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Internal Assessment Test – II January 2024

Sub:	AV Mathematics-III for EC Engineering						Code:	BMATEC301
Date:	17/01/2024	Duration:	90 mins	Max Marks:	50	Sem: III	Branch:	ECE

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Solve the difference equation of $y_{n+2}+6y_{n+1}+9y_n=2^n$ with $y_0=y_1=0$ using Z-transforms.	[8]	CO3	L3
2	Find the Fourier sine and cosine transform of $f(x)=e^{-\alpha x}$, $\alpha>0$.	[7]	CO2	L3
3	Obtain the Fourier cosine transform of the function $f(x)=\begin{cases}4x & ,0<x<1\\ 4-x & ,1<x<4\\ 0 & ,x>4\end{cases}$	[7]	CO2	L3
4	Obtain the Z-transform of $\cos n\theta$.	[7]	CO3	L3
5	Find the Z- transform of $2n+\sin(\frac{n\pi}{4})+1$.	[7]	CO3	L3
6	If $\bar{u}(z)=\frac{2z^2+3z+12}{(z-1)^4}$, find the values of u_0, u_1, u_2 and u_3.	[7]	CO3	L3
7	Find $Z_T^{-1}\left[\frac{8z-z^3}{(4-z)^3}\right]$.	[7]	CO3	L3
8	Solve $(4D^4-4D^3-23D^2+12D+36)y=0$.	[7]	CO4	L3

① Given, the Difference Equation as:-

$y_{n+2}+6y_{n+1}+9y_n=2^n$ with $y_0=y_1=0$.

From the given,

$y_{n+2}+6y_{n+1}+9y_n=2^n$

Applying Z-Transform on Both sides.

$\therefore Z_T[y_{n+2}+6y_{n+1}+9y_n]=Z_T[2^n]$ — ①

$\therefore$ By applying Linearity Property ($\because Z_T[c_1u_n+c_2v_n]$
$= c_1 Z_T[u_n]+c_2 Z_T[$

eqn ① can be written as,

$\Rightarrow Z_T[y_{n+2}]+6Z_T[y_{n+1}]+9Z_T[y_n]=Z_T[2^n]$

$\Rightarrow z^2[\bar{y}(z)-y_0-y_1z^{-1}]+6z[\bar{y}(z)-y_0]+9[\bar{y}(z)]=\dfrac{z}{z-2}$

$\left[\because Z_T[u_n]=\bar{u}(z),\ Z_T[u_{n+1}]=z[\bar{u}(z)-u_0],\ Z_T[u_{n+2}]=z^2[\bar{u}(z)-u_0-u_1z^{-1}],\ Z_T[k^n]=\dfrac{z}{z-k}\right]$

$\Rightarrow z^2[\bar{y}(z)-0-0]+6z[\bar{y}(z)-0]+9[\bar{y}(z)]=\dfrac{z}{z-2}$

($\because$ Given, $y_0=y_1=0$)

$\Rightarrow z^2[\bar{y}(z)]+6z[\bar{y}(z)]+9[\bar{y}(z)]=\dfrac{z}{z-2}$

$\therefore \bar{y}(z)[z^2+6z+9]=\dfrac{z}{z-2}$

$\bar{y}(z)=\dfrac{z}{(z-2)(z^2+6z+9)}$

$z^2+6z+9=0$

$z^2+3z+3z+9=0$

$z(z+3)+3(z+3)$

$(z+3)(z+3)$

$\bar{y}(z)=\dfrac{z}{(z-2)(z+3)^2}$

we get,

$$\bar{y}(z) = \frac{z}{(z-2)(z+3)^2}$$

$$\frac{z}{(z-2)(z+3)^2} = \frac{A}{(z-2)} + \frac{B}{(z+3)} + \frac{C}{(z+3)^2}$$ ($\because$ By using partial fraction method)

$$\frac{z}{(z-2)(z+3)^2} = \frac{Az}{(z-2)} + \frac{Bz}{(z+3)} + \frac{C(-3z)}{(z+3)^2} \quad - (2)$$

