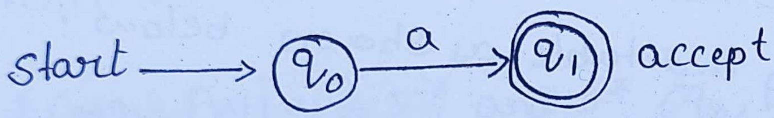
DFA

1 Draw a DFA to accept string of a's having at least one a

Step 1 :- $\Sigma = \{a\}$ Minimum string = "a" with one Symbol. Since it has to accept one Symbol.

It requires two states q_0 and q_1 and the initial DFA is shown below:



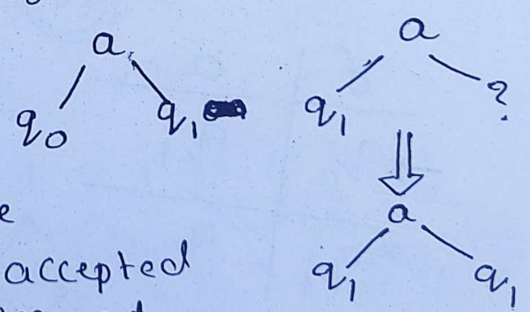
$\Sigma = \{a\}$

Step 2 : Identify the transition not defined in Step 1.

$\delta(q_1, a) = ?$

Go from q_0 to q_1 on a

we need to find



The Sequence aa should be accepted Since it is having at least one a. So, go to final state q_1

Step 3: The DFA can be obtained by initial DFA and transitions obtained from previous step.

The DFA is defined as:

$$M = (Q, \Sigma, \delta, q_0, F)$$

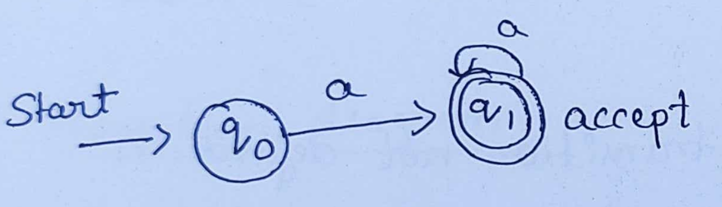
where $Q = \{q_0, q_1\}$

$\Sigma = \{a\}$

q_0 is the start state

$F = \{q_1\}$

δ is shown below using the transition diagram and table as shown below:



Transition diagram

δ	a
$\rightarrow q_0$	q_1
$*q_1$	q_1

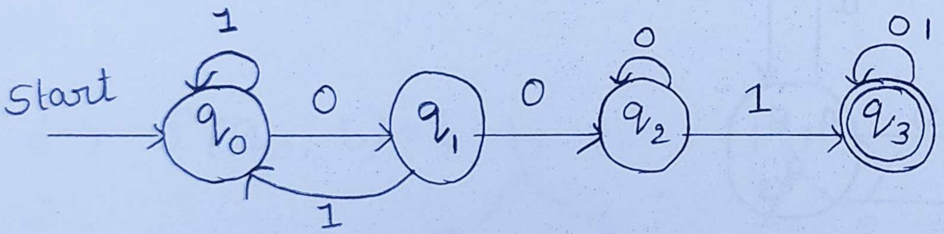
2. Define DFSM. Design a DFSM to accept each of the following languages:

(i) $L = \{W \in \{0,1\}^* :$ w has 001 as a substring$\}$

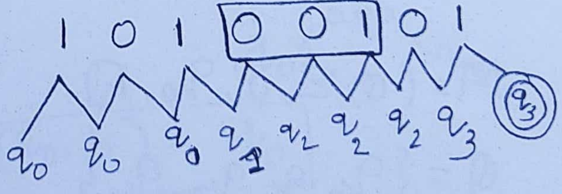
Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA. A string w is accepted by the machine M, if it takes the machine from initial state q_0 to final state i.e $\delta^*(q_0, w)$ is in F. Thus the language accepted by DFA represented as L(M) can be formally written as:

$$L(M) = \{W \mid W \in \Sigma^* \text{ and } \delta^*(q_0, w) \text{ is in } F\}$$

