CBCS SCHEME

USN | | | | | | | | | | **18ME62**

Sixth Semester B.E. Degree Examination, June/July 2023

Design of Machine Elements – II

Time: 3 hrs. Max. Marks: 100

Note:1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Using design data hand book is permitted.
3. Assume missing data suitably.

Module-1

1 a. Discuss about the following terms :
(i) Active coils
(ii) Deflection
(iii) Solid length
(iv) Free length
(v) Resilience **(05 Marks)**

b. Derive an expression for energy stored in a spring. **(05 Marks)**

c. Design a helical compression spring to carry a load of 500 N with a deflection of 20 mm. The allowable shear stress in the spring material is 350 MN/m^2 and the modulus of rigidity is 82.7×10^3 MN/m^2. The spring index is 6. **(10 Marks)**

OR

2 a. A leather belt 125 mm wide and 6 mm thickness transmits power from a pulley 750 mm diameter which runs at 500 rpm. The angle of lap is 150° and the coefficients of friction between the belt and the pulley is 0.3. If the belt density is 1000 kg/m^3 and the stress in the belt is not to exceed 2.75 N/mm^2, find the power that can be transmitted by the belt. Also find the initial tension in the belt. **(10 Marks)**

b. An oil well has to be drilled to a depth of 900 mm using 100 drill pipe. Assume 200 N for every 15 m length of pipe. The rope sheaves are of 80 mm diameter and acceleration is 2.5 m/s^2. Determine the size of 6×37 wire rope for lifting the string of pipes using a FOS as 3 and ultimate stress as 1800 MPa. **(10 Marks)**

Module-2

3 Design a pair of spur gear to transmit 27 kW for an oil pump with the gear ratio of 3 : 1, the rpm of the pinion is 1200, the centre distance is 400 mm, and the gears are to be forged steel untreated with $14\frac{1}{2}^{\circ}$ FDI. Check the design for dynamic and wear condition. **(20 Marks)**

OR

4 A pair of helical gears are used to transmit 15 kW. The teeth are 20° full depth in normal plane and have a helix angle of 30°. The pinion has 24 teeth and operates at 1000 rpm. The velocity ratio is 5 to 1. The pinion is made of cast steel $[\sigma_d = 50\,\text{MPa}]$ and the gear is of bronze $[\sigma_d = 40\,\text{MPa}]$. The pinion material is hardened to 200 BHN. Design the gear pair. **(20 Marks)**

Module-3

5 A pair of straight tooth bevel gear at right angle is to transmit 5 kW at 1200 rpm of the pinion. The diameter of the pinion is 80 mm and the velocity ratio is 3.5 to 1. The tooth form is $14\frac{1}{2}^\circ$ composite type. Both pinion and gear are made of CI $\left[\sigma_d = 55\ N/mm^2\right]$. Determine the face width and the required module from the stand point of strength using Lewis equation and check for design from the stand point of dynamic load and wear load.

(20 Marks)

OR

6 Design a worm gear to transmit 2 kW at 1000 rpm, speed ratio is 20 and centre distance is 200 mm.

(20 Marks)

Module-4

7 a. A cone clutch with a face angle of 14° has to transmit 286 N-m of torque at a speed of 600 rev/min. The larger diameter of the clutch is 250 mm, face width is 60 mm and co-efficient of friction is 0.18. Determine (i) Axial force to transmit the torque (ii) Average normal pressure (iii) Maximum normal pressure. Assume uniform wear condition.

(10 Marks)

b. A single plate friction clutch of both sides effective has 0.3 m outer diameter and 0.16 m inner diameter. The coefficient of friction is 0.2 and it runs at 1000 rpm. Find the power transmitted for uniform wear and uniform pressure distribution cases if the allowable maximum pressure is 0.08 MPa.

(10 Marks)

OR

8 a. Fig. Q8 (a) shows a CI brake shoe. The coefficient of friction is 0.30. The breaking torsional moment is to be 346 N. Determine

(i) The force P, for anti-clock wise rotation.

(ii) The force P, for clockwise direction.

(iii) Where must the pivot be placed to make the brake self energizing with the counter clockwise direction.

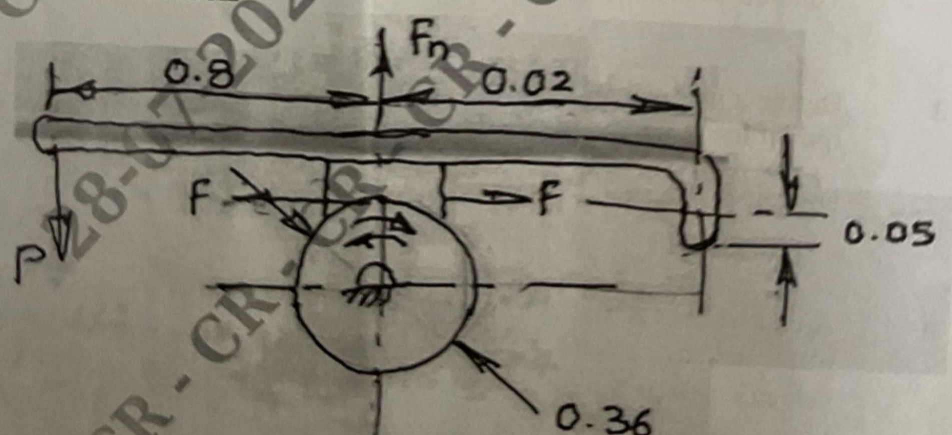


Fig. Q8 (a)

(10 Marks)

b. In a simple band break, the length of the lever is 440 mm, the tight end of the hand is attached to the fulcrum of the lever and the slack end to a pin 50 mm from the fulcrum. The diameter of the break drum is 1 mm and arc of contact is 300°, the co-efficient of friction between the band and the drum is 0.35. the break drum is attached to a hoisting drum of diameters 0.65 m that sustains a load of 20 kN (Fig. Q8(b)),

(i) Force required at the end of lever to support the load.

(ii) Width of steel band if the tensile stress is limited to 50 N/mm^2.

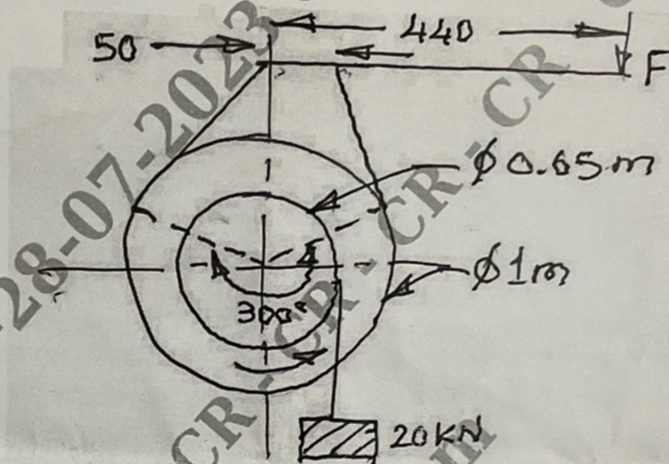


Fig. Q8 (b)

(10 Marks)

Module-5

9 a. Derive Petroff's equation for lightly loaded bearing. **(12 Marks)**

b. For a full journal bearing has the following specification : Shaft diameter 45 mm, bearing length 66 mm, Clearance ratio 0.0015, Speed 2800 rpm, Load 800 N and absolute viscosity 8.27×10^{-3} Pa-S. Determine (a) frictional torque (b) Co-efficient of friction (c) Power loss. **(08 Marks)**

OR

10 a. A full journal bearing of 50 mm diameter and 100 mm long has a bearing pressure of 1.4 N/mm^2. The speed of journal is 900 rpm and the ratio of journal diameter to the diametrical clearance is 1000. The bearing is lubricated with oil whose absolute viscosity at the operating temperature of 75°C may be taken as 0.011 kg/m. The room temperature is 35°C. Determine :

(i) The amount of artificial cooling required.

