CMR INSTITUTE OF TECHNOLOGY	USN										

Internal Assesment Test I – June - 2024							
Sub:	Power System Analysis - II					Code:	21EE62
Date:	06/06/2024	Duration:	90 Min	Max Marks:	50	Sem: 6	Section: A & B

Note: Answer any **FIVE FULL** Questions & Sketch Neat Figures Wherever Necessary.

		Mark	OBE CO	OBE RBT
1	A power system consists of 4 buses. Generators are connected at buses 1 and 3 reactance's of which are j0.2 and j0.1, respectively. The transmission lines are connected between buses 1-2, 1-4, 2-3, and 3-4 and have reactance's j0.25, j0.5, j0.4, and j0.1, respectively. Find the bus admittance matrix using Singular Transformation Method and verify the same using direct inspection method.	[10]	CO1	L3

Taking bus (1) as reference, the graph is drawn as shown in Fig. E.4.13.

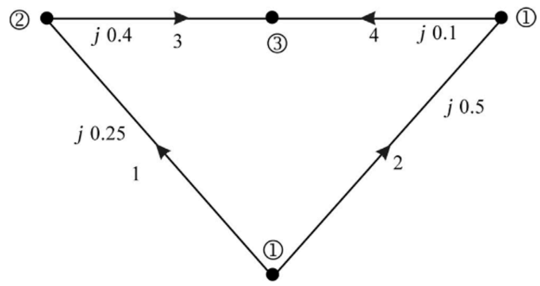


FIGURE E.4.13

Graph for E4.6.

Only the network reactances are considered. Generator reactances are not considered.
By direct inspection:

$Y_{BUS} =$

	(1)	(2)	(3)	(4)
1	$\frac{1}{j0.25}+\frac{1}{j0.5}$	$-\frac{1}{j0.25}$	0	$-\frac{1}{j0.5}$
2	$-\frac{1}{j0.25}$	$\frac{1}{j0.4}+\frac{1}{j0.25}$	$-\frac{1}{j0.4}$	0
3	0	$-\frac{1}{j0.4}$	$\frac{1}{j0.4}+\frac{1}{j0.1}$	$-\frac{1}{j0.1}$
4	$-\frac{1}{j0.5}$	0	$-\frac{1}{j0.1}$	$\frac{1}{j0.1}+\frac{1}{j0.5}$

This reduces to

$Y_{BUS} =$

	(1)	(2)	(3)	(4)
1	$-j6.0$	$j4.0$	0	$j2.0$
2	$j4.0$	$-j6.5$	$j2.5$	0
3	0	$j2.5$	$-j12.5$	$j10$
4	$+j2$	0	$j10$	$-j12$

Deleting the reference bus (1)

$Y_{BUS} =$

	(2)	(3)	(4)
(2)	$-j6.5$	$j2.5$	0
(3)	$j2.5$	$-j12.5$	$j10$
(4)	0	$j10$	$-j12.0$

By singular transformation

The primitive impedance matrix

$$[z] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.25 & 0 & 0 & 0 \\ 0 & j0.5 & 0 & 0 \\ 0 & 0 & j0.4 & 0 \\ 0 & 0 & 0 & j0.1 \end{bmatrix} \end{array}$$

The primitive admittance matrix is obtained by taking the reciprocals of z elements since there are no matrices.

$$[y] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -j4 & & & \\ & j2 & & \\ & & -j2.5 & \\ & & & -j10 \end{bmatrix} \end{array}$$

The bus incidence matrix is from the graph:

$$A = \begin{array}{c|c|c|c|} & (2) & (3) & (4) \\ \hline 1 & -1 & 0 & 0 \\ \hline 2 & 0 & 0 & -1 \\ \hline 3 & +1 & -1 & 0 \\ \hline 4 & 0 & -1 & +1 \\ \hline \end{array}$$

and

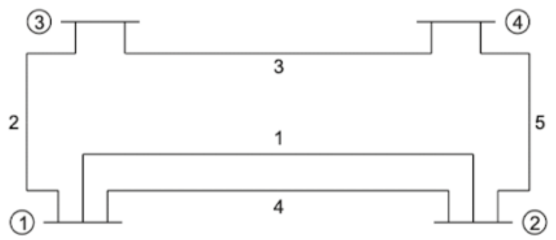
$$y \cdot A = \begin{bmatrix} j4 & 0 & 0 \\ 0 & 0 & j2 \\ -j2.5 & j2.5 & 0 \\ 0 & j10 & -j10 \end{bmatrix}$$

$$A^t y A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} j4 & 0 & 0 \\ 0 & 0 & j2 \\ -j2.5 & j2.5 & 0 \\ 0 & j10 & -j10 \end{bmatrix} = