicular to the direction of motion. Lift force occurs only when the axis of the body is inclined to the direction of fluid flow. If the axis of the body is parallel to the direction of fluid flow, lift force is zero. In that case only drag force acts.

If the fluid is assumed ideal and the body is symmetrical such as a sphere or cylinder, both the drag and lift will be zero.

b. Deduce an expression for pressure drop (dp) in a pipe flow using Buckingham's π - theorem if fluid has velocity (V), viscosity (μ) and density (ρ). Consider pipe diameter (D) and length (L). **(10 Marks)**

Solution. This problem is similar to problem 12.10. The only difference is that Δp is to be calculated for viscous flow. Then in the repeating variable instead of ρ, the fluid property μ is to be chosen.

Now Δp is a function of D, l, V, μ, ρ or $\Delta p = f(D, l, V, \mu, \rho)$

or $f_1 (\Delta p, D, l, V, \mu, \rho) = 0$ ...(*i*)

Total number of variables, $n = 6$

Number of fundamental dimensions, $m = 3$

Number of π-terms $= n - 3 = 6 - 3 = 3$

Hence equation (*i*) is written as $f_1(\pi_1, \pi_2, \pi_3) = 0$ ...(*ii*)

Each π-term contains $m + 1$ variables, *i.e.*, $3 + 1 = 4$ variables. Out of four variables, three are repeating variables.

Choosing D, V, μ as repeating variables, we have π-terms as

$$\pi_1 = D^{a_1} . V^{b_1} . \mu^{c_1} . \Delta p$$

$$\pi_2 = D^{a_2} . V^{b_2} . \mu^{c_2} . l$$

$$\pi_3 = D^{a_3} . V^{b_3} . \mu^{c_3} . \rho$$

First π-term $\pi_1 = D^{a_1} . V^{b_1} . \mu^{c_1} . \Delta p$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-1}T^{-1})^{c_1} . ML^{-1}T^{-2}$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1 + 1,$ $\therefore\ c_1 = -1$

Power of L, $0 = a_1 + b_1 - c_1 - 1,$ $\therefore\ a_1 = -b_1 + c_1 + 1 = 1 - 1 + 1 = 1$

Power of T, $0 = -b_1 - c_1 - 2,$ $\therefore\ b_1 = -c_1 - 2 = 1 - 2 = -1$

Substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = D^1 . V^{-1} . \mu^{-1} . \Delta p = \frac{D\Delta p}{\mu V}.$$

Second π-term $\pi_2 = D^{a_2} . V^{b_2} . \mu^{c_2} . l$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-1}T^{-1})^{c_2} . L.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_2,$ $\therefore\ c_2 = 0$

Power of L, $0 = a_2 + b_2 - c_2 + 1,$ $\therefore\ a_2 = -b_2 + c_2 - 1 = -1$

Power of T, $0 = -b_2 - c_2,$ $\therefore\ b_2 = -c_2 = 0$

Substituting the values of a_2, b_2 and c_2 in π_2,

$$\pi_2 = D^{-1} \cdot V^0 \cdot \mu^0 \cdot l = \frac{l}{D}.$$

Third π-term $\pi_3 = D^{a_3} \cdot V^{b_3} \cdot \mu^{c_3} \cdot \rho$

Substituting the dimension on both sides,

$$M^0L^0T^0 = L^{a_3} \cdot (LT^{-1})^{b_3} \cdot (ML^{-1}T^{-1})^{c_3} \cdot ML^{-3}.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_3 + 1$, $\therefore \; c_3 = -1$

Power of L, $0 = a_3 + b_3 - c_3 - 3$, $\therefore \; a_3 = -b_3 + c_3 + 3 = -1 - 1 + 3 = 1$

Power of T, $0 = -b_3 - c_3$, $\therefore \; b_3 = -c_3 = -(-1) = 1$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = D^1 \cdot V^1 \cdot \mu^{-1} \cdot \rho = \frac{\rho DV}{\mu}.$$

Substituting the values of π_1, π_2 and π_3 in equation (*ii*),

$$f_1\left(\frac{D\Delta p}{\mu V}, \frac{l}{D}, \frac{\rho DV}{\mu}\right) = 0 \quad \text{or} \quad \frac{D\Delta p}{\mu V} = \phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right] \quad \text{or} \quad \Delta p = \frac{\mu V}{D}\phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right]$$

Experiments show that the pressure difference Δp is a linear function $\frac{l}{D}$. Hence $\frac{l}{D}$ can be taken out of the functional as

$$\Delta p = \frac{\mu \mathbf{V}}{\mathbf{D}} \times \frac{l}{\mathbf{D}}\phi\left[\frac{\rho \mathbf{DV}}{\mu}\right]. \textbf{ Ans.}$$