(ii) The mass of lubricating oil required, if the difference between outlet and inlet temperature of the oil is 10°C.

Take specific heat of 1850 J/kg°K. **(10 Marks)**

b. A bearing for an axial flow compressor is to carry a radial load of 4905 N and thrust load of 2452 N. The service imposes light shock and the bearing is used for 40 hours/week for 5 years. The speed of the shaft is 300 rpm and diameter of the shaft is 60 mm. Select a suitable bearing. **(10 Marks)**

* * * * *

Solutions

1a) Discuss the following terms

i) **Active coils**

Active coils are within the spring. An inactive coil is at the end of the spring on each side, therefore, active coils are all of the coils that aren't on the ends of a spring. If the spring has an open end, meaning the coil at the end is cut in half, that means it has a half inactive coil.

Deflection

Spring deflection is the action or motion that results from the application or release of a load. It is the response to a force when it is applied or removed. Spring deflection is also known as spring travel, which is a reference to the distance a spring moves between the loaded and the preloaded position.

Solid length

i) Solid length : When the compression spring is compressed until coils come in contact with each other, then the spring is said to be solid.

$$\text{Solid length} = i' d,$$

where i' => Total number of turns

d = spring wire diameter

Free length

ii) Free length: The free length of a compression spring is the length of the spring during unloaded condition.

$$\text{Free length } l_f \geq (i + n)d + y + a$$

Where, a=> clearance

i = no. of active turns or coils

y = maximum deflection.

Resilience

Resilience of a spring is equal to the energy absorbed by it. The energy absorbed is equal to the work done on it by the external force.

1b) Energy stored in a spring

$$W = \int F\,dx$$

Spring force $= kx$, k is a constant

$$\therefore W_{spring} = \int_0^x kx\,dx$$

$$\Rightarrow W = k \int_0^x x\,dx$$

$$\Rightarrow W = \frac{1}{2} k \left[x^2\right]_0^x$$

$$\Rightarrow W = \frac{1}{2} k (x^2 - 0)$$

$$\therefore \boxed{W = \frac{1}{2} kx^2} \Rightarrow \text{Potential Energy of a Spring}$$

1c)

$F : 500N$; $y : 20mm$; $\tau : 350\ MN/m^2 : 350\ N/mm^2$

$G : 82.7 \times 10^3\ N/mm^2$; $c : 6$

Stress factor $K : \dfrac{4c-1}{4c-4} + \dfrac{0.615}{c}$

$\Rightarrow K : 1.2525$

Shear stress $\tau = \frac{8FDK}{\pi d^3} = \frac{8FcK}{\pi d^2}$ $\left[c = \frac{D}{d} \right]$

$\Rightarrow d = 5.23\,mm \simeq 5.3\,mm$

$\therefore$ wire diameter $= 5.3\,mm$

Mean coil dia $\Rightarrow D = cd = 31.8\,mm$

$D_o = D + d = 37.1\,mm$

$D_i = D - d = 26.5\,mm$

Deflection $y = \frac{8FD^3 i}{Gd^4}$

$\Rightarrow i = 10.15$ (Active no: of coils)

Assuming ends are squared and ground

Total coils $i' = i + 2 = 12.15$

Solid height $= h = i'd = 64.395\,mm$

Free length $l_0 = h + y + a$

$= h + y + (i' - 1) \times 2$

$l_0 = 106.7\,mm$

pitch $p = \frac{l_0 - 2d}{i} = 9.47mm$.

2) a) $b = 125mm$; $t = 6mm$; $n_1 = 500rpm$

$d_1 = 750mm$

$\theta = 150^\circ$; $\mu = 0.3$; $w = 1000 kg/m^3$

$\sigma_1 = 2.75 N/mm^2$ $P = ?$ $T_0 = ?$

Soln.

$v = \frac{\pi n_1 d_1}{60000} m/s = \frac{\pi \times 500 \times 750}{60000}$

$v = 19.63 m/s$

Centrifugal stress $\sigma_c = \frac{w}{g} \times v^2 \times 10^{-6}$

$= \frac{10 \times 10^{-6} \times (19.63)^2 \times 10^{6}}{9810}$

$\sigma_c = 0.3928 N/mm^2$

Capacity

$$\theta_s = 150^\circ = 150 \times \frac{\pi}{180} = 2.61 \text{ rad}$$

$$e^{\mu\theta} = e^{\{0.3 \times 2.61\}} = 2.188$$

Constant k $= \dfrac{e^{\mu\theta} - 1}{e^{\mu\theta}} = \dfrac{1.188}{2.188} = 5.42$

Power transmitted / mm² area

$$P/_{mm^2} = \frac{(\sigma_1 - \sigma_c)}{1000} k v$$

$$= \frac{(2.75 - 0.3298)}{1000} \times 5.42 \times 19.63$$

$$P = 0.257 \text{ kW/mm}^2$$

Total power $= 0.257 \times 125 \times 6$

$= 193$ kW

$$2\sqrt{T_0} = \sqrt{T_1} + \sqrt{T_2}$$

$$\frac{\sigma_1 - \sigma_c}{\sigma_2 - \sigma_c} = e^{\mu\theta}$$

$$\Rightarrow \frac{2.75 - 0.3298}{\sigma_2 - 0.3298} = 2.186$$

$$\Rightarrow \sigma_2 = 1.435 \ N/mm^2$$

$$T_2 = \sigma_2 A = 1.435 \times 125 \times 6$$

$$T_2 = 1076.94 \ N$$

$$T_1 = 2.75 \times 125 \times 6$$

$$T_1 = 2062.5 N$$

$$2\sqrt{T_0} = \sqrt{2062.5} + \sqrt{1076.94}$$

$$\sqrt{T_0} = 39.11$$

$$T_0 = 1529.72 N \leftarrow \text{Initial tension}$$

Given data

$P = N = 20kW$

$n_1 = 1000$ rpm

$n_2 = 310$ rpm

No. of teeth on pinion, $z_1 = 31$

Tooth profile : 20^0 Full depth

Material for pinion : C45 steel untreated

Material for gear: cast steel 0.2%C untreated.

We need the number of teeth on gear to find out Lewis form factor 'y'.

Step 1: Identify the weaker member

Gear ratio

$$i = \frac{n_1}{n_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1}$$

$$i = \frac{n_1}{n_2} = \frac{1000}{310} = 3.2258$$

$z_2 = iz_1 = 3.2258 \text{ x } 31 = 99.999 = 100$ tooth.

Lewis form factor for given tooth profile,

Using equation 23.116,

$$y = 0.154 - \frac{0.912}{z}$$

$y_1 = 0.12458$ - for pinion with z = 31

$y_2 = 0.14458$ - for gear with z = 100

From table 23.18 (page 23.71) , the value of allowable static stress σ_0 is chosen.

Pinion - C45 Untreated (SAE1045 steel) - $\sigma_{01} = 207MPa$

Gear - Cast steel 0.2% C untreated - $\sigma_{02} = 138MPa$

Particulars	σ	y	$\sigma . y$	Remarks
Pinion	$\sigma_{01} = 207MPa$	$y_1 = 0.12458$	$\sigma_{01} . y_{01} = 25.788$ MPa	
Gear	$\sigma_{02} = 138MPa$	$y_2 = 0.14458$	$\sigma_{02} . y_{02} = 19.95$ MPa	Weaker

From the above values, it is found that gear is the weaker member and hence design should be based on gear only.

Step 2: Design

i) Tangential tooth load, use equation 23.87 (page 23.17)

$$F_t = \frac{9550PC_s}{nr}(1000)$$

P = 20 kW (Given)
n = n_2 = 310 rpm
r = $r_2 = \frac{mz_2}{2} = \frac{m100}{2} = 50m$
C_s = *service factor,* Table 23.13 - Assume medium shock and 8-10 hrs/day => C_s = 1.5

substituting the above values,

$F_t = \frac{18483.87}{m}$ ————————————-> (a)

ii) Tangential load using Lewis equation 23.93

$$F_t = \sigma_0 bypk_v$$

for weaker member

$$F_{t2} = \sigma_{02} y_2 bpk_v$$

$\sigma_{02} \cdot y_{02}$ = 19.95 MPa
b = Equation 23.132 - Page 23.29 - Assume $10m$
$p = \pi m$
k_v — *based on pitch line velocity of weaker member*

$$v_m = \frac{\pi d_2 n_2}{60000} = \frac{\pi m z_2 n_2}{60000} = \frac{(\pi)(m)(100)(310)}{60000} = 1.623m$$

Trial 1
From table 23.3
Choose module m = 4mm
v_m = 1.623(4) = 6.5m/s which is less than 7.5 m/s
using equation 23.134a, calculate k_v

$$k_v = \frac{3}{3 + 6.5} = 0.316$$

$F_t = (19.95)(10m)(\pi m)(0.316)$ ————————————> (b)
from (a) and (b)
$m^3 . k_v \geq 29.4887$
$(4)^3(0.316) \geq 29.4887$
$20.224 \geq 29.4887$
Not suitable.

Trial 2
From table 23.3 , Choose module m = 5mm
v_m = 1.623(5) = 8.115m/s > 7.5m/s

Use equation 23.135a for calculating velocity factor k_v

$$k_v = \frac{4.5}{4.5 + 8.115} = 0.35672$$

$F_t = (19.95)(10m)(\pi m)(0.35672)$

from Equations (a) and (b)

$(5)^3 \,.\, (0.35672) \geq 29.4887$

$44.59 \geq 29.4887$ (Suitable)

iii) Check for stress

$$\sigma_{allowable} = (\sigma_{02} \,.\, k_v) = (138)(0.35672) = 49.227\text{MPa}$$

Induced stress (σ_{ind}) is calculated as follows,

$$\sigma_{ind} = \frac{F_{t_2}}{b y_2 p} = \frac{(\frac{18487.87}{5})}{(10)(5)(0.14458)(\pi)(5)} = 32.488 < 49.227$$

Hence design is safe

Step 3 Dimensions

Note: Only module and face-width to be calculated as we have to check for dynamic load and wear also.

module m = 5mm

Face-width b = 10m = 50mm

Step 4 Check for Dynamic load and Wear load

i) Use equation 23.155 (Page no 23.33) for dynamic load

$$F_d = F_t + \frac{21 v_m (F_t + bC)}{21 v_m + \sqrt{(F_t + bC)}}$$

$$F_t = \frac{18483.87}{5} = 3696.774$$

b = 50mm

For calculation of 'C'

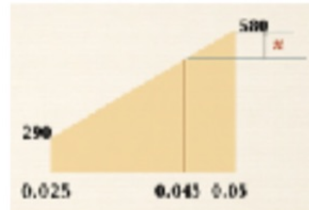
From fig 23.35a Page 23.35, corresponding to pitch line velocity 8.115m/s, error 'f' = 0.045 mm

From Table 23.32 (Page 23.74), for f = 0.045 mm, (between f =0.025 and f = 0.05) we get two values of C

For Steel-Steel combination, 20 degree full depth,

f (0.025) => C = 290kN/m = 290 N/mm
f (0.05) => C = 580 kN/m = 580 N/mm

By interpolation method, we need to find the actual value of 'C' corresponding to f (0.045)



From figure,

$$\frac{x}{(580-290)} = \frac{(0.045-0.025)}{(0.05-0.025)}$$

$x = 232 N/mm$

Therefore C = 290+ x = 522 N/mm

Now, dynamic load can be calculated from the equation.

$F_d = 18499.51N$

ii) Use equation 23.160 for wear load calculation

$$F_w = d_1 bQK = mz_1 bQK$$

$$Q = \frac{2Z_2}{Z_1 + Z_2} = 1.52672$$

for safe design,

$F_w \geq F_d$

$d_1 bQK \geq 18499.57$

$(5)(31)(50)(1.52672)(K) \geq 18499.57$

$K \geq 1.5634$

from table 23.37B (Page 23.80), corresponding to 20 Degree F.D tooth profile,

K = 1.6069 > 1.5634, design safe

corresponding BHN for pinion and gear are as follows
BHN for pinion = 350
BHN for gear = 300

———————————————All the best ———————————————

Given Data:

P = 15 kW; n_1 = 3200 rpm; $\beta = 26^0$; $z_1 = 20$; $i = 4$;

Pinion material : 0.4% Carbon Un-treated;

Gear material : High grade C.I

$$i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1}$$

$z_2 = iz_1 = (4)(20) => z_2 = 80.$

From Table 23.18, Allowable static stress for the given material,

σ_{01} = 69.6MPa (0.4% C untreated)

σ_{02} = 31MPa (High grade C.I)

Virtual number of teeth , (Equation 23.285)

$$z_v = \frac{z}{cos^3\beta}$$

for pinion, $z_{1v} = \dfrac{20}{cos^3 26} = 27.545$

for Gear $z_{2v} = \dfrac{80}{cos^3 26} = 110.18$

Assume tooth profile along normal plane as 20^0 Full Depth

Lewis form factor is given by the equation 23.116

$y = 0.154 - \dfrac{0.912}{z}$ (Use z = z_v)

Lewis Form factor for pinion ,

$$y_1 = 0.154 - \frac{0.912}{z_{1v}} = 0.1208$$

Lewis form factor for gear,

$$y_2 = 0.154 - \frac{0.912}{z_{2v}} = 0.1457$$

Step 1 Identify the Weaker member

Particulars	σ (MPa)	y	$\sigma . y$ (MPa)	Remarks
Pinion	69.6	0.1208	8.4077	
Gear	31	0.1457	4.516	Weaker

As evident from the above table, Gear is weaker and hence design should be based on gear only.

Step 2. Design

ii) Tangential tooth load, (Equation 23.87b)

$$F_t = \frac{9550PC_s}{nr} X1000$$

Tangential tooth load on the weaker member Gear

$$F_{t2} = \frac{9550PC_s}{n_2 r_2} X1000$$

Assume medium shock and 8-10 hours per day duty cycle, $C_s = 1.5$

$$r_2 = \frac{d_2}{2} = \frac{m_n z_2}{2cos\beta} = \frac{80m_n}{2cos26} = 44.5\ m_n$$

$n_2 = 3200/4 = 800$ rpm

Substituting all above values,

$$F_{t2} = \frac{6035.8}{m_n} \text{ ——————————————> (a)}$$

ii) Lewis equation for tangential tooth load (Equation 23.286)

$$F_t = \frac{\sigma_0 b y p_n k_v}{C_w}$$

Since gear is the weaker member,

$$F_{t2} = \frac{\sigma_{02} b y_2 p_n k_v}{C_w} \text{ ———————————————> (b)}$$

$\sigma_{02} y_2$ =4.516 ;   b = $10m_n$; $p_n = \pi m_n$

$$v_m = \frac{\pi d_2 n_2}{60000} = \frac{\pi m_n z_2 n_2}{60000 cos\beta}$$

v_m = $3.7284 m_n$ m/s

C_w - Refer Table 23.37, Assuming Scant lubrication but frequent inspection,

C_w = 1.25

from (a) and (b)

$$F_{t2} = \frac{6035.8}{m_n} = \frac{(4.512)(10m_n)(\pi m_n)(k_v)}{1.25}$$

On simplifying the above equations,

$$m_n^3 k_v \geq 53.171 \text{ ———————————————> (c)}$$

Trial 1

Assume module m_n = 5 mm (Table 23.3)

v_m = $3.7284 m_n$ = 3.7284 x 5 = 18.642 m/s < 20 m/s

Velocity factor is given by the equation 23.289a

$$k_v = \frac{6}{6 + v_m} = \frac{6}{6 + 18.642} = 0.2435$$

$(5)^3(0.2345) \geq 53.171$

$29.3125 \geq 53.171$ (Not suitable)

Trial 2

Assume module $m_n = 6$ mm

$v_m = 3.7284m_n = 3.7284 \text{ x } 6 = 22.3704 \text{ m/s} > 20 \text{ m/s}$

$$k_v = \frac{5.6}{5.6 + \sqrt{v_m}} = 0.542$$

from equation (c),

$(6)^3 0.542 \geq 53.171$

$117.072 \geq 53.171$ (Suitable)

Hence,

Normal module $m_n = 6$ mm

ii) Check for stress

Allowable stress for gear,

$\sigma_{all} = (\sigma_0 k_v)_{allowable} = (31 \text{ x } 0.54) = 16.74\text{MPa}$

Induced stress for gear,

$$\sigma_{ind} = (\sigma_0 k_v)_{induced} = \frac{F_t C_w}{b y p_n} = \frac{F_{t2} C_w}{b y_2 p_n}$$

$$= \frac{\frac{6035.8}{6} X 1.25}{(10X6)(0.1457)(6\pi)}$$

$$= 7.63\text{MPa}$$

Since $(\sigma_0 k_v)_{allowable} > (\sigma_0 k_v)_{induced}$

Design is safe.

Normal Module m_n = 6mm

Step 3 Dimensions

Normal Module $m_n = 6$ mm

Transverse module $m_t = \frac{m_n}{cos\beta} = 6.6756$ mm

Face width b = 60mm

$$b_{min} = \frac{p_t}{tan\beta} \text{ (Equation 23.277/Page 23.53)}$$

But $p_t = \pi m_t$ (Equation 23.213/Page 23.48)

Therefore,

$$b_{min} = \frac{\pi m_t}{tan\beta} = 43 \text{ mm} < 60 \text{ mm, so safe}$$

Transverse pitch = $p_t = \pi m_t$=20.972 mm

Normal pitch $p_n = \pi m_n = 18.85$ mm

PCD of pinion $d_1 = \frac{m_n z_1}{cos\beta} = 133.5$ mm

PCD of gear $d_2 = \frac{m_n z_2}{cos\beta} = 534$ mm

Centre distance 'a' $= \frac{d_1 + d_2}{2} = 333.75$ mm

Other dimensions to be calculated as per table 23.1

Step 4: Check for Dynamic and wear load

i) Dynamic load is given by equation 23.309a

$$F_d = F_t + \frac{21V(F_t + bCcos^2\beta)cos\beta}{21V + \sqrt{(F_t + bCcos^2\beta)}}$$

where

v = v_m =22.3704 m/s

b = 60 mm

$$F_t = \frac{6035.8}{6} = 1006 \text{ N}$$

From Figure 23.5a, for v_m =22.3704 m/s, error 'f' = 0.015mm

From Table 23.32, for 20 Degree FD teeth and Steel-C.I combination,

f(0.0125), C = 99.57 N/mm

0.0125	*99.57*
0.015	*x*

$$x = \frac{99.57X0.015}{0.0125} = 119.484\ N/mm$$

i.e C = 119.484 N/mm

Now F_d = 6155.27 N

ii) Wear load is given by equation 23.310

$$F_w = \frac{d_1 bQK}{cos^2\beta} = \frac{mz_1 bQK}{cos^2\beta}$$

$$Q = Q = \frac{2Z_2}{Z_1 + Z_2} = 1.6$$

For safe design,

$$\frac{d_1 bQK}{cos^2\beta} \geq F_d$$

$$\frac{(133.5)(60)(1.6)(K)}{cos^2 26} \geq 6155.27$$

K ≥0.388

From table 23.37B, corresponding to α = 20 Degree and K≥0.388

Surface hardness of Pinion = 200 BHN

Surface hardness of Gear = 150 BHN

Diameter of pinion shaft $D = \left(\frac{16\,M_{te}}{\pi\tau}\right)^{1/3}$ ..... (14.1)

$$= \left(\frac{16\times 291861.2}{\pi\times 50}\right)^{1/3} = 30.98 \text{ mm} \simeq 32 \text{ mm}$$

Example 5 : **A pair of straight tooth bevel gears at right angles is to transmit 5 kW at 1200 rpm of the pinion. The diameter of the pinion is 75 mm and the velocity ratio is 3.5 to 1. The tooth form is 14½° composite type. Both the pinion and gear are cast iron (σ_o = 55 MN/m²).**

1. **Determine the face width and the required module from the standpoint of strength using Lewis equation.**
2. **Check the design from the standpoint of dynamic load and wear load.**

Data : $\Sigma = 90°$, $N = 5$ kW, $n_1 = 1200$ rpm, $d_1 = 80$ mm, $i = 3.5$, $\alpha = 14½°$, $\sigma_{o1} = \sigma_{o2} = 55$ N/mm²

Solution :

Transmission ratio $i = \frac{n_1}{n_2} = \frac{d_2}{d_1}$ ..... (2.393)

∴ Speed of the gear $n_2 = \frac{n_1}{i} = \frac{1200}{3.5} = 342.857$ rpm

Pitch diameter of gear $d_2 = i\,d_1 = 3.5 \times 80 = 280$ mm

The pinion and gear are made of same material, the pinion is the weaker member. Therefore the design is based on pinion strength.

Pitch cone angle of pinion, $\tan\delta_1 = \frac{1}{i} = \frac{1}{3.5}$ ..... (2.402)

∴ $\delta_1 = 15.95°$

Pitch cone angle of gear $\delta_2 = \Sigma - \delta_1 = 90 - 15.95 = 74.05°$

Formative number of teeth on pinion $z_{v1} = \frac{z_1}{\cos\delta_1}$ ..... (2.418)

$$= \frac{d_1}{m\cos\delta_1} = \frac{80}{m\cos 15.95} = \frac{83.203}{m} \qquad \left(\because z_1 = \frac{d_1}{m}\right)$$

Lewis form factor for 14½° tooth form $y = 0.124 - \frac{0.684}{z_v}$ ..... (2.97)

$$\therefore\ y_1 = 0.124 - \frac{0.684\times m}{83.203} = 0.124 - 8.2209\times 10^{-3}\, m$$

Torque on pinion $M_{t1} = \dfrac{9550\,N}{n_1}$

$$= \frac{9550 \times 5}{1200} = 39.792 \text{ N-m} = 39792 \text{ N-mm}$$

Tangential force $F_t = \dfrac{2\,M_{t1}}{d_1} = \dfrac{2 \times 39792}{80} = 994.8 \text{ N}$

Cone distance $R = \dfrac{1}{2}\sqrt{d_1^2 + d_2^2}$ ..... (2.414)

$$= \frac{1}{2}\sqrt{80^2 + 280^2} = 145.602 \text{ mm}$$

Let the face width $b = \dfrac{R}{3} = \dfrac{145.602}{3} = 48.534 \text{ mm}$

Take the face width $b = 48 \text{ mm}$ $\left(\because \dfrac{R}{4} \le b \le \dfrac{R}{3}\right)$

$\therefore$ Ratio $\dfrac{R-b}{R} = \dfrac{145.602 - 48}{145.602} = 0.6703$

Velocity $v_m = \dfrac{\pi d_1 n_1}{60 \times 1000} = \dfrac{\pi \times 80 \times 1200}{60 \times 1000} = 5.026 \text{ m/sec}$

For generated teeth, velocity factor $C_v = \dfrac{5.55}{5.55 + v_m}$ ..... (2.429)

$$= \frac{5.55}{5.55 + 5.026} = 0.5248$$

By Lewis equation, tangential force $F_t = \sigma_o\, C_v\, b\, Y\, m \left(\dfrac{R-b}{R}\right)$ ..... (2.426a)

i.e., $994.8 = 55 \times 0.5248 \times 48 \times \pi (0.124 - 8.2209 \times 10^{-3}\, m) \times m \times 0.6703$

$8.2209 \times 10^{-3}\, m^2 - 0.124\, m + 0.341 = 0$

$\therefore$ Module $m = \dfrac{0.124 \pm \sqrt{0.124^2 - 4 \times 8.2209 \times 10^{-3} \times 0.341}}{2 \times 8.2209 \times 10^{-3}}$

$$= 11.47 \text{ mm or } 3.618 \text{ mm}$$

Take $m = 3.618 \text{ mm}$

From table (2.84), the standard module m is 4 mm.