($\because$ $Z_T[k^n \cdot n] = \frac{kz}{z-k}$, $Z_T[k^n] = \frac{z}{z-k}$)

$$\frac{z}{(z-2)(z+3)^2} = \frac{Az(z+3)^2 + Bz(z+3)(z-2) - 3Cz(z-2)}{(z-2)(z+3)^2}$$

$$Az(z+3)^2 + Bz(z+3)(z-2) - 3Cz(z-2) = z$$

$$Az(z^2+6z+9) + Bz(z^2-2z+3z-6) - 3C(z^2-2z) = z$$

$$Az^3 + 6Az^2 + 9Az + Bz(z^2+z-6) - 3Cz^2 + 6Cz = z$$

$$Az^3 + 6Az^2 + 9Az + Bz^3 + Bz^2 - 6Bz - 3Cz^2 + 6Cz = z$$

$$Az^3 + 6Az^2 + 9Az + Bz^3 + Bz^2 - 6Bz - 3Cz^2 + 6Cz = z$$

$$z^3(A+B) + z^2(6A+B-3C) + z(9A-6B+6C) = z$$

$\therefore$ By comparing the coefficients on both the sides.

$A+B=0$,	$6A+B-3C=0$,	$9A-6B+6C=1$
$A=-B$	$\therefore$ substitue 'A'	substitute 'A'
	$6(-B)+B-3C=0$	$9(-B)-6B+6C=1$
	$-5B-3C=0$	$-9B-6B+6C=1$
	$5B+3C=0 \quad - (3)$	$-15B+6C=1 \quad - (4)$

Multiply eq^n (2) with 3

$$(5B+3C)3=0$$

$$\therefore \quad 15B+9C=0 \quad - (5)$$

Solving eqn. (3), (4)

$$15B + 9C = 0$$
$$-15B + 6C = 1$$
$$15C = 1$$

$$C = \frac{1}{15}$$

Substitue 'C' in eqn (3)

$$5B + 3C = 0$$
$$5B + \cancel{3}\left(\frac{1}{\cancel{15}_{5}}\right) = 0$$
$$5B + \frac{1}{5} = 0$$
$$5B = -\frac{1}{5}$$

$$B = \frac{-1}{25}$$

As, $A = -B$

$$A = -\left(\frac{-1}{25}\right)$$

$$A = \frac{1}{25}$$

Substitute A, B, C in eqn (2).

$$\therefore \frac{z}{(z-2)(z+3)^2} = \frac{\frac{1}{25}(z)}{z-2} + \frac{\left(\frac{-1}{25}\right)z}{z+3} + \frac{\left(\frac{1}{15}\right)(-3z)}{(z+3)^2}.$$

$$\bar{y}(z) = \frac{1}{25}\left[\frac{z}{z-2}\right] - \frac{1}{25}\left[\frac{z}{z+3}\right] + \frac{1}{15}\left[\frac{-3z}{(z+3)^2}\right]$$

$\therefore$ Apply Inverse Z - Transform on both sides.

$$\therefore z_T^{-1}[\bar{y}(z)] = z_T^{-1}\left(\frac{1}{25}\left[\frac{z}{z-2}\right] - \frac{1}{25}\left[\frac{z}{z+3}\right] + \frac{1}{15}\left[\frac{-3z}{(z+3)^2}\right]\right)$$

$\therefore$ By Linearity Property.

$$y_n = \frac{7}{25} z^{-1}_T\left[\frac{z}{z-2}\right] - \frac{1}{25} z^{-1}_T\left[\frac{z}{z-(-3)}\right] + \frac{1}{15} z^{-1}_T\left[\frac{-3z}{(z-(-3))^2}\right]$$

$$y_n = \frac{1}{25}\left[2^n\right] - \frac{1}{25}\left[(-3)^n\right] + \frac{1}{15}\left[(-3)^n . n\right]$$

$$\left(\because z^{-1}_T\left[\frac{z}{z-k}\right] = k^n ,\quad z^{-1}_T\left[\frac{kz}{(z-k)^2}\right] = k^n . n\right]$$

$$y_n = \frac{1}{5}\left[\frac{2^n}{5} - \frac{(-3)^n}{5} + \frac{1}{3}\left((-3)^n . n\right)\right].$$