Expression for difference of pressure head for viscous flow

$$h_f = \frac{\Delta p}{\rho g} = \frac{\mu V}{D} \times \frac{l}{D} \times \frac{1}{\rho g}\phi\,[R_e] \qquad \left\{\because \; \frac{\rho DV}{\mu} = R_e\right\}$$

$$= \frac{\mu \mathbf{Vl}}{\mathbf{wD^2}}\phi\,[\mathbf{R_e}]. \textbf{ Ans.}$$

OR

8 a. Explain the following terms:
(i) Reynold's number (ii) Froude's number (iii) Euler's number
(iv) Weber's number (v) Mach number **(10 Marks)**

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force. As this is a ratio of one force to the other force, it will be a dimensionless number. These dimensionless numbers are also called non-dimensional parameters. The followings are the important dimensionless numbers :

1. Reynold's number,
2. Froude's number,
3. Euler's number,
4. Weber's number,
5. Mach's number.

12.8.1 Reynold's Number (R_e). It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained as

Inertia force (F_i) = Mass × Acceleration of flowing fluid

$$= \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{Time}} = \rho \times \frac{\text{Volume}}{\text{Time}} \times \text{Velocity}$$

$$= \rho \times AV \times V \quad \{\because \text{ Volume per sec} = \text{Area} \times \text{Velocity} = A \times V\}$$

$$= \rho AV^2 \qquad \text{...(12.11)}$$

Viscous force (F_v) = Shear stress × Area $\left\{\because \ \tau = \mu \frac{du}{dy} \ \therefore \ \text{Force} = \tau \times \text{Area}\right\}$

$$= \tau \times A$$

$$= \left(\mu \frac{du}{dy}\right) \times A = \mu \,.\, \frac{V}{L} \times A \qquad \left\{\because \ \frac{du}{dy} = \frac{V}{L}\right\}$$

By definition, Reynold's number,

$$R_e = \frac{F_i}{F_v} = \frac{\rho AV^2}{\mu \,.\, \frac{V}{L} \times A} = \frac{\rho VL}{\mu}$$

$$= \frac{V \times L}{(\mu / \rho)} = \frac{V \times L}{\nu} \qquad \left\{\because \ \frac{\mu}{\rho} = \nu = \text{Kinematic viscosity}\right\}$$

In case of pipe flow, the linear dimension L is taken as diameter, d. Hence Reynold's number for pipe flow,

$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho Vd}{\mu}. \qquad \text{...(12.12)}$$

12.8.2 Froude's Number (F_e). The Froude's number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force. Mathematically, it is expressed as

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

where F_i from equation (12.11) $= \rho AV^2$

and F_g = Force due to gravity

= Mass × Acceleration due to gravity

$= \rho \times \text{Volume} \times g = \rho \times L^3 \times g$ $\{\because \ \text{Volume} = L^3\}$

$= \rho \times L^2 \times L \times g = \rho \times A \times L \times g$ $\{\because \ L^2 = A = \text{Area}\}$

$$\therefore \quad F_e = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\rho AV^2}{\rho ALg}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}} \qquad \text{...(12.13)}$$

12.8.3 Euler's Number (E_u). It is defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force. Mathematically, it is expressed as

$$E_u = \sqrt{\frac{F_i}{F_P}}$$

where F_P = Intensity of pressure × Area = $p \times A$

and $F_i = \rho AV^2$

$$\therefore \quad E_u = \sqrt{\frac{\rho AV^2}{p \times A}} = \sqrt{\frac{V^2}{p / \rho}} = \frac{V}{\sqrt{p / \rho}} \qquad \text{...(12.14)}$$

12.8.4 Weber's Number (W_e). It is defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

Weber's Number, $$W_e = \sqrt{\frac{F_i}{F_s}}$$

where F_i = Inertia force = ρAV^2

and F_s = Surface tension force

= Surface tension per unit length × Length = $\sigma \times L$

$$\therefore \quad W_e = \sqrt{\frac{\rho AV^2}{\sigma \times L}} = \sqrt{\frac{\rho \times L^2 \times V^2}{\sigma \times L}} \qquad \{\because \quad A = L^2\}$$

$$= \sqrt{\frac{\rho L \times V^2}{\sigma}} = \sqrt{\frac{V^2}{\sigma / \rho L}} = \frac{V}{\sqrt{\sigma / \rho L}}. \qquad ...(12.15)$$