Transition diagram



δ	a(0)	b(1)
$\rightarrow q_0$	q_1	q_0
q_1	q_2	q_0
q_2	q_2	q_3
$*q_3$	q_3	q_3



$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

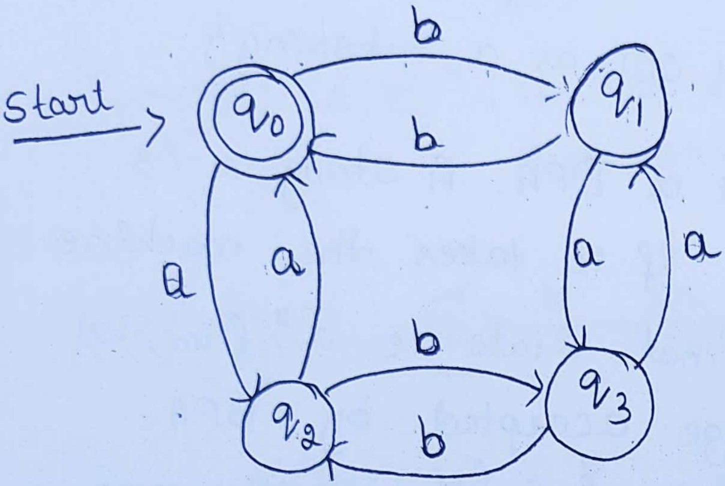
$\Sigma = \{0, 1\}$

$\delta = Q \times \Sigma = Q$

q_0 = Start state

F = Final state

ii: L = {W ∈ {0,1}* : W has Even number of a's and Even number of b's }



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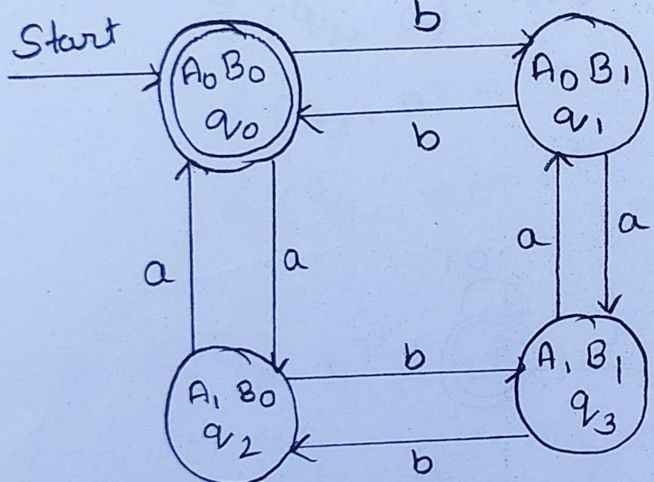


$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a\ b\}$

$\delta = Q \times \Sigma = Q$

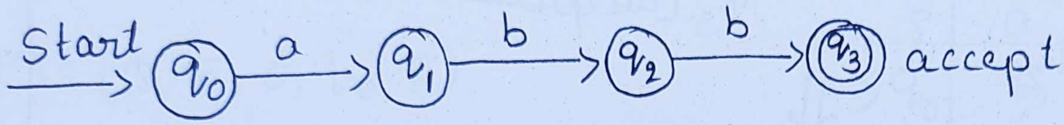
$q_0 =$ Start State

$F = \{q_0\}$

3 Draw to accept string of a's and b's ending with the string abb

(or)

$L = \{w \in \{a,b\}^* ;$ w has all strings that ends with sub string abb$\}$



$\delta(q_0, b) = ?$ $= (i)$

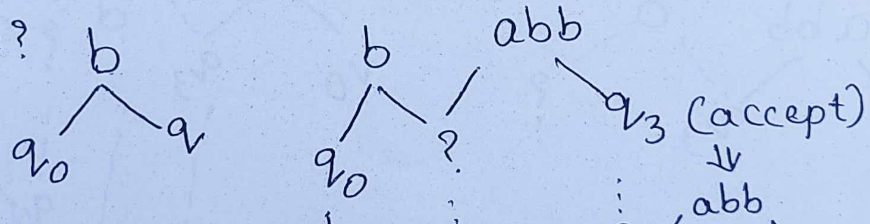
$\delta(q_1, a) = ?$ $= (ii)$

$\delta(q_2, a) = ?$ $= (iii)$

$\delta(q_3, a) = ?$ $= (iv)$

$\delta(q_3, b) = ?$ $= (v)$

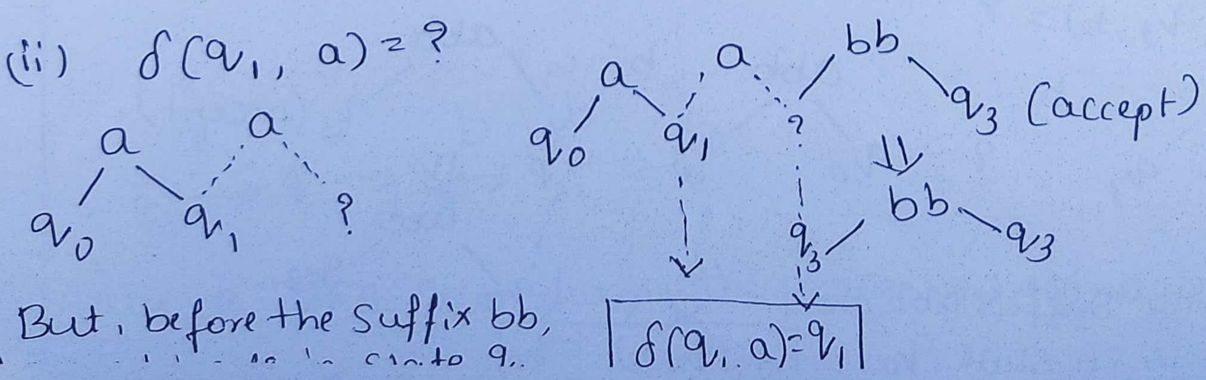
(i) $\delta(q_0, b) = ?$



But, before the suffix abb, the machine is in state q_0

$\boxed{\delta(q_0, b) = q_0}$

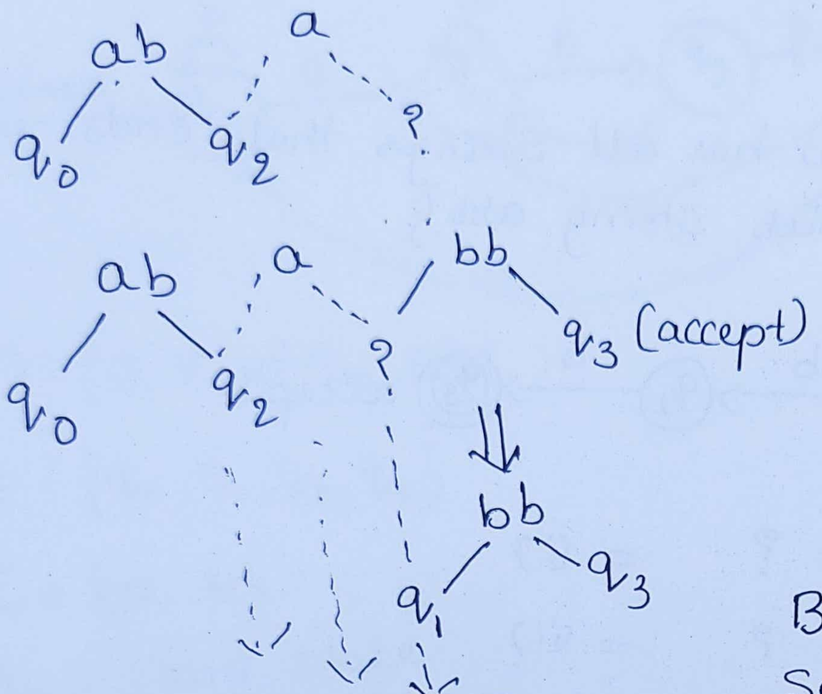
(ii) $\delta(q_1, a) = ?$



But, before the suffix bb,

$\boxed{\delta(q_1, a) = q_1}$

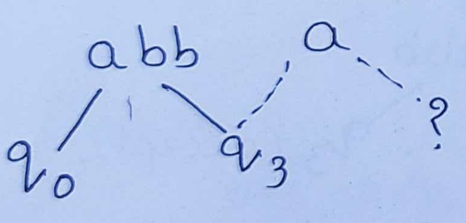
(iii) $\delta(q_2, a) = ?$

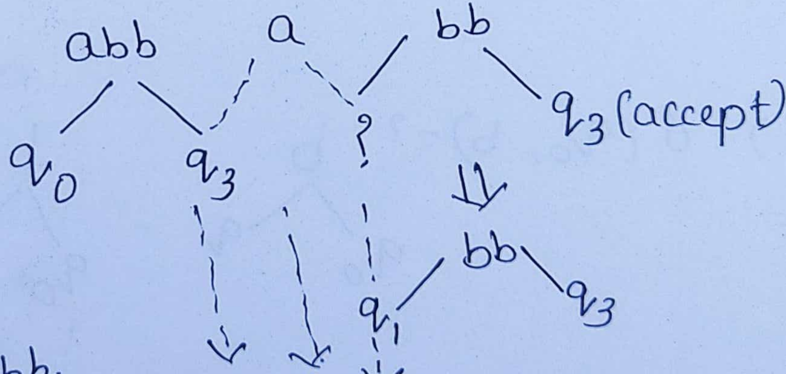


But, before the suffix bb, the machine is in state q_1

$\boxed{\delta(q_2, a) = q_1}$

(iv) $\delta(q_3, a) = ?$

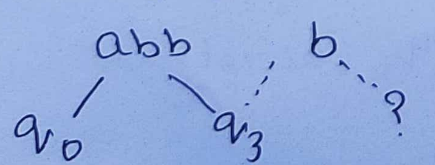


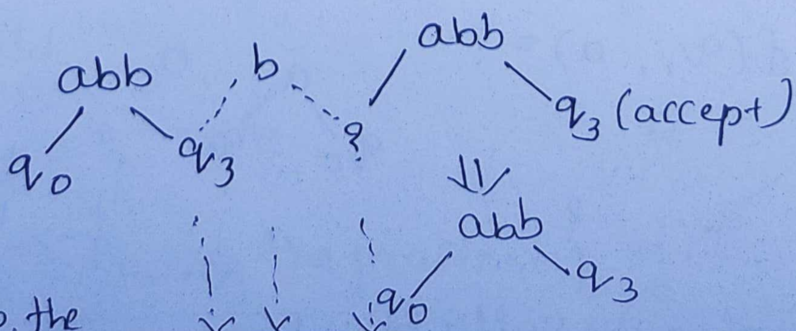


But, before the suffix bb, the machine is in state q_1

$\boxed{\delta(q_3, a) = q_1}$

(v) $\delta(q_3, b) = ?$

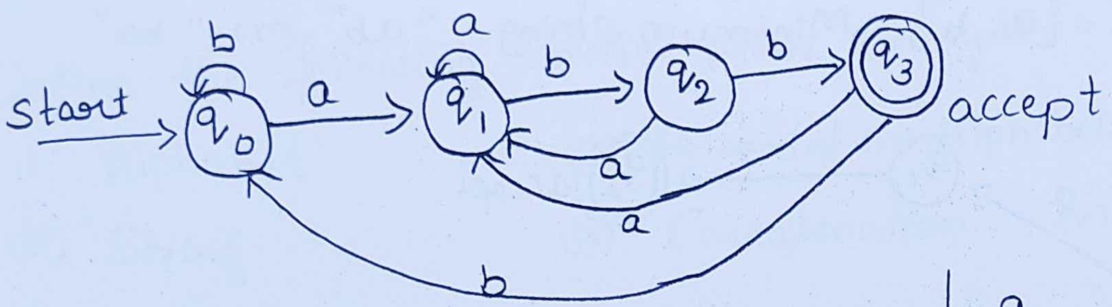




