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Internal Assessment Test – I December 2023

Sub:	Mathematics for Computer Science								
Date:	18/12/2023	Duration:	90 mins	Max Marks:	50	Sem:	III	Code:	BCS301
								Branch:	AIML/AIDS/ CSE/ISE/ CS DS/CS ML

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Find the mean and standard deviation of the Binomial distribution	[8]	CO1	L1
2	The probability distribution of a finite random variable X is given by the following data. Find the value of K, mean and variance.	[7]	CO1	L1
3	The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto 70?	[7]	CO1	L2
4	The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given. Fit a poisson distribution for the data and calculate the theoretical frequencies.	[7]	CO1	L2

Table for Question 2:

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	K

Table for Question 4:

X	0	1	2	3	4	5
F	173	168	37	18	3	1

5	Find k such that $f(x) = \begin{cases} k\,x\,e^{-x} & , 0 < x < 1 \\ 0 & otherwise \end{cases}$ is a p.d.f. And find the mean.	[7]	CO1	L3
6	If x is normally distributed with mean 12 and S.D 4, find the following =. (i) $P(x \geq 20)$ (ii) $P(x \leq 20)$ [Note: $\Phi(2) = 0.4772$]	[7]	CO1	L3
7	In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. It is given that if $P(z) = \frac{1}{\sqrt{2\pi}}\int_0^z e^{-z^2/2}\,dz$ then $P(1.2263 = 0.39)$ and $P(1.4757) = 0.43$	[7]	CO1	L3
8	The joint distribution of two random variables X and Y is as follows. X \ Y: -4, 2, 7 1: 1/8, 1/4, 1/8 5: 1/4, 1/8, 1/8 Compute the following: (a) E(x) and E(Y) (b) E(XY) (c) ρ(X,Y)	[7]	CO2	L3

Joint distribution for question 8:

X \ Y	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Q1.

3.41 Mean and Standard Deviation of the Binomial Distribution

$$\text{Mean } (\mu) = \sum_{x=0}^{n} x\,P(x)$$

$$\mu = \sum_{x=0}^{n} x \cdot {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n \cdot (n-1)!}{(x-1)!(n-x)!} p \cdot p^{x-1} q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)![(n-1)-(x-1)]!} p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = n p \sum_{x=1}^{n} {}^{(n-1)}C_{(x-1)}\ p^{x-1} q^{(n-1)-(x-1)}$$

$$\mu = n p\ (q + p)^{n-1} = n p$$

Mean (μ) = np

$$\text{Variance } (V) = \sum_{x=0}^{n} x^2 P(x) - \mu^2$$

$$\text{Now, } \sum_{x=0}^{n} x^2 P(x) = \sum_{x=0}^{n} [x(x-1) + x] P(x)$$

$$= \sum_{x=0}^{n} x(x-1) P(x) + \sum_{x=0}^{n} x P(x)$$

$$= \sum_{x=0}^{n} x(x-1)\ {}^{n}C_{x}\ p^{x} q^{n-x} + n p$$

$$= \sum_{x=0}^{n} x(x-1)\ \frac{n!}{x!(n-x)!} p^{x} q^{n-x} + n p$$

$$= \sum_{x=0}^{n} \frac{n \cdot (n-1) \cdot (n-2)!}{(x-2)!(n-x)!} p^2\, p^{x-2} q^{n-x} + n p$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)![(n-2)-(x-2)]!}\ p^{x-2} q^{(n-2)-(x-2)} + n$$

$$= n(n-1)p^2 \sum_{x=2}^{n} {}^{(n-2)}C_{(x-2)}\ p^{x-2} q^{(n-2)-(x-2)} + n p$$

$$= n(n-1)p^2 (q + p)^{n-2} + n p$$

$$\therefore \quad \sum x^2 P(x) = n(n-1)p^2 + n p$$

Using this result in (1) along with $\mu = n p$ we have,

$$\text{Variance } (V) = \{n(n-1)p^2 + n p\} - (np)^2$$

$$= n^2p^2 - np^2 + np - n^2p^2 = np(1-p) = npq$$

Hence, variance $(V) = npq$

$$S.D\ (\sigma) = \sqrt{V} = \sqrt{npq}$$

Thus we have for the Binomial Distribution,

Mean $(\mu) = np$ and $S.D\ (\sigma) = \sqrt{npq}$

2

[6] *The probability distribution of a finite random variable X is given by the following table.*

x_i	−2	−1	0	1	2	3
$P(x_i)$	0.1	K	0.2	$2K$	0.3	K

Find the value of K, mean and variance.