2) Given,

$f(x) = e^{-\alpha x}$, $\alpha > 0$

Fourier Sine Transform,

$$F_s[u] = \int_0^\infty f(x) . \sin ux \, dx$$

$$F_s[u] = \int_0^\infty e^{-\alpha x} . \sin ux \, dx$$

$$F_s[u] = \left[\frac{e^{-\alpha x}}{\alpha^2+u^2}\left[(-\alpha)\sin ux - u\cos ux\right]\right]_0^\infty$$

$$\left(\because \int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin bx - b\cos bx\right]\right.$$

$$F_s[u] = \left[\frac{e^{-\alpha x}}{\alpha^2+u^2}\left[-\alpha\sin ux - u\cos ux\right]\right]_0^\infty$$

$$F_s[u] = \left(\frac{e^{-\infty}}{\alpha^2+u^2}\left[-\alpha\sin(\infty) - u\cos u(\infty)\right] - \frac{e^0}{\alpha^2+u^2}\left[-\alpha\sin 0 - u\cos\right.\right.$$

$$F_s[u] = \left[0 - \frac{1}{\alpha^2+u^2}[-u(1)] \right] \quad (\because e^{-\infty}=0, \sin 0 = 0, \cos 0 = 1)$$

$$F_s[u] = \left[\frac{u}{\alpha^2+u^2} \right].$$

$\therefore$ Fourier sine transform of $f(x) = e^{-\alpha x}$ is:-

$$F_s[u] = \frac{u}{\alpha^2+u^2}$$

Fourier cosine transform of $f(x) = e^{-\alpha x}$ is:-

$$F_c[u] = \int_0^\infty f(x) \cdot \cos ux \, dx$$

$$F_c[u] = \int_0^\infty e^{-\alpha x} \cos ux \, dx \Rightarrow \left[\frac{e^{-\alpha x}}{\alpha^2+u^2} [(-\alpha)\cos ux + u \sin ux] \right]_0^\infty$$

$$\left(\because \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}[a \cos bx + b \sin bx] \right)$$

$$F_c[u] = \left[\frac{e^{-\infty}}{\alpha^2+u^2}[(-\alpha)\cos(\infty) + u \sin(\infty)] - \frac{e^0}{\alpha^2+u^2}[(-\alpha)\cos 0 + u \sin 0] \right]$$

$$F_c[u] = \left[0 - \frac{1}{\alpha^2+u^2}(-\alpha) \right]$$

$$F_c[u] = \left[\frac{\alpha}{\alpha^2+u^2} \right].$$

$\therefore$ Fourier Cosine Transform of $f(x) = e^{-\alpha x}$ is:-

$$F_c[u] = \frac{\alpha}{\alpha^2+u^2} \,//$$

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Q3. $f(x) = \begin{cases} 4x & 0 < x < 1 \\ 4-x & 1 < x < 4 \\ 0, & x > 4 \end{cases}$

$$F_c[(f(x)] = \int_0^\infty f(x) \cos ux \, dx$$

$$= \int_0^1 4x \cos ux \, dx + \int_1^4 (4-x) \cos ux \, dx + 0$$

$$= 4\left(x\left[\frac{\sin ux}{u}\right] - 1\left[\frac{-\cos ux}{u^2}\right]\right)_0^1 + \left(4-x\left[\frac{\sin ux}{u}\right] - (-1)\left[\frac{-\cos ux}{u^2}\right]\right)_1^4$$