before the suffix abb, the machine is in state q_0

$\boxed{\delta(q_3, b) = q_0}$

to accept strings of a's and b's Ending with abb



$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$q_0 =$ start state

$F = \{q_3\}$

	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
$* q_3$	q_1	q_0

Input string:

a a b b

$\delta(q_0, a) = q_1$

$(q_1, a) = q_1$

$\delta(q_1, b) = q_2$

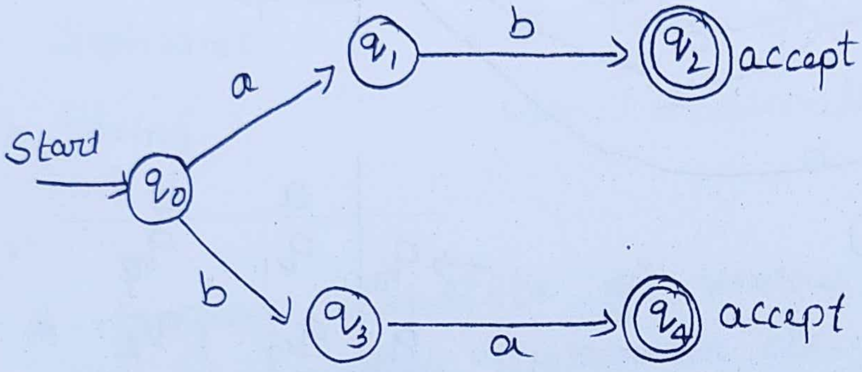
$\delta(q_2\ b) = q_3$

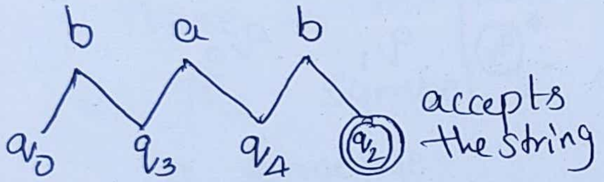
$q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{b} q_3$

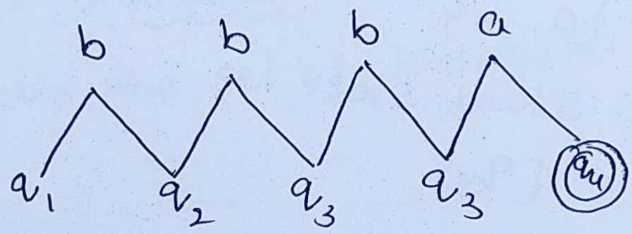
States a DFA is in during the processing of aabb

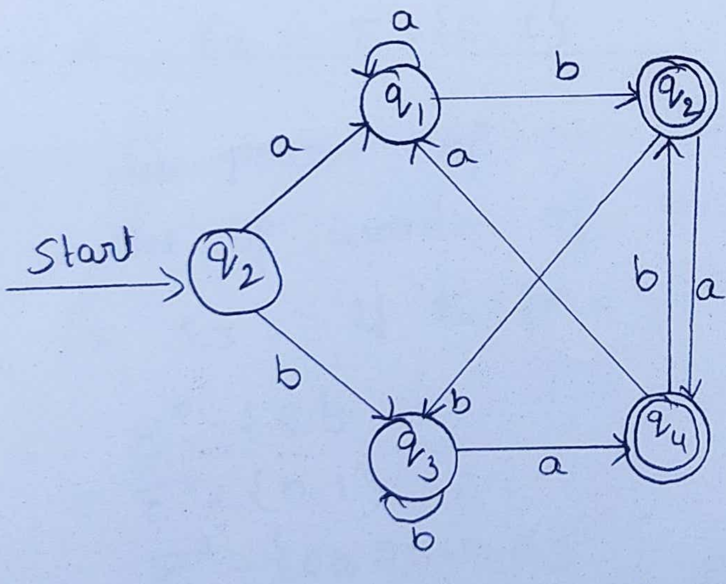
5. To accept $L = \{W(ab+ba) \mid W \in \{a,b\}^*\}$

Here, $\Sigma = \{a, b\}$ Minimum string = "ab" (or) "ba"









	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_1	q_2
q_2	q_4	q_3
q_3	q_4	q_3
$* q_4$	q_1	q_2

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$M = \{Q, \Sigma, \delta, q_0, F\}$.