☞ We must have $P(x_i) \geq 0$ and $\sum P(x_i) = 1$ for a probability distribution.

$$\sum P(x_i) = 1 \text{ requires } 4K + 0.6 = 1 \quad \therefore \boxed{K = 0.1}$$

Mean $(\mu) = \sum x_i P(x_i) = -0.2 - 0.1 + 0.2 + 0.6 + 0.3 = 0.8$

Variance $V = (\sigma^2) = \sum x_i^2 P(x_i) - \mu^2$

$$V = \sigma^2 = (0.4 + 0.1 + 0.2 + 1.2 + 0.9) - (0.8)^2 = 2.16$$

Thus, $\boxed{\text{Mean} = 0.8 \text{ and Variance} = 2.16}$

3 **[15]** *The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 atleast 7 of them will live upto* **70.**

☞ Let x be the number of persons aged 60 years living upto 70 years.

For this variate we have by data,

$$p = 0.65. \quad \text{Hence, } q = 0.35$$

Consider, $P(x) = {}^nC_x\, p^x\, q^{n-x}$. Here $n = 10$

We have to find $P(x \geq 7)$.

$$P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$$

$$= {}^{10}C_7\,(0.65)^7(0.35)^3 + {}^{10}C_8\,(0.65)^8(0.35)^2 + {}^{10}C_9(0.65)^9(0.35) + (0.65)^{10}$$

But, ${}^{10}C_7 = {}^{10}C_3 = \dfrac{10\cdot 9\cdot 8}{1\cdot 2\cdot 3} = 120\,; = {}^{10}C_8 = {}^{10}C_2 = \dfrac{10\cdot 9}{1\cdot 2} = 45\,; = {}^{10}C_9 = {}^{10}C_1 = 10$

Hence, $\boxed{P(x \geq 7) = 0.5138}$

$$121.3\frac{(0.5)^0}{0!} \approx 121,\ 121.3\frac{(0.5)^1}{1!} \approx 61,$$

$$121.3\frac{(0.5)^2}{2!} \approx 15,\ \frac{121.3(0.5)^3}{3!} \approx 3,\ 121.3\frac{(0.5)^4}{4!} \approx 0$$

Thus the required theoretical frequencies are 121, 61, 15, 3, 0.

[25] *The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given. Fit a Poisson distribution for the data and calculate the theoretical frequencies.*

x	0	1	2	3	4	5
f	173	168	37	18	3	1

☞ We have for the Poisson distribution :

$$\text{Mean} \quad (\mu) = \frac{\sum f x}{\sum f} = \frac{0+168+74+54+12+5}{400} = 0.7825$$

The Poisson distribution is $P(x) = \dfrac{m^x e^{-m}}{x!}$

Let, $\quad f(x) = 400\,P(x)$

$$= 400\frac{(0.7825)^x e^{-0.7825}}{x!} \quad \text{But, } e^{-0.7825} \approx 0.4573$$

$$\therefore \quad f(x) = 182.9\frac{(0.7825)^x}{x!}$$

Theoretical frequencies are got by substituting $x = 0, 1, 2, 3, 4, 5$ in $f(x)$ and they are as follows.

$(182.9)1 \approx 183$; $(182.9)(0.7825) \approx 143$

$$(182.9)\frac{(0.7825)^2}{2} \approx 56 \quad ; \quad \frac{(182.9)(0.7825)^3}{6} \approx 15$$

$$(182.9)\frac{(0.7825)^4}{24} \approx 3 \quad ; \quad \frac{(182.9)(0.7825)^5}{120} \approx 0$$

Thus the theoretical frequencies are 183, 143, 56, 15, 3, 0.

5

[34] *Find k such that* $f(x) = \begin{cases} k\,x\,e^{-x}, & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ *is a p.d.f. Find the mean*

☞ $f(x) \geq 0$ if $k \geq 0$. Also we must have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

ie., $\int_0^1 k\,x\,e^{-x}\,dx = 1$

Applying Bernoulli's rule we have,

$$k\left[x(-e^{-x}) - (1)(e^{-x})\right]_0^1 = 1$$

ie., $k\left[-\frac{1}{e} - \left(\frac{1}{e} - 1\right)\right] = 1,$

ie., $k\left(1 - \frac{2}{e}\right) = 1$

$\therefore$ $\boxed{k = \frac{e}{e-2}}$

$$\text{Mean} = \mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_0^1 x \cdot \frac{e}{e-2}\,x\,e^{-x}\,dx$$

$$= \frac{e}{e-2}\int_0^1 x^2 e^{-x}\,dx$$

$$= \frac{e}{e-2}\left[x^2(-e^{-x}) - (2x)(e^{-x}) + 2(-e^{-x})\right]_0^1$$

$$\mu = \frac{e}{e-2}\left[-\frac{1}{e} - \frac{2}{e} - 2\left(\frac{1}{e} - 1\right)\right]$$

$$\mu = \frac{e}{e-2}\left[2 - \frac{5}{e}\right] = \boxed{\frac{2e-5}{e-2}}$$

6 **[46]** *If x is normally distributed with mean 12 and S.D 4, find the following.*

(*i*) $P(x \geq 20)$ (*ii*) $P(x \leq 20)$

☞ We have s.n.v, $z = \dfrac{x-\mu}{\sigma} = \dfrac{x-12}{4}$

If $x = 20$, $z = 2$

We have to find $P(z \geq 2)$ and $P(0 \leq z \leq 2)$.

Now, $P(z \geq 2) = P(z>0) - P(0 \leq z \leq 2)$

$= 0.5 - \phi(2)$

$= 0.5 - 0.4772 = 0.0228$

Also, $P(z \leq 2) = P(-\infty \leq z \leq 0) + P(0 \leq z \leq 2)$

$= 0.5 + \phi(2)$

$= 0.5 + 0.4772 = 0.9772$

Thus, $\boxed{P(x \geq 20) = 0.0228 \text{ and } P(x \leq 20) = 0.9772}$

[50] In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. It is given that if

$$P(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz \text{ then } P(1.2263) = 0.39 \text{ and } P(1.4757) = 0.43$$

☞ Let μ and σ be the mean and S.D of the normal distribution.

By data we have, $P(x<35) = 0.07$ and $P(x<60) = 0.89$

We have, s.n.v $z = \dfrac{x-\mu}{\sigma}$

When $x = 35,\ z = \dfrac{35-\mu}{\sigma} = z_1\ (say)$

$x = 60,\ z = \dfrac{60-\mu}{\sigma} = z_2\ (say)$

Hence we have

$$P(z<z_1) = 0.07 \text{ and } P(z<z_2) = 0.89$$

ie., $0.5+\phi(z_1) = 0.07$ and $0.5+\phi(z_2) = 0.89$

$\therefore$ $\phi(z_1) = -0.43$ and $\phi(z_2) = 0.39$

Using the given data in the RHS of these we have,

$$\phi(z_1) = -\phi(1.4757) \text{ and } \phi(z_2) = \phi(1.2263)$$

$\Rightarrow$ $z_1 = -1.4757$ and $z_2 = 1.2263$

ie., $\dfrac{35-\mu}{\sigma} = -1.4757$ and $\dfrac{60-\mu}{\sigma} = 1.2263$

or $\mu - 1.4757\sigma = 35$ and $\mu + 1.2263\sigma = 60$

By solving we get, $\mu = 48.65$ and $\sigma = 9.25$

Thus, $\boxed{\text{Mean} = 48.65 \text{ and S.D} = 9.25}$

[1] *The joint distribution of two random variables X and Y is as follows*

X \ Y	−4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute the following.

(*a*) $E(X)$ *and* $E(y)$ (*b*) $E(XY)$ (*c*) σ_X *and* σ_y

(*d*) $COV(X, Y)$ (*e*) $\rho(X, Y)$ **[June 2017, 18, Dec 18]**

☞ The distribution (*marginal distribution*) of X and Y is as follows. This distribution is obtained by adding the all the respective row entries and also the respective column entries.

Distribution of X :

x_i	1	5
$f(x_i)$	1/2	1/2

Distribution of Y :

y_j	−4	2	7
$g(y_j)$	3/8	3/8	1/4

(a) $E(X) = \sum x_i f(x_i) = (1)(1/2) + (5)(1/2) = 3$

$$E(Y) = \sum y_j\, g(y_j)$$

$$= (-4)(3/8) + (2)(3/8) + (7)(1/4) = 1$$

Thus, $\boxed{\mu_X = E(X) = 3 \text{ and } \mu_Y = E(Y) = 1}$

(b) $E(XY) = \sum x_i y_j J_{ij}$

$$= (1)(-4)(1/8) + (1)(2)(1/4) + (1)(7)(1/8)$$

$$+ (5)(-4)(1/4) + (5)(2)(1/8) + (5)(7)(1/8)$$

$$= \frac{-1}{2} + \frac{1}{2} + \frac{7}{8} - 5 + \frac{5}{4} + \frac{35}{8} = \frac{3}{2}$$

Thus, $\boxed{E(XY) = 3/2}$

(c) $\sigma_X^2 = E(X^2) - \mu_X^2$ and $\sigma_Y^2 = E(Y^2) - \mu_Y^2$

Now, $E(X^2) = \sum x_i^2 f(x_i)$

ie., $E(X^2) = (1)(1/2) + (25)(1/2) = 13$

Also, $E(Y^2) = \sum y_j^2\, g(y_j)$

ie., $E(Y^2) = 16(3/8) + (4)(3/8) + (49)(1/4) = 79/4$

Hence, $\sigma_X^2 = 13 - (3)^2 = 4$; $\sigma_Y^2 = (79/4) - (1)^2 = 75/4$

Thus, $\boxed{\sigma_X = 2 \;\text{ and }\; \sigma_Y = \sqrt{75/4} = 4.33}$

(d) $COV(X,Y) = E(XY) - \mu_X \mu_Y$

$$= (3/2) - (3)(1) = -3/2$$

$\therefore \quad \boxed{COV(X,Y) = -3/2}$

(e) $$\rho(X,Y) = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

$$= \frac{-3/2}{(2)\sqrt{75/4}} = \frac{-3}{2\sqrt{75}}$$

Thus, $\boxed{\rho(X,Y) = -0.1732}$