$$= 4\left[\frac{\sin u}{u} + \frac{\cos u}{u^2} - \frac{1}{u^2}\right] + \left(-\frac{\cos 4u}{u^2}\right)\left(-\left[\frac{3\sin u}{u} - \frac{\cos u}{u^2}\right]\right)$$

$$= \frac{4\sin u}{u} + \frac{4\cos u}{u^2} - \frac{4}{u^2} - \frac{\cos 4u}{u^2} - \frac{3\sin u}{u} + \frac{\cos u}{u^2}$$

$$\frac{\sin u}{u} + \frac{5\cos u}{u^2} - \frac{4}{u^2} - \frac{\cos 4u}{u^2}$$

The fourier cosine transform of $f(x)$ is

$$\frac{\sin u}{u} + \frac{5\cos u}{u^2} - \frac{4}{u^2} - \frac{\cos 4u}{u^2}$$

4. Given,

Obtain z-Transform of $\cos n\theta$

∴ As WKT,

$e^{in\theta} = \cos n\theta + i\sin n\theta$ — ① $\left(\because e^{i\theta} = \cos\theta + i\sin\theta\right)$

∴ Apply z-Transform on '$e^{in\theta}$'.

$\therefore Z_T[e^{in\theta}] = Z_T[(e^{i\theta})^n]$ $\left(\because a^{mn} = (a^m)^n\right)$

let, $k = e^{i\theta}$

$\Rightarrow Z_T[k^n]$ $\left[\because Z_T[k^n] = \frac{z}{z-k}\right]$

$\Rightarrow \frac{z}{z-k}$

$\Rightarrow \frac{z}{z - e^{i\theta}}$

∴ By Rationalising,

$$\frac{z}{z - e^{i\theta}} \times \frac{z - e^{-i\theta}}{z - e^{-i\theta}}$$

$$\Rightarrow \frac{z(z - e^{-i\theta})}{(z - e^{i\theta})(z - e^{-i\theta})} = \frac{z\left[z - (\cos\theta - i\sin\theta)\right]}{z^2 - ze^{-i\theta} - ze^{i\theta} + e^{i\theta}.e^{-i\theta}}$$

$$\Rightarrow \frac{z\left[z - \cos\theta + i\sin\theta\right]}{z^2 - z(e^{i\theta} + e^{-i\theta}) + e^{i\theta - i\theta}}$$ $\left(\because a^m.a^n = a^{m+n}\right)$

$$\Rightarrow \frac{z^2 - z\cos\theta + iz\sin\theta}{z^2 - z(2\cos\theta) + 1}$$

$\left(\because \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}\right)$

$\left(\because e^0 = 1\right)$

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$$\Rightarrow \frac{z^2 - z\cos\theta + iz\sin\theta}{z^2 - 2z\cos\theta + 1}$$

$$\Rightarrow \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta + 1} + \frac{i\, z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

$$\therefore Z_T[e^{in\theta}] = \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta + 1} + \frac{i\, z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

$$Z_T[\cos n\theta + i\sin n\theta] = \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta + 1} + i\frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

($\because$ from eq^n ①)

$\therefore$ By linearity Property,

$$Z_T[c_1 u_n + c_2 v_n] = c_1 Z_T[u_n] + c_2 Z_T[v_n]$$

$$Z_T[\cos n\theta] + i Z_T[\sin n\theta] = \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta + 1} + i\frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

$\therefore$ By comparing Real and imaginary Parts,

$$Z_T[\cos n\theta] = \frac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta + 1}, \quad Z_T[\sin n\theta] = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

$\therefore$ Z-Transform of $\cos n\theta$ is:- $Z_T[\cos n\theta] = \dfrac{z^2 - z\cos\theta}{z^2 - 2z\cos\theta + 1}$ //