$\delta : Q \times \Sigma = Q$

q_0 : Start state

$F = \{q_2, q_4\}$

Automata Theory and Computability

1 Define the following terms with Example:
(i) Alphabet (ii) Power of an Alphabet
(iii) String (iv) Concatenation (v) Languages

Ans:-
(i) * A language consists of various symbols from which the words, statements etc... can be obtained These symbols are called "Alphabets".
* The symbol Σ denotes the set of Alphabets of a language.
* Ex :- $\Sigma = \{0, 1\}$

(ii) * The power of an alphabet denoted by Σ^i ~~is th~~ set of words of length i.
* Ex: if $\Sigma = \{0,1\}$ then

$\Sigma^0 = \{\varepsilon\}$ .... is set of words length 0
$\Sigma^1 = \{0,1\}$ .... — 11 — 1
$\Sigma^2 = \{00, 01, 10, 11\}$ .... — 11 — 2
$\Sigma^3 = \{000, 001, 010, 100, 011, 101, 110, 111\}$ ... — 11 — 3.

(iii) * The sequence of symbols obtained form the alphabets of a Language is called a String
$\Sigma = \{0,1\}$ is set of alphabets.

The various strings that can be obtained from Σ
{0, 1, 00, 01, 10, 11, 010101, 1010, ...}

(iv) * The Concatenation of two strings u and v is the string obtained by writing the letters of string u followed by the letters of string v (i.e, appending the symbols of v to the right of u) i.e,
if $u = a_1 a_2 a_3 \ldots a_n$ and $v = b_1 b_2 b_3 \ldots b_m$, then the Concatenation of u and v is denoted by

$$uv = a_1 a_2 a_3 \ldots a_n b_1 b_2 b_3 \ldots b_m$$

* Ex :- Let u = " Computer" and v = " Science" The Concatenation of u and v denoted by uv is shown below:

uv = Computer Science.

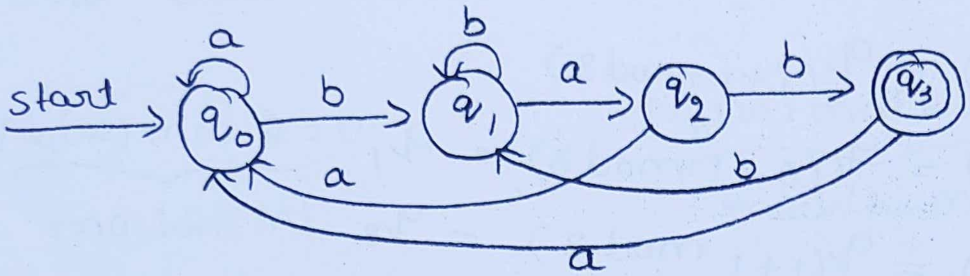
(v) * A Language can be defined as a set of strings obtained from Σ^* where Σ is set of alphabets of a particular language.
Formally, a language L over Σ is subset of Σ^* which is denoted by $L \subseteq \Sigma^*$

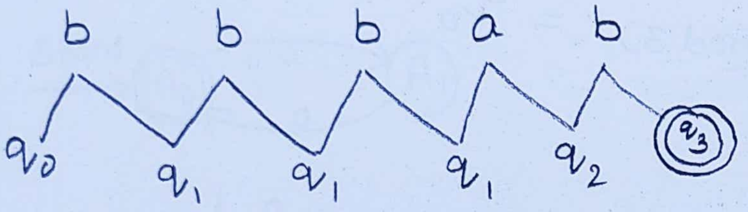
* Ex :- A language of strings consisting of equal number of 0's and 1's can be represented as

$$\Sigma = \{\epsilon, 01, 10, 0011, 1010, 0101, 0011, \ldots\}$$

* An Empty language is denoted by ϕ

6 To accept $L = \{wbab \mid w \in \{a, b\}\}$





$Q = \{q_0, q_1, q_2, q_3$

$M = \{Q, \Sigma, \delta, q_0, F\}$

$\delta = Q \times \Sigma = Q$

$q_0 =$ Start State

$F = \{q_3\}$

	a	b
$\to q_0$	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_3
$*q_3$	q_0	q_1

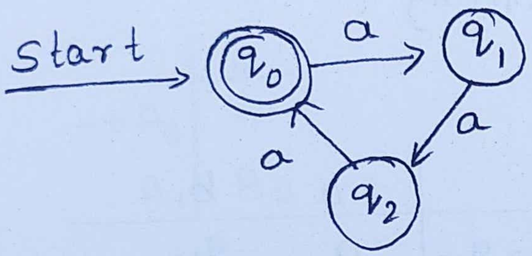
7 Obtain a DFA to accept $L = \{W : |W| \bmod 3 = 0\}$ where $\Sigma = \{a\}$

$L = \{W$ where $|W| \bmod 3 = 0\}$ with q_0 as a Start State and q_0 as final State

The transitions can be obtained using the relation "$\delta(q_i, a) = q_{(i+1 \bmod k)}$"

where $k = 3$ (divisor) and $i = 0, 1, 2$ (remainders after dividing by 3)

i	$\delta(q_i, a) = q_{(i+1 \bmod 3)}$
0	$\delta(q_0, a) = q_{(0+1 \bmod 3)} = q_1$
1	$\delta(q_1, a) = q_{(1+1 \bmod 3)} = q_2$
2	$\delta(q_2, a) = q_{(2+1 \bmod 3)} = q_0$



The language accepted by above DFA can also be written as:

$L = \{w : |w| \bmod 3 = 0\}$ where $\Sigma = \{a\}$

or

$L = \{w : n_a(w)$ are divisible by $3\}$ where $\Sigma = \{a\}$

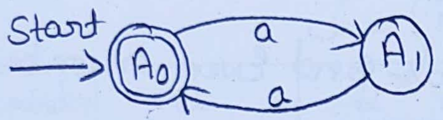
or

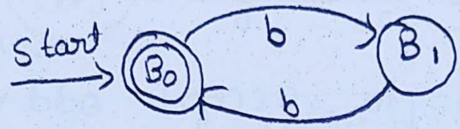
$L = \{a^{3n} : n \geq 0\}$

8 To accept strings having Even number of a's and odd number b's

$N_a(w) \bmod 2 = 0$ — machine M1

$N_b(w) \bmod 2 = 0$ — machine M2





	a
$\rightarrow A_0$	A_1
A_1	A_0

	b
$\rightarrow B_0$	B_1
B_1	B_0

The states Q of combined machine M that accepts a given string w can be obtained by taking the cross product of Q1 and Q2 as shown below:

$Q = Q1 \times Q2 = \{ (A_0, B_0), (A_0, B_1)$ ⇒ // horizontal transition for input symbol b

$(A_1, B_0), (A_1, B_1)$

$\}$ ⇓

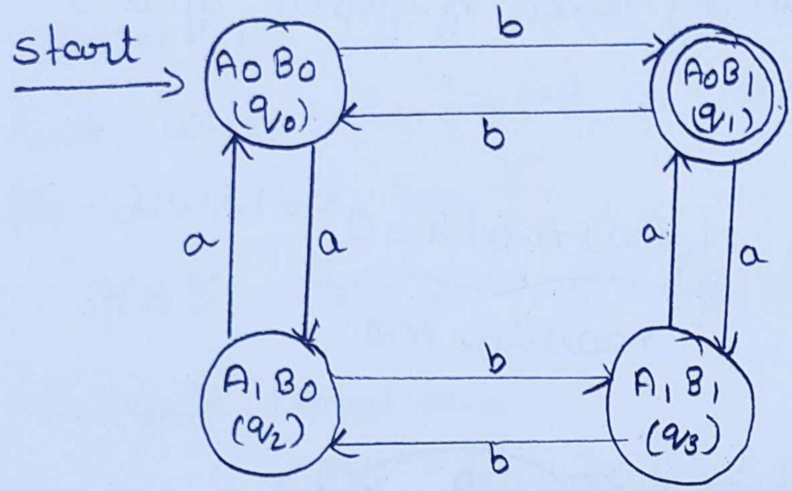
// vertical transition for input symbol a

$M = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

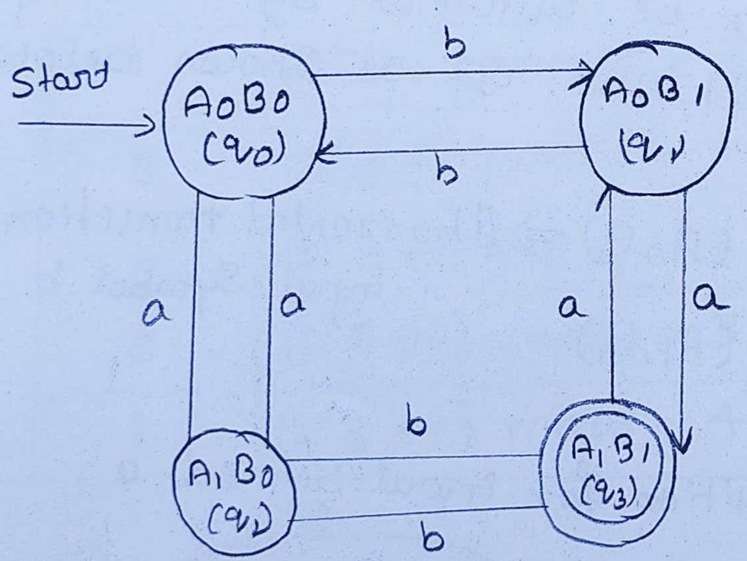
$\delta = Q \times \Sigma = Q$



The DFA to accept odd number of b's and Even number of a's

$F = \{q_1\}$

(i) DFA to accept odd number of a's and odd number of b's



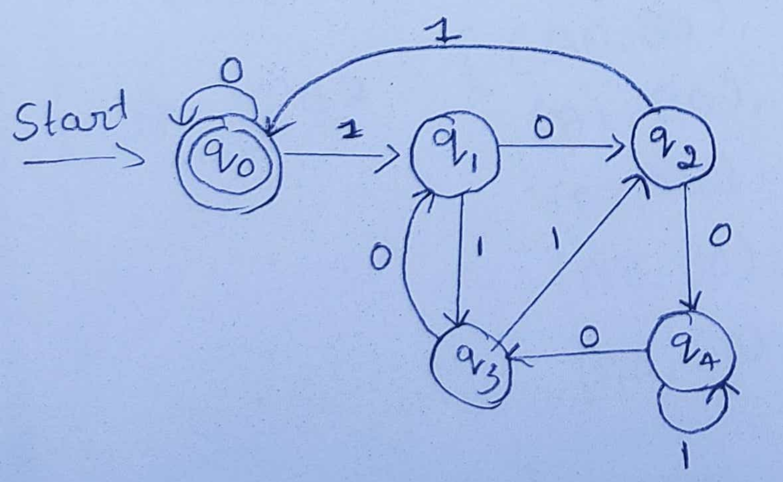
Obtain a DFA that accepts set of all strings that, when interpreted in reverse as a binary integer, is divisible by 5.