Allowable stress $\sigma_{all} = \sigma_{o1} C_v = 55 \times 0.5248 = 28.864 \text{ N/mm}^2$

Tangential force $$F_t = \sigma_{o1} C_v b Y m \left(\frac{R-b}{R}\right) = \sigma_{in} b \pi y m \left(\frac{R-b}{R}\right) \quad \text{..... (2.426a)}$$

i.e., $$994.8 = \sigma_{in} \times 48 \times \pi (0.124 - 8.2209 \times 10^{-3} \times 4) \times 4 \times 0.6703$$

∴ Induced stress $\sigma_{in} = 27 \text{ N/mm}^2$

Since the induced stress is less than the allowable stress, the design is safe.

Number of teeth on pinion $z_1 = \dfrac{d_1}{m} = \dfrac{80}{4} = 20$

Number of teeth on gear $z_2 = \dfrac{d_2}{m} = \dfrac{280}{4} = 70$

Dynamic load :

Assume the drive uses carefully cut gears, from fig. (2.29), corresponding to 4 mm module, the expected error f is 0.025 mm. From table (2.35), for $f = 0.025$ mm, the value $C = 139.7$ kN/m = 139.7 N/mm

Dynamic load $$F_d = F_t + \frac{21 v_m (F_t + bC)}{21 v_m + \sqrt{F_t + bC}} \quad \text{..... (2.148a)}$$

$$= 994.8 + \frac{21 \times 5.026 (994.8 + 48 \times 139.7)}{21 \times 5.026 + \sqrt{994.8 + 48 \times 139.7}} = 5199.43 \text{ N}$$

Wear load :

Wear load $$F_w = \frac{d_1 b Q K}{\cos \delta_1} \quad \text{..... (2.441a)}$$

Formative number of teeth on pinion $z_{v1} = \dfrac{z_1}{\cos \delta_1} = \dfrac{20}{\cos 15.95} = 20.8$

Formative number of teeth on gear $z_{v2} = \dfrac{z_2}{\cos \delta_2} = \dfrac{70}{\cos 74.05} = 254.73$

Ratio factor $Q = \dfrac{2 z_{v2}}{z_{v2} + z_{v1}} = \dfrac{2 \times 254.73}{254.73 + 20.8} = 1.849$

From table (2.40), for 14½° cast iron gears, the load stress factor $K = 1.0487$

∴ Wear load $F_w = \dfrac{80 \times 48 \times 1.849 \times 1.0487}{\cos 15.95} = 7744.07 \text{ N}$

Since $F_w > F_d$, the design is safe.

Solution :

i) Dimension of worm and worm gear

$$\text{Velocity ratio } i = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{\pi d_2}{p_z}$$

$$\text{Speed of worm gear } n_2 = \frac{n_1}{i} = \frac{1000}{20} = 50 \text{ rpm}$$

$$\text{Pitch diameter of worm } d_1 = \frac{a^{0.875}}{3.5} \qquad \text{---- 2.519 b (DDHB)}$$

where a = Centre distance in meters = 0.2 m

$$\therefore\ d_1 = \frac{(0.2)^{0.875}}{3.5} = 0.069876 \text{ m} = 69.876 \text{ mm} \approx 70 \text{ mm}$$

$$\text{Centre distance } a = \frac{d_1 + d_2}{2}$$

$$\text{i.e., } 200 = \frac{70 + d_2}{2}$$

$\therefore$ Pitch diameter of worm gear $d_2 = 330$ mm, based on the first estimate

Also $d_1 \approx 3\,p_c$

i.e., $70 = 3\pi m$

$\therefore$ Axial module $m = 7.42 \text{ mm} = 8 \text{ mm}$ [select standard module from Table 2.3]

$$\text{Now } i = \frac{\pi d_2}{p_z} = \frac{\pi d_2}{\pi m_x z_1} = \frac{d_2}{m z_1}$$

$$\therefore\ d_2 = i m z_1 = 20 \times 8 \times z_1 = 160$$

Thus for various values of z_1 the value of d_2 is tabulated as below

Z_1	1	2	3	4
d_2 mm	160	320	480	640

Since 320 mm is closest to 330 mm, take the pitch circle diameter of worm gear $d_2 = 320$ mm

$\therefore$ Pitch diameter of worm $d_1 = 2a - d_2 = 2 \times 200 - 320 = 80$ mm

Number of starts on worm $z_1 = 2$

Number of teeth on worm wheel $z_2 = iz_1 = 20 \times 2 = 40$

Lead angle $\gamma = \tan^{-1}\dfrac{mz_1}{d_1} = \dfrac{8 \times 2}{80} = 11.31°$

Assume pressure angle $\alpha = 20°$ full depth involute.

Axial module $m = 8\,mm$

Normal module $m_n = m\cos\gamma = 8 \times \cos 11.31 = 7.8446\,mm$

Dimensions of worm

Number of starts on worm $z_1 = 2$

Pitch diameter of worm $d_1 = 80\,mm$

From Table 2.95 (DDHB)

Face length of worm $L_1 = (4.5 + 0.02\,z_1)\pi m$

$= (4.5 + 0.02 \times 2)\pi \times 8 = 114.1\,mm \approx 115\,mm$

Depth of tooth $h_1 = 0.686\pi m = 0.686 \times \pi \times 8 = 17.24\,mm$

Addendum $h_{a_1} = 0.318\pi m = 1\,m = 1 \times 8 = 8\,mm$

Outside diameter of worm $d_{a_1} = d_1 + 2h_{a_1} = 80 + 2 \times 8 = 96\,mm$

Dedendum $h_{f_1} = (2.2\cos\gamma - 1)\,m = (2.2\cos 11.31 - 1)\,8 = 9.26\,mm$

Root diameter of worm $d_{f_1} = d_1 - 2h_{f_1} = 80 - 2 \times 9.26 = 61.48\,mm$

Diametral quotient $q = \dfrac{d_1}{m} = \dfrac{80}{8} = 10$

Dimensions of worm wheel

Number of teeth on worm wheel $z_2 = 40$

Pitch circle diameter of worm wheel $d_2 = 320\,mm$

From Table 2.95 (DDHB)

Face width of worm wheel $b = 2.38\pi m + 6.25 = 66.1\,mm$

From Table 2.103 (DDHB)

$b \le 0.75\,d_1$; i.e., $b \le 0.75 \times 80 \;\therefore\; b \le 60\,mm$

$\therefore$ Take face width $b = 60\,mm$

Addendum $h_{a_2} = m(2\cos\gamma - 1) = 8\,[2\cos 11.31 - 1] = 7.7\,mm$

Outside diameter of worm wheel $d_{a_2} = d_2 + 2h_{a_2} = 320 + 2 \times 7.7 = 335.4\,mm$

Dedendum $h_{f_2} = m\,[1 + 0.2\cos\gamma] = 8\,[1 + 0.2\cos 11.31] = 9.6\,mm$

Root diameter of worm wheel $d_{f_2} = d_2 - 2h_{f_2} = 320 - 2 \times 9.6 = 300.8\,mm$

Checking

From Table 2.98 (DDHB) for $i = 20$ and centre distance $a = 200\,mm$.