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(5) Given,

$[2n + \sin(\frac{n\pi}{4}) + 1]$; let, $u_n = 2n + \sin(\frac{n\pi}{4}) + 1$

∴ To obtain the z-Transform of u_n, i.e., $Z_T[u_n] = \bar{u}(z)$

Apply z-Transform on 'u_n'

$$\therefore \quad Z_T[u_n] = Z_T[2n + \sin(\frac{n\pi}{4}) + 1]$$

$$Z_T[2n] + Z_T[\sin(\frac{n\pi}{4})] + Z_T[1]$$

(∵ By Linearity Property, $Z_T[c_1 u_n + c_2 v_n] = c_1 Z_T[u_n] + c_2 Z_T[v_n]$)

$$\Rightarrow 2 Z_T[n] + Z_T[\sin(\frac{n\pi}{4})] + Z_T[1]$$

$$\Rightarrow 2\left[\frac{z}{(z-1)^2}\right] + \left[\frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}\right]_{\theta \to \frac{\pi}{4}} + \left[\frac{z}{z-1}\right]$$

$$\left(\because Z_T[n] = \frac{z}{(z-1)^2}, \; Z_T[\sin n\theta] = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}, \; Z_T[1] = \frac{z}{z-1}\right)$$

$$\Rightarrow \frac{2z}{(z-1)^2} + \frac{z\sin(\frac{\pi}{4})}{z^2 - 2z\cos(\frac{\pi}{4}) + 1} + \frac{z}{z-1}$$

$$\Rightarrow \frac{2z}{(z-1)^2} + \frac{z(\frac{1}{\sqrt{2}})}{z^2 - 2z(\frac{1}{\sqrt{2}}) + 1} + \frac{z}{z-1} \quad \left(\because \sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}, \; \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \frac{2z}{(z-1)^2} + \frac{z}{\sqrt{2}(z^2 - 2z(\frac{1}{\sqrt{2}}) + 1)} + \frac{z}{z-1}$$

$$\Rightarrow \frac{2z}{(z-1)^2} + \frac{z}{\sqrt{2}z^2 - 2z + \sqrt{2}} + \frac{z}{z-1}.$$

$$\therefore Z_T[u_n] = \frac{2z}{(z-1)^2} + \frac{z}{\sqrt{2}z^2 - 2z + \sqrt{2}} + \frac{z}{z-1}$$

$$\bar{u}(z) = Z_T\left[2n + \sin\left(\frac{n\pi}{4}\right) + 1\right] = \frac{2z}{(z-1)^2} + \frac{z}{\sqrt{2}z^2 - 2z + \sqrt{2}} + \frac{z}{z-1} \quad //$$

⑥ Given,

$$\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$$

$\therefore$ By Initial Value Theorem, $\lim_{z\to\infty} \bar{u}(z) = u_0$

$$\therefore u_0 = \lim_{z\to\infty} \bar{u}(z)$$

$$u_0 = \lim_{z\to\infty}\left[\frac{2z^2 + 3z + 12}{(z-1)^4}\right]$$

$$u_0 = \lim_{z\to\infty}\left[\frac{\cancel{z^2}\left(2 + \frac{3}{z} + \frac{12}{z^2}\right)}{z^4\left(1 - \frac{1}{z}\right)^4}\right]$$

$$u_0 = \lim_{z\to\infty}\left[\frac{2 + \frac{3}{z} + \frac{12}{z^2}}{z^2\left(1 - \frac{1}{z}\right)^4}\right]$$

$$u_0 = \left[\frac{2 + \frac{3}{\infty} + \frac{12}{\infty^2}}{(\infty)^2\left(1 - \frac{1}{\infty}\right)^4}\right] \quad \left(\because \frac{1}{\infty} = 0\right)$$

$$u_0 = \frac{1}{\infty}\left[\frac{2 + 0 + 0}{(1-0)^4}\right]$$

$$u_0 = 0\left(\frac{2}{1}\right)$$

$$\boxed{u_0 = 0}$$

$\therefore$ As WKT, $Z_T[u_{n+1}] = z[\bar{u}(z) - u(0)]$

$\therefore$ If, $n=0$

$Z_T[u_1] = z[\bar{u}(z) - u_0]$

$\therefore \quad u_1 = \lim\limits_{z \to \infty} z\left[\bar{u}(z) - u_0\right]$