12.8.5 Mach's Number (*M*). Mach's number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force. Mathematically, it is defined as

$$M = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{F_i}{F_e}}$$

where $F_i = \rho AV^2$

and F_e = Elastic force = Elastic stress × Area

$= K \times A = K \times L^2$ $\{\because \quad K = \text{Elastic stress}\}$

$$\therefore \quad M = \sqrt{\frac{\rho AV^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K \times L^2}} = \sqrt{\frac{V^2}{K/\rho}} = \frac{V}{\sqrt{K/\rho}}$$

But $\sqrt{\frac{K}{\rho}} = C$ = Velocity of sound in the fluid

$$\therefore \quad M = \frac{V}{C}. \qquad ...(12.16)$$

(iv) Weber's number (v) Mach number

b. A flat plate 1.5 m × 1.5 m moves at 50 km/hr in stationary air of density 1.15 kg/m^3. The coefficients of drag and lift are 0.15 and 0.75 respectively. Compute:

(i) Lift force

(ii) Drag force

(iii) Resultant force

(iv) Power required to keep the plate in motion. (10 Marks)

Problem 14.1 *A flat plate 1.5 m × 1.5 m moves at 50 km/hour in stationary air of density 1.15 kg/m³. If the co-efficients of drag and lift are 0.15 and 0.75 respectively, determine :*

(i) The lift force, *(ii) The drag force,*

(iii) The resultant force, and

(iv) The power required to keep the plate in motion.

Solution. Given :

Area of the plate, $A = 1.5 \times 1.5 = 2.25 \text{ m}^2$

Velocity of the plate, $U = 50 \text{ km/hr} = \frac{50 \times 1000}{60 \times 60} \text{ m/s} = 13.89 \text{ m/s}$

Density of air $\rho = 1.15 \text{ kg/m}^3$

Co-efficient of drag, $C_D = 0.15$

Co-efficient of lift, $C_L = 0.75$

(*i*) **Lift Force (F_L).** Using equation (14.4),

$$F_L = C_L A \times \frac{\rho U^2}{2} = 0.75 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \mathbf{187.20 \text{ N. Ans.}}$$

(*ii*) **Drag Force (F_D).** Using equation (14.3),

$$F_D = C_D \times A \times \frac{\rho U^2}{2} = 0.15 \times 2.25 \times \frac{1.15 \times 13.89^2}{2} \text{ N} = \mathbf{37.44. \text{ N. Ans.}}$$

(*iii*) **Resultant Force (F_R).** Using equation (14.5),

$$F_R = \sqrt{F_D^2 + F_L^2} = \sqrt{37.44^2 + 187.20^2} \text{ N}$$

$$= \sqrt{1400 + 35025} = \mathbf{190.85 \text{ N. Ans.}}$$

(*iv*) **Power Required to keep the Plate in Motion**

$$P = \frac{\text{Force in the direction of motion} \times \text{Velocity}}{1000} \text{ kW}$$

$$= \frac{F_D \times U}{1000} = \frac{37.425 \times 13.89}{1000} \text{ kW} = \mathbf{0.519 \text{ kW. Ans.}}$$

Module-5

9 a. Show that velocity of elastic wave propagation in an adiabatic medium is given by $C = \sqrt{\gamma RT}$. **(10 Marks)**

15.4.4 Velocity of Sound for Adiabatic Process.

Adiabatic process is given by equation (15.4), as

$$\frac{p}{\rho^k} = \text{Constant or } p\rho^{-k} = \text{Constant}$$

Differentiating the above equation, we get

$$p\,(-k)\,\rho^{-k-1}\,d\rho + \rho^{-k}\,dp = 0$$

Dividing by ρ^{-k}, we get $-pk\rho^{-1}\,d\rho + dp = 0$ or $dp = \frac{pk}{\rho}\,d\rho$

$$\therefore \quad \frac{dp}{d\rho} = \frac{p}{\rho}\,k = RTk \qquad \left(\because \frac{p}{\rho} = RT\right)$$

$$= kRT$$

Substituting the value of $\frac{dp}{d\rho}$ in equation (15.15), we get $C = \sqrt{kRT}$. ...(15.18)

Note 1. For the propagation of the minor disturbances through air, the process is assumed to be adiabatic. The velocity of the disturbances (pressure waves) through air is very high and hence there is no time for any appreciable heat transfer.