$K=5 \quad r=2 \quad d=\{0,1\} \quad i=0,1,2,3,4$

Compute transitions

$j=(Ri+d)$ module K

Remainder	d	2* i+d mod 5	$\delta(q_i,d)=q_j$
i=0	0	$(2*0+0) \bmod 5 = 0$	$\delta(q_0,0)=q_0$
	1	$(2*0+1) \bmod 5 = 1$	$\delta(q_0,1)=q_1$
i=1	0	$(2*1+0) \bmod 5 = 2$	$\delta(q_1,0)=q_2$
	1	$(2*1+1) \bmod 5 = 3$	$\delta(q_1,1)=q_3$
i=2	0	$(2*2+0) \bmod 5 = 4$	$\delta(q_2,0)=q_4$
	1	$(2*2+1) \bmod 5 = 0$	$\delta(q_2,1)=q_0$
i=3	0	$(2*3+0) \bmod 5 = 1$	$\delta(q_3,0)=q_1$
	1	$(2*3+1) \bmod 5 = 2$	$\delta(q_3,1)=q_2$
i=4	0	$(2*4+0) \bmod 5 = 3$	$\delta(q_4,0)=q_3$
	1	$(2*4+1) \bmod 5 = 4$	$\delta(q_4,1)=q_4$



	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

(ii) Construct a DFA which accept decimal Strings divisible by 3

$r=10 \quad K=3 \quad d=\{0,1,2,3,4,5,6,7,8,9\} \quad i=0,1,2$

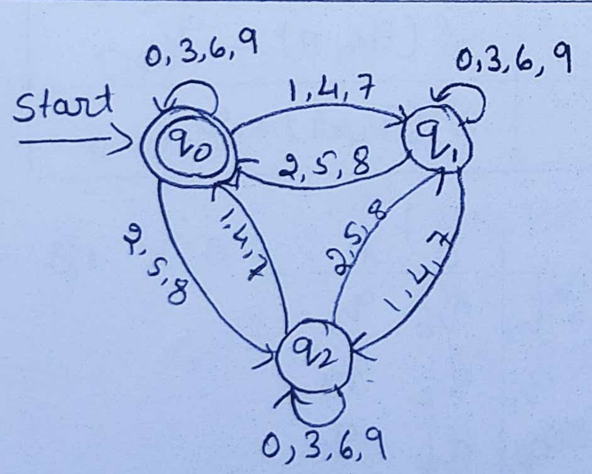
$\delta=\{0,3,6,9\}=\{0\}$

$\delta=\{1,4,7\}=\{1\}$

$\delta=\{2,5,8\}=\{2\}$

remainder	d	$(r^* i+d) \bmod 3 = i$	$\delta(q_i,d)=q_j$	
$i=0$	0	$(10^*0+0) \bmod 3 = 0$	$\delta(q_0,0)=q_0$	$\Rightarrow \delta(q_0,\{0,3,6,9\})=q_0$
	1	$(10^*0+1) \bmod 3 = 1$	$\delta(q_0,1)=q_1$	$\Rightarrow \delta(q_0,\{1,4,7\})=q_1$
	2	$(10^*0+2) \bmod 3 = 2$	$\delta(q_0,2)=q_2$	$\Rightarrow \delta(q_0,\{2,5,8\})=q_2$
$i=1$	0	$(10^*1+0) \bmod 3 = 1$	$\delta(q_1,0)=q_1$	$\Rightarrow \delta(q_1,\{0,3,6,9\})=q_1$
	1	$(10^*1+1) \bmod 3 = 2$	$\delta(q_1,1)=q_2$	$\Rightarrow \delta(q_1,\{1,4,7\})=q_2$
	2	$(10^*1+2) \bmod 3 = 0$	$\delta(q_1,2)=q_0$	$\Rightarrow \delta(q_1,\{2,5,8\})=q_0$
$i=2$	0	$(10^*2+0) \bmod 3 = 2$	$\delta(q_2,0)=q_2$	$\Rightarrow \delta(q_2,\{0,3,6,9\})=q_2$
	1	$(10^*2+1) \bmod 3 = 0$	$\delta(q_2,1)=q_0$	$\Rightarrow \delta(q_2,\{1,4,7\})=q_0$
	2	$(10^*2+2) \bmod 3 = 1$	$\delta(q_2,2)=q_1$	$\Rightarrow \delta(q_2,\{2,5,8\})=q_1$

Transitions of DFA



10 Obtain a DFA to accept strings of a's and b's such that the number of a's is divisible by 5 and number of b's is divisible by 3.