$\therefore \quad u_1 = \lim\limits_{z \to \infty} z\left[\dfrac{2z^2+3z+12}{(z-1)^4} - 0\right]$

$u_1 = \lim\limits_{z \to \infty} z\left[\dfrac{2z^2+3z+12}{(z-1)^4}\right]$

$u_1 = \lim\limits_{z \to \infty} z(z^2)\left[\dfrac{2+\frac{3}{z}+\frac{12}{z^2}}{z^4(1-\frac{1}{z})^4}\right]$

$u_1 = \lim\limits_{z \to \infty} z^3\left[\dfrac{2+\frac{3}{z}+\frac{12}{z^2}}{z^4(1-\frac{1}{z})^4}\right]$

$u_1 = \lim\limits_{z \to \infty}\left[\dfrac{2+\frac{3}{z}+\frac{12}{z^2}}{z(1-\frac{1}{z})^4}\right]$

$u_1 = \dfrac{1}{\infty}\left[\dfrac{2+\frac{3}{\infty}+\frac{12}{\infty}}{\left(1-\frac{1}{\infty}\right)^4}\right] \qquad \left(\because \frac{1}{\infty} = 0\right)$

$u_1 = 0$

Similarly,

$u_2 = \lim\limits_{z \to \infty} z^2[\bar{u}(z) - u_0 - u_1 z^{-1}]$

$u_2 = \lim\limits_{z \to \infty} z^2[\bar{u}(z) - u_0 - u_1 z^{-1}]$

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$$u_2 = \lim_{z\to\infty} z^2\left[\frac{2z^2+3z+12}{(z-1)^4} - 0 - 0\right]$$

$$u_2 = \lim_{z\to\infty} z^2\left[\frac{2z^2+3z+12}{(z-1)^4}\right]$$

$$u_2 = \lim_{z\to\infty} z^2(z^2)\left[\frac{2+\frac{3}{z}+\frac{12}{z^2}}{z^4(1-\frac{1}{z})^4}\right]$$

$$u_2 = \lim_{z\to\infty} \cancel{z^4}\left[\frac{2+\frac{3}{z}+\frac{12}{z^2}}{\cancel{z^4}(1-\frac{1}{z})^4}\right]$$

$$u_2 = \lim_{z\to\infty}\left(\frac{2+\frac{3}{z}+\frac{12}{z^2}}{(1-\frac{1}{z})^4}\right)$$

$$u_2 = \left[\frac{2+\frac{3}{\infty}+\frac{12}{\infty}}{(1-\frac{1}{\infty})^4}\right] \qquad \left(\because \frac{c}{\infty}=0\right)$$

$$u_2 = \left[\frac{2}{(1)^4}\right]$$

$$u_2 = 2$$

Similarly,

$$u_3 = \lim_{z\to\infty} z^3\left[\bar{u}(z) - u_0 - u_1 z^{-1} - u_2 z^{-2}\right]$$

$$u_3 = \lim_{z\to\infty} z^3\left[\frac{2z^2+3z+12}{(z-1)^4} - 0 - 0 - 2(z^{-2})\right]$$

$$\cancel{u_3 = \lim_{z\to\infty}\left[\frac{2z^5+3z^4+12z^3}{(z-1)^4} - 2z\right]}$$

$$u_3 = \lim_{z\to\infty} z^3\left[\frac{2z^2+3z+12}{(z-1)^4} - \frac{2}{z^2}\right]$$