2. Isothermal process is considered for the calculation of the velocity of the sound waves (or pressure waves) only when it is given in the numerical problem that process is isothermal. If no process is mentioned, it is assumed to be adiabatic.

b. A projectile travels in air of pressure 100 kPa at 10°C with a speed of 1500 km/hr. Compute the Mach number and Mach angle. Take $\gamma = 1.4$ and $R = 287$ J/kg-K. **(10 Marks)**

Problem 15.9 *A projectile travels in air of pressure 10.1043 N/cm² at 10°C at a speed of 1500 km/hour. Find the Mach number and the Mach angle. Take k = 1.4 and R = 287 J/kg°K.*

Solution. Given :

Pressure, $p = 10.1043 \text{ N/cm}^2 = 10.1043 \times 10^4 \text{ N/cm}^2$

Temperature, $t = 10°\text{C}$

$\therefore$ $T = 10 + 273 = 283°\text{K}$

Speed of projectile, $V = 1500 \text{ km/hour} = \frac{1500 \times 1000}{60 \times 60} \text{ m/s} = 416.67 \text{ m/s}$

$k = 1.4$, $R = 287$ J/kg°K

For adiabatic process, the velocity of sound is given by

$$C = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 283} = 337.20 \text{ m/s}$$

$\therefore$ Mach number, $M = \frac{V}{C} = \frac{416.67}{337.20} = $ **1.235. Ans.**

∴ Mach angle is obtained from equation (15.20) as

$$\sin \alpha = \frac{C}{V} = \frac{1}{M} = \frac{1}{1.235} = 0.8097$$

∴ Mach angle, $\alpha = \sin^{-1} 0.8097 = \mathbf{54.06°}$. **Ans.**

OR

10 a. Explain the necessity, applications and limitations of CFD. (10 Marks)

Necessity of CFD (Computational Fluid Dynamics):

1. Cost-Effective Analysis: CFD enables cost-effective analysis and design improvements in various fields without the need for physical prototypes.

2. Complex System Understanding: It helps in understanding and predicting the behavior of fluid flow in complex systems like aerodynamics, weather patterns, combustion, HVAC systems, and more.

3. Optimization and Design: CFD aids in optimizing designs, predicting performance, and understanding the impact of changes in various scenarios, leading to efficient and effective designs.

Applications of CFD:

1. Aerospace and Automotive Industry: Used for aircraft design, engine optimization, aerodynamics, and vehicle design to improve efficiency and performance.

2. Environmental Studies: Assessing air and water pollution, weather patterns, and the impact of various factors on the environment.

3. Civil and Architectural Engineering: Evaluating HVAC systems, wind patterns around buildings, and other fluid-related aspects in construction and infrastructure.

Limitations of CFD:

1. Simplifications and Assumptions* Models are based on certain assumptions and simplifications that might not capture all real-world complexities accurately.

2. Computational Resources: High computational requirements, especially for high-fidelity simulations, can limit the application of CFD in some cases.

3. Validation and Accuracy: Results need to be validated with experimental data, and the accuracy heavily depends on the quality of inputs, models, and simulations.

4. Complexity of Fluid Dynamics: Modeling turbulence, multiphase flows, and other complex phenomena accurately remains a challenge.

Understanding the necessity, applications, and limitations of CFD is crucial for leveraging its benefits effectively while being aware of its constraints for accurate and reliable simulations.

10 a. Explain the necessity, applications ...

b. A projectile travels with a speed of 1500 km/hr at 20°C temperature and 0.1 MPa air pressure. Calculate the Mach number and Mach angle. Take $\gamma = 1.4$ and R = 287 J/kg-K.

(10 Marks)

Problem 15.9 *A projectile travels in air of pressure 10.1043 N/cm² at 10°C at a speed of 1500 km/hour. Find the Mach number and the Mach angle. Take k = 1.4 and R = 287 J/kg°K.*

Solution. Given :

Pressure, $p = 10.1043 \text{ N/cm}^2 = 10.1043 \times 10^4 \text{ N/cm}^2$

Temperature, $t = 10°C$

$\therefore$ $T = 10 + 273 = 283°K$

Speed of projectile, $V = 1500 \text{ km/hour} = \dfrac{1500 \times 1000}{60 \times 60} \text{ m/s} = 416.67 \text{ m/s}$

$k = 1.4,\ R = 287 \text{ J/kg°K}$

For adiabatic process, the velocity of sound is given by

$$C = \sqrt{kRT} = \sqrt{1.4 \times 287 \times 283} = 337.20 \text{ m/s}$$

$\therefore$ Mach number, $M = \dfrac{V}{C} = \dfrac{416.67}{337.20} = \mathbf{1.235.\ Ans.}$

$\therefore$ Mach angle is obtained from equation (15.20) as

$$\sin \alpha = \frac{C}{V} = \frac{1}{M} = \frac{1}{1.235} = 0.8097$$

$\therefore$ Mach angle, $\alpha = \sin^{-1} 0.8097 = \mathbf{54.06°.\ Ans.}$