The selected worm gear designation is 2/40/10/8

i.e., $z_1 = 2$; $z_2 = 40$; $q = 10$; $m = 8$

Therefore the design is satisfactory.

ii) Check the gear for gear tooth strength

From Lewis equation permissible transmitted load $F_{t_2} = \sigma_{02}\, b\, Y\, m_n\, C_v$

Assume hardened steel worm and phosphor bronze worm wheel

$\therefore$ From Table 2.106 (DDHB) for phosphor bronze worm wheel $\sigma_{02} = 55\ \text{N/mm}^2$

$$\text{Approximate } y_2 = 0.154 - \frac{0.912}{40} = 0.1312$$

$$Y = \pi y_2 = \pi \times 0.1312 = 0.4122$$

Mean pitch line velocity of worm wheel $v_m = \dfrac{\pi d_2 n_2}{60000} = \dfrac{\pi \times 320 \times 50}{60000} = 0.838\ \text{m/sec}$

Considering dynamic effect, velocity factor $C_v = \dfrac{6}{6 + v_m} = \dfrac{6}{6 + 0.838} = 0.8775$

$\therefore$ Permissible transmitted load $F_{t_2} = (55)(60)(0.4122)(7.8446)(0.8775) = 9363.5\ \text{N}$

$$\text{Transmitted load } F_{t_2} = \frac{9550 \times 1000 \times N}{n_2 r_2} = \frac{9550 \times 1000 \times 2}{50 \times \left(\dfrac{320}{2}\right)} = 2387.5\ \text{N}$$

$$\text{Estimated dynamic load } F_d = \frac{F_t}{C_v} = \frac{2387.5}{0.8775} = 2720.8\ \text{N}$$

Allowable wear load $F_w = d_2 b K$ → 23.572

From Table 2.111 (DDHB) for hardened steel worm and phosphor bronze worm wheel and $\gamma = 11.31°$

Load stress factor $K = 0.69\ \text{MPa} = 0.69\ \text{N/mm}^2$ $\therefore$ $F_w = (320)(60)(0.69) = 13248\ \text{N}$

As the allowable wear load is greater than the estimated dynamic load, and the allowable transmitted load is greater than the required transmitted load, the design is satisfactory from stand point of strength and wear of the gear teeth. In fact the permissible power is 7.844 kW.

$$\text{i.e., } 9363.5 = \frac{9550 \times 1000 \times N}{(50)\left(\dfrac{320}{2}\right)}$$

$\therefore$ Permissible power $N = 7.844\ \text{kW}$

iii) Efficiency

Considering worm as the driver

$$\eta = \frac{\tan\gamma\left[\cos\alpha_n \cos\gamma - \mu \sin\gamma\right]}{\cos\alpha_n \sin\gamma + \mu\cos\gamma} \quad \text{---- 2.587 (DDHB)}$$ → 23.604h

$$v_r = \text{Rubbing velocity} = \frac{\pi d_1 n_1}{60000\cos\gamma} \quad \text{---- 2.544 a (DDHB)}$$ → 23.571a

$$= \frac{\pi \times 80 \times 1000}{60000 \cos 11.31} = 4.272\ \text{m/sec}$$ → 23.507c

$$\mu = 0.025 + \frac{v_r}{305} \text{ since } 2.75 < v_r < 20\ \text{m/sec} \quad \text{---- 2.543 b (DDHB)}$$

$$= 0.025 + \frac{4.272}{305} = 0.039$$

$$\therefore \eta = \frac{\tan 11.31[\cos 20 \cos 11.31 - 0.039 \sin 11.31]}{\cos 20 \sin 11.31 + 0.039 \cos 11.31} = 0.8212 = 82.12\%$$

iv) Heat balance

a) Heat generated $H_g = \frac{\mu F_n v_r}{1000}$ kW → 23.603a ---- 2.576 a (DDHB)

$$F_n = \text{Normal force} = \frac{2Mt_2}{d_2 \cos\gamma \cos\alpha} = \frac{F_{t_2}}{\cos\gamma \cos\alpha}$$ ---- 2.546 (DDHB)

$$\therefore \text{Permissible normal force } F_n = \frac{9363.5}{\cos 11.31 \cos 20} = 10161.8 \text{ N}$$

$$\therefore \text{Permissible heat generated } H_g = \frac{(0.039)(10161.8)(4.272)}{1000} = 1.693 \text{ kW}$$

b) Heat dissipated $H_d = \frac{h_{cr} A (t_g - t_a)}{1000}$ kW → 23.603e ---- 5.77 (DDHB)

A = Radiating area of housing in $m^2 = 14.4\,a^{1.7}$

$= 14.4\,(0.2)^{1.7} = 0.9335\ m^2$

t_g = Temperature of gear = 65° C (assume) = 273 + 65 = 338° K

t_a = Ambient temperature = 25° C (assume) = 273 + 25 = 298° K

∴ Temperature difference = $t_g - t_a = 338 - 298 = 40°$ K

From Fig. 2.73 (DDHB) for $A = 0.9335\ m^2$

Coefficient of heat transfer h_{cr} = 320 W/m² K

$$\therefore \text{Heat dissipated } H_d = \frac{(320)(0.9335)(40)}{1000} = 11.95 \text{ kW}$$

Since heat generated is less than heat dissipation capacity, artificial cooling arrangement is not necessary.

6) worm gear design

30 and the center distance is 160 mm.

Data : $N = 2$ kW, $n_1 = 1200$ rpm, $i = 30$, $a = 160$ mm

Solution :

Speed of worm gear $n_2 = \frac{n_1}{i} = \frac{1200}{30} = 40$ rpm

From table (2.98), for 160 mm center distance and transmission ratio of 30, the values of $z_1 / z_2 / q / m = 1 / 30 / 10 / 8$

i.e., Number of threads on worm $z_1 = 1$

Number of teeth on worm gear $z_2 = 30$

Diametral quotient $q = \frac{d_1}{m} = 10$

Module $m = 8$ mm

$\therefore$ Pitch diameter of worm $d_1 = q\,m = 10 \times 8 = 80$ mm

Pitch diameter of worm gear $d_2 = m\,z_2 = 8 \times 30 = 240$ mm

Correct center distance $a = \frac{d_1 + d_2}{2} = \frac{80+240}{2} = 160$ mm

Speed ratio $$i = \frac{\pi d_2}{p_z} \qquad \text{..... (2.502)}$$

i.e., $$30 = \frac{\pi \times 240}{p_z}$$

$\therefore$ Lead $p_z = 25.13$ mm

Lead angle, $$\tan\gamma = \frac{p_z}{\pi d_1} \qquad \text{..... (2.527)}$$

$\therefore$ $$\gamma = \tan^{-1}\left(\frac{25.13}{\pi \times 80}\right) = 5.7^{\circ}$$

Torque on worm gear $$M_{t2} = \frac{9550\,N}{n_2}$$

$$= \frac{9550 \times 2}{40} = 477.5 \text{ N-m} = 477.5 \times 10^3 \text{ N-mm}$$

Tangential load on worm gear $$F_t = \frac{2\,M_{t2}}{d_2} = \frac{2 \times 477.5 \times 10^3}{240} = 3979.2 \text{ N}$$

Mean velocity $$v_m = \frac{\pi\, d_2\, n_2}{60 \times 1000} = \frac{\pi \times 240 \times 40}{60 \times 1000} = 0.5026 \text{ m/sec}$$

Velocity factor $$C_v = \frac{6}{6 + v_m} \qquad \text{..... (2.542 b)}$$

$$= \frac{6}{6 + 0.5026} = 0.923$$

Lewis equation for permissible load

Take the face width $b = 50$ mm

Dynamic load $F_d = C_v F_t$

$= 0.923 \times 3979.2 = 3672.8$ N

Wear load $F_w = d_2 \, b \, K$ ..... (2.557)

From table (2.111), corresponding to the materials steel-phospor bronze and the lead angle of 5.7°, the load stress factor $K = 0.414$ MPa $= 0.414$ N/mm².