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$$u_3 = \lim_{z\to\infty} z^{3}\left[\frac{(2z^2+3z+12)z^2 - 2(z-1)^4}{(z-1)^4(z^2)}\right]$$

$$u_3 = \lim_{z\to\infty} z\left[\frac{2z^4+3z^3+12z^2 - 2(z^4+1^4-4z^3-4z+6z^2)}{(z-1)^4}\right]$$

$$u_3 = \lim_{z\to\infty} z\left[\frac{\cancel{2z^4}+3z^3+12z^2-\cancel{2z^4}-2+8z^3+8z-\cancel{12z^2}}{(z-1)^4}\right]$$

$$u_3 = \lim_{z\to\infty} z\left[\frac{11z^3+8z-2}{(z-1)^4}\right]$$

$$u_3 = \lim_{z\to\infty} z(z^3)\left[\frac{11+\frac{8}{z^2}-\frac{2}{z^3}}{z^4(1-\frac{1}{z})^4}\right]$$

$$u_3 = \lim_{z\to\infty} \cancel{z^4}\left[\frac{11+\frac{8}{z^2}-\frac{2}{z^3}}{\cancel{z^4}(1-\frac{1}{z})^4}\right]$$

$$u_3 = \frac{11+\frac{8}{\infty}+\frac{2}{\infty}}{(1-\frac{1}{\infty})^4} \qquad \left(\because \frac{1}{\infty}=0\right)$$

$$u_3 = \frac{11}{(1)^4}$$

$$u_3 = 11$$

$\therefore u_0 = 0,\ u_1 = 0,\ u_2 = 2,\ u_3 = 11.$

(7) Given,

$$z^{-1}_T\left[\frac{8z-z^3}{(4-z)^3}\right]$$

$\Rightarrow$ let, $\overline{u}(z) = \dfrac{8z-z^3}{(4-z)^3}$

$$\Rightarrow \frac{\cancel{-}(z^3-8z)}{\cancel{-}(z-4)^3}$$

$$\Rightarrow \frac{z^3-8z}{(z-4)^3}$$

$$\therefore \frac{z^3-8z}{(z-4)^3} = \frac{A}{(z-4)} + \frac{B}{(z-4)^2} + \frac{C}{(z-4)^3}$$

$$\Rightarrow \frac{Az}{(z-4)} + \frac{B4z}{(z-4)^2} + \frac{C(4z^2+16z)}{(z-4)^3} \quad \text{--- (1)}$$

$$\Rightarrow \frac{Az(z-4)^2 + B\,4z(z-4) + C(4z^2+16z)}{(z-4)^3} = \frac{z^3-8z}{\cancel{(z-4)^3}}$$

$$\Rightarrow Az(z^2+16-8z) + B(4z^2-16z) + C4z^2+16Cz = z^3-8z$$

$$\Rightarrow Az^3+16Az-8Az^2+4Bz^2-16Bz+4Cz^2+16Cz = z^3-8z$$

$\therefore$ By comparing coefficients on both sides

$$z^3\cancel{(A)}\; Az^3 + z^2(-8A+4B+4C) + z(16A-16B+16C) = z^3-8z$$

$A=1$, $\quad -8A+4B+4C=0$

$-8+4B+4C=0$

$4(B+C) = 8 \quad \therefore B+C=2$ --- (2)

$16A + 16C - 16B = -8$

$16 + 16C - 16B = -8$

$16(1 - B + C) = -8$

$1 - B + C = \frac{-8}{16}$

$1 - B + C = -\frac{1}{2}$

$-B + C = -\frac{1}{2} - 1$

$-B + C = -\frac{3}{2}$ — ③

∴ solve eqⁿ ②, ③

$B + C = 2$

$-B + C = -\frac{3}{2}$

$2C = 2 - \frac{3}{2}$

$2C = \frac{1}{2}$

$C = \frac{1}{4}$

∴ Substitute 'C' in eqⁿ ②

$B + C = 2$

$B = 2 - C$

$B = 2 - \frac{1}{4}$

$B = \frac{8-1}{4}$

$B = \frac{7}{4}$

$\therefore$ Substitute 'A, B, C' in eqn ①

$$\therefore \frac{z}{z-4} + \frac{\left(\frac{7}{4}\right)4z}{(z-4)^2} + \frac{\frac{1}{4}(4z^2+16z)}{(z-4)^3} = \frac{z^3-8z}{(z-4)^3}$$