$L = \{ W | W \in (a+b)^* \; N_a(w) \bmod 5 = 0 \text{ and } N_b(w) \bmod 3 = 0 \}$

The $N_a(w)$ mod 5 gives the remainder after dividing number of a's by 5. The possible remainders are $\{0, 1, 2, 3, 4\}$ and can be represented as:

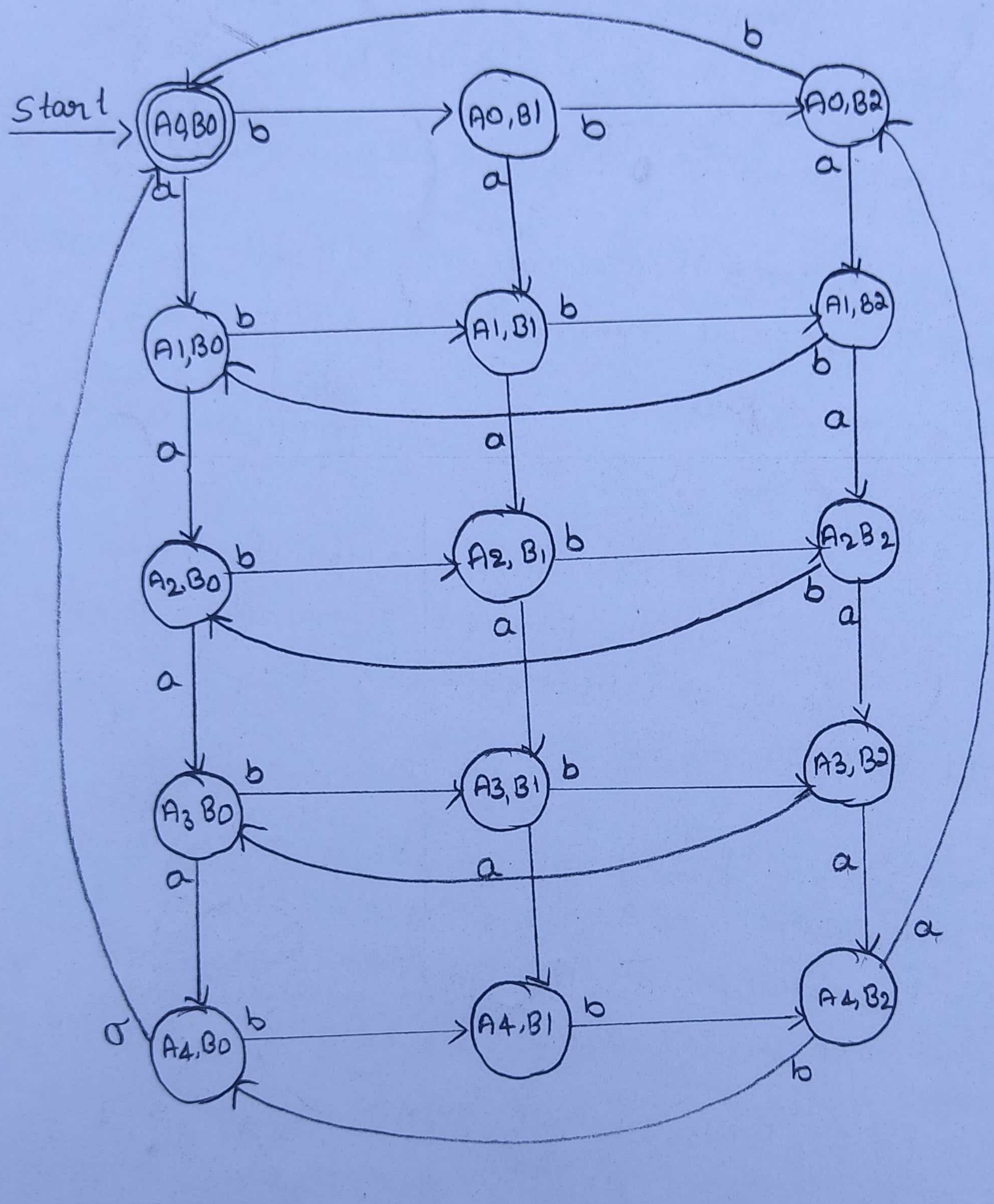
$Q = \{A0, A1, A2, A3, A4\}$ — (1)

$N_b(w)$ mod 3 gives the remainder after dividing number of b's by 3 The possible remainders are $\{0, 1, 2\}$ and can be represented as:

$Q_2 = \{B0, B1, B2\}$ — (2)

Since each state of DFA should keep track of $N_a(w)$ mod 3 and $N_b(w)$ mod 2, the possible states of the DFA can be obtained by $Q1 \times Q2$ (cross product) and can be represented as shown below:

$Q1 \times Q2 = \{ (A0, B0), (A0, B1), (A0, B2),$
$(A1, B0), (A1, B1), (A1, B2)$
$(A2, B0), (A2, B1), (A2, B2),$
$(A3, B0), (A3, B1), (A3, B2),$
$(A4, B0), (A4, B1), (A4, B2),$
$\}$



Start
A0,B0
A0,B1
A0,B2
A1,B0
A1,B1
A1,B2
A2,B0
A2,B1
A2B2
A3B0
A3,B1
A3,B2
A4,B0
A4,B1
A4,B2
a
b