∴ Wear load $F_w = 240 \times 50 \times 0.414 = 4968$ N

Since $F_w > F_d$, the design is safe

Length of worm $L_1 = (11 + 0.06\, z_2)\, m_x$ ..... Table (2.104)

$= (11 + 0.06 \times 30) \times 8 = 102.4$ mm

Efficiency :

Rubbing velocity
$$v_r = \frac{\pi d_1 n_1}{60 \times 1000 \times \cos\gamma} \quad \text{..... (2.544)}$$

$$= \frac{\pi \times 80 \times 1200}{60 \times 1000 \times \cos 5.7} = 5.05 \text{ m/sec}$$

Coefficient of friction
$$\mu = 0.025 + \frac{v_r}{305} \quad \text{..... (2.543 b)}$$

$$= 0.025 + \frac{5.05}{305} = 0.0416$$

Efficiency
$$\eta = \frac{\tan\gamma\,(\cos\alpha_n \cos\gamma - \mu \sin\gamma)}{\cos\alpha_n \sin\gamma + \mu \cos\gamma} \quad \text{..... (2.587)}$$

$$= \frac{\tan 5.7\,(\cos 20 \cos 5.7 - 0.0416 \times \sin 5.7)}{\cos 20 \sin 5.7 + 0.0416 \times \cos 5.7} \quad (\text{let } \alpha_n \simeq \alpha)$$

$$= 0.6897 = 68.97\ \%$$

Thermal capacity :

Normal force
$$F_n = \frac{2\, M_{t2}}{d_2 \cos\gamma \cos\alpha} \quad \text{..... (2.546)}$$

$$= \frac{2 \times 477.5 \times 10^3}{240 \times \cos 5.7 \cos 20} = 4255.6 \text{ N}$$

Heat generated
$$H_g = \frac{\mu F_n v_r}{1000} \quad \text{..... (2.576a)}$$

$$= \frac{0.0416 \times 4255.6 \times 5.05}{1000} = 0.894 \text{ kW}$$

Module: 4

7a) $M_t = 286 \times 10^3$ Nmm $n = 600$ rpm

$D_2 = 250$ mm ; $b = 60$ mm $\mu = 0.18$

From $M_t = \dfrac{\mu F_a D_m}{2 \sin\alpha}$; $\alpha = 14^\circ$ (Face angle)

$D_m = D_2 - b \sin\alpha$

$= 250 - 60 \sin 14^\circ$

$D_m = 235.48$ mm

$286 \times 10^3 = \dfrac{0.18 \times F_a \times 235.48}{2 \times \sin 14^\circ}$

$F_a = 3264.71$ N Ans

Also $F_a = \pi D_m p b \sin\alpha$

$3264.71 = \pi \times 235.48 \times p \times 60 \sin 14^\circ$

$p = 0.03$ N/mm^2 ← Avg Normal pressure.

Max^m pressure occur at inner diameter

$D_1 = D_m - b \sin\alpha = 220.96$ mm

$F_a = \pi D_1 P_{max} b \sin\alpha$

$\Rightarrow P_{max} = 0.0324$ N/mm^2 ← P_{max}

7b)

Data : $i = 2$, $D_2 = 0.3$ m = 300 mm, $D_1 = 0.16$ m = 160 mm, $\mu = 0.2$, $n = 1000$ rpm,
$p = 0.08$ MPa = 0.08 N/mm²

Uniform wear :

Mean diameter $$D_m = \frac{D_2 + D_1}{2} = \frac{300+160}{2} = 230 \text{ mm} \quad \text{..... (19.85}b\text{)}$$

Axial force $$F_a = \frac{1}{2}\pi p D_1 (D_2 - D_1) \quad \text{..... (19.83)}$$

$$= \frac{1}{2} \times \pi \times 0.08 \times 160\,(300 - 160) = 2814.87 \text{ N}$$

Torque transmitted $$M_t = i \times \frac{1}{2}\mu F_a D_m \quad \text{..... (19.84)}$$

$$= 2 \times \frac{1}{2} \times 0.2 \times 2814.87 \times 230 = 129484 \text{ N-mm} = 129.484 \text{ N-m}$$

Power transmitted $$N = \frac{M_t n}{9550} = \frac{129.484 \times 1000}{9550} = 13.56 \text{ kW} \quad \text{..... (19.3}c\text{)}$$

Uniform Pressure :

Mean diameter $$D_m = \frac{2}{3}\left(\frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right) \quad \text{..... (19.85}a\text{)}$$

$$= \frac{2}{3}\left(\frac{300^3 - 160^3}{300^2 - 160^2}\right) = 237.1 \text{ mm}$$

Axial load $$F_a = \frac{\pi p (D_2^2 - D_1^2)}{4} \quad \text{..... (19.86}a\text{)}$$

$$= \frac{\pi \times 0.08(300^2 - 160^2)}{4} = 4046.4 \text{ N}$$

8 a)

(J

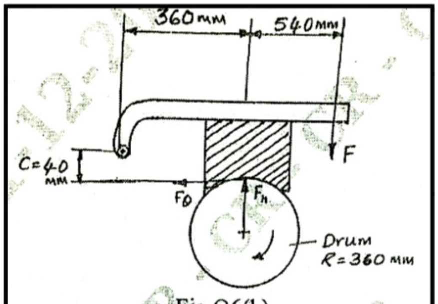


Fig.Q6(b)

Given data:

a = 360 mm; b = 540 mm; c = 40 mm; r = 360 mm;

n = 1000 rpm; N = 23.5 kW

Solution:

Torque transmitted ,

$$M_t = 9550 \text{ X } 1000 \text{ X } \frac{N}{n} \quad \text{N-mm}$$

$$= 9550 \text{ X } 1000 \text{ X } \frac{23.5}{1000}$$

$$= 224425 \text{ N-mm}$$

i) Normal force on the shoe (F_n)

$$M_t = F_\theta \text{ X r}$$

$$224425 = F_\theta \text{ X } 360$$

$$F_\theta = 623.40$$

Again, $F_\theta = \mu F_n$ (Assume $\mu = 0.25$ and $2\theta < 60^o$)

$$623.40 = 0.25 \text{ X } F_n$$

$$F_n = \mathbf{2493.60\ N}$$

ii) Tangential force (F_θ)

From previous step,

$$F_\theta = \mathbf{623.40N}$$

iii) Operating force for CW rotation

From table 19.4, Equation 19.147

Applied force $F = F_\theta.(\frac{a}{(a+b)})(\frac{1}{\mu}+\frac{c}{a})$

$$F = 623.40\ (\frac{360}{(360+540)})(\frac{1}{0.25}+\frac{40}{360})$$

F =1025.15 N

iv) Operating force for CCW rotation

From table 19.4, Equation 19.148

Applied force $F = F_\theta.(\frac{a}{(a+b)})(\frac{1}{\mu}-\frac{c}{a})$

$$F = 623.40\ (\frac{360}{(360+540)})(\frac{1}{0.25}-\frac{40}{360})$$

F =969.73 N

v) The value of ‘c’ for the brake to be self locking

for self locking, $\frac{a}{c} \leq \mu$

$$\frac{360}{c} \leq 0.25$$

$$c \geq \frac{360}{0.25}$$

c ≥ 1440 mm

8b)

Data : $a = 440$ mm, $b = 50$ mm, $D = 1$ m $= 1000$ mm, $D_b = 0.65$ m $= 650$ mm, $\theta = 300°$, $\mu = 0.35$, $W = 20$ kN $= 20 \times 10^3$ N, $\sigma_d = 50$ N/mm^2

Solution :

Torque on hoisting drum $M_t = W R_b = \frac{W D_b}{2}$

$$= \frac{20 \times 10^3 \times 650}{2} = 6.5 \times 10^6 \text{ N-mm}$$

The hoisting drum and the brake drum are mounted on same shaft.