$\therefore$ Apply Inverse z-Transform:

$$\therefore z_T^{-1}\left[\frac{z^3-8z}{(z-4)^3}\right] = z_T^{-1}\left[\frac{z}{z-4} + \frac{7}{4}\left(\frac{4z}{(z-4)^2}\right) + \frac{1}{4}\left(\frac{4z^2+16z}{(z-4)^3}\right)\right]$$

$\therefore$ By linearity Property

$$z_T^{-1}[\bar{u}(z)] = z_T^{-1}\left[\frac{z}{z-4}\right] + \frac{7}{4} z_T^{-1}\left[\frac{4z}{(z-4)^2}\right] + \frac{1}{4} z_T^{-1}\left[\frac{4z^2+16z}{(z-4)^3}\right]$$

$$u_n = 4^n + \frac{7}{4}(4^n \cdot n) + \frac{1}{4}[4^n \cdot n^2]$$

$$\left(\because z_T^{-1}\left[\frac{z}{z-k}\right] = k^n,\ z_T^{-1}\left[\frac{kz}{(z-k)^2}\right] = k^n \cdot n,\ z_T^{-1}\left[\frac{k^2z+kz^2}{(z-k)^3}\right] = k^n \cdot n^2\right)$$

$$u_n = 4^n\left[1 + \frac{7}{4}(n) + \frac{1}{4}(n^2)\right].$$

$$\therefore z_T^{-1}\left[\frac{8z-z^3}{(4-z)^3}\right] = 4^n\left[1 + \frac{7}{4}(n) + \frac{1}{4}(n^2)\right].$$

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(8) Given,

$$(4D^4-4D^3-23D^2+12D+36)y=0$$

Given an Homogeneous equation in the form of $f(D)y=0$

$$\therefore (4D^4-4D^3-23D^2+12D+36)y=0 \quad -(1)$$

$\therefore$ The Auxiallary eq^n of (1) is:-

$$(4m^4-4m^3-23m^2+12m+36)y=0$$

$$\therefore \quad 4m^4-4m^3-23m^2+12m+36=0.$$

let $m=2$

$$4(2)^4-4(2)^3-23(2)^2+12(2)+36=0$$

$$64-32-92+24+36=0$$

$$124-124=0$$

$$0=0(\checkmark)$$

$\therefore$ By synthetic Division method

2	4	-4	-23	12	36
	0	8	8	-30	-36
	4	4	-15	-18	0

$$4m^3+4m^2-15m-18=0$$

let $m=2$

$$4(2)^3+4(2)^2-15(2)-18=0$$

$4(8)+4(4)-30-18=0$

$32+16-48=0$

$48-48=0$

$0=0$ (T)

$\therefore m=2$

$$\begin{array}{c|cccc} 2 & 4 & 4 & -15 & -18 \\ & 0 & 8 & 24 & 18 \\ \hline & 4 & 12 & 9 & 0 \end{array}$$

$4m^2+12m+9=0$

$4m^2+6m+6m+9=0$

$2m(2m+3)+3(2m+3)=0$

$(2m+3)(2m+3)=0$

$m=-\frac{3}{2}, -\frac{3}{2}$

$\therefore m=2,2,-\frac{3}{2},-\frac{3}{2}$

$\therefore m_1=2,2, \quad m_2=-\frac{3}{2},-\frac{3}{2}$

$\therefore$ The complementary function is:-

$y=(c_1+c_2x)e^{m_1x}+(c_3+c_4x)e^{m_2x}$

$y=(c_1+c_2x)e^{2x}+(c_3+c_4x)e^{\left(-\frac{3}{2}\right)x}$

$\therefore$ solution of $(4D^4-4D^3-23D^2+12D+36)y=0$ is:

$y=(c_1+c_2x)e^{2x}+(c_3+c_4x)e^{\left(-\frac{3}{2}\right)x}$