$\therefore$ Torque on brake drum $M_t = 6.5 \times 10^6$ N-mm

Braking force $F_\theta = \frac{M_t}{R} = \frac{2 M_t}{D}$

$$= \frac{2 \times 6.5 \times 10^6}{1000} = 13000 \text{ N}$$

Ratio of tensions $\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.35 \times 300 \times \pi/180} = 6.25$

Clockwise direction :

Force at the end of lever $F = \frac{F_\theta b}{a}\left[\frac{1}{e^{\mu\theta} - 1}\right]$ ..... (19.151)

$$= \frac{13000 \times 50}{440}\left[\frac{1}{6.25 - 1}\right] = 281.38 \text{ N}$$

Counter clockwise rotation :

Force $F = \frac{F_\theta b}{a}\left[\frac{e^{\mu\theta}}{e^{\mu\theta} - 1}\right]$ ..... (19.152)

$$= \frac{13000 \times 50}{440}\left[\frac{6.25}{6.25 - 1}\right] = 1758.66 \text{ N}$$

Thickness of band $h = 0.005\, D$ ..... (19.160)

$= 0.005 \times 1000 = 5$ mm

Braking force $F_\theta = T_1 - T_2 = T_1 - \frac{T_1}{e^{\mu\theta}} = T_1\left(1 - \frac{1}{e^{\mu\theta}}\right)$

i.e., $13000 = T_1\left(1 - \frac{1}{6.25}\right)$

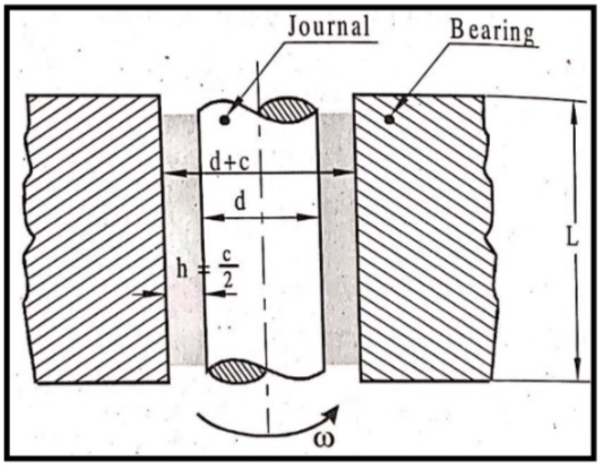
$\therefore$ Tight side tension $T_1 = 15476.2$ N

width of band $w = \frac{T_1}{h\sigma_d} = \frac{15476.2}{5 \times 50} = 62$ mm

9a)

1. Petroff's Equation

Petroff's equation is used to find the coefficient of friction in journal bearings.



Consider a vertical shaft rotating in a guide bearing as shown in figure.
Assumptions:

1. The bearing is lightly loaded.
2. The clearance 'c' is completely filled with oil.
3. There is no end leakage.
4. Viscosity of oil used is very high.
5. The journal rotates at very high speed.
6. There is no eccentricity between the journal and bearing.

Let

d = diameter of journal or shaft
c = diametral clearance
n' = Speed of the journal or shaft in rev/sec

$$n' = \frac{n}{60}$$

L = Length of the bearing

$$\psi = \frac{c}{d} = \text{diametral clearance ratio}$$

η = Viscosity of oil , Pas
v = velocity = $\pi d n'$, m/s

shear stress, $\tau = \eta \,.\, \frac{v}{h} = \eta \,.\, \frac{\pi d n'}{\frac{c}{2}} = \frac{2\pi d\eta \,.\, n'}{c}$

Surface area A = πdL

Therefore,

Force, $F = \tau A = \frac{2\pi^2 d^2 n' \eta \,.\, L}{c}$

Torque, M_t = F x r

$= F.(\frac{d}{2})$

$= \frac{\pi^2 d^2 n' \eta \,.\, L}{(\frac{c}{d})}$

$$M_t = \frac{\pi^2 d^2 n' \eta \,.\, L}{\psi} \text{ ——————————————> (a)}$$

But, $M_t = (\mu \,.\, W)(\frac{d}{2})$ $\qquad$ $((\mu \,.\, W)$ = Frictional Force)

and W = PA = P.L.d = Load

where P = Bearing Pressure in Pa

$$M_t = (\mu \,.\, PL \,.\, d)(\frac{d}{2}) \text{ ——————————————> (b)}$$

Equating (a) and (b)

$$(\mu \,.\, PL \,.\, d)(\frac{d}{2}) = \frac{\pi^2 d^2 n' \eta \,.\, L}{\psi}$$

$$\mu = 2\pi^2 (\frac{\eta \,.\, n'}{P}) \,.\, \frac{1}{\psi}$$

The above equation is Petroff's Equation

9b)

$d = 45mm = 0.045m$

$L = 66mm = 0.066m$

$\psi = 0.0015$; $n = 2800$ rpm ; $W = 800N$

$\eta = 8.27 \times 10^{-3}$ Pa·s

Sol

$$p = \frac{W}{L \cdot d} = \frac{800}{0.066 \times 0.045} = 2[illegible]8.36 \times 10^{3} \, N/m^2$$

$$\mu = 2\pi^2 \left(\frac{\eta n'}{p}\right) \frac{1}{\psi} = 2\pi^2 \times \left(\frac{8.27 \times 10^{-3} \times 2800}{60 \times 268.36 \times 10^{3}}\right) \times \frac{1}{0.0015}$$

$$\mu = 2\pi^2 \times \frac{8.27 \times 10^{-3} \times 46.67 \times 0.066 \times 0.045}{800 \times 0.0015}$$

$$\mu = 0.0188$$

Frictional Torque $= \mu W \times \frac{d}{2}$

$= 0.0188 \times 800 \times \frac{0.045}{2}$ Nm

$= 0.339$ Nm

Power loss $= \mu W V = 0.0022$ kW

10a) $d = 50\,mm = 0.05\,m$ $p = 1.4\,N/mm^2$

$L = 100\,mm = 0.1\,m.$ $n = 900\,rpm.$

$t_0 = 75^\circ C$; $t_a = 35^\circ C$ $\frac{d}{c} = 1000 \Rightarrow \psi = \frac{c}{d} = 1\times10^{-3}$

$\eta = 0.011\,kg/ms = 0.011\,Pa.s$

To Find (i) Amount of artificial cooling

(ii) m' of oil.

Using McKee equation

$$\mu = k_a\left(\frac{\eta n'}{p}\right)\left(\frac{1}{\psi}\right)10^{-10} + \Delta\mu$$

$$= 1.95\times10^{11} \times \left(\frac{0.011\times900}{1.4\times60\times10^{6}}\right)\left(\frac{1}{0.001}\right)10^{-10} + 0.002$$

$$\mu = 0.00429$$

Heat generated $H_g = \mu W v = \mu(pLd)v$

$$= 0.00429\times1.4\times10^{6}\times0.1\times0.05\times\frac{\pi\times0.05\times900}{60}$$

$$H_g = 70.75\text{ Watts}$$

Heat dissipated $H_d = \frac{(\Delta T + 18)^2 L \cdot d}{K'}$

$K' = 0.475$ for bearing located in still air

$\Delta T = t_b - t_a = \frac{t_0 - t_a}{2} = \frac{75 - 35}{2} = 20°C$

$H_d = \frac{(20+18)^2 \times 0.1 \times 0.05}{0.475}$

$= 15.20$ W

Amount of artificial cooling $= H = H_g - H_d$

$= (70.75 - 15.20)$

$H = 55.55$ W

Mass flow rate of oil

$55.55 = \gamma C_{sp} Q \Delta T$

$= 8.83 \times 10^3 \times 0.19 \times 10^3 \times Q \times 20$

$\Rightarrow Q = \frac{55.55}{8.83 \times 10^3 \times 0.19 \times 10^3 \times 20}$

$Q = 1.655 \times 10^{-6}$ m^3/sec ← Mass flow rate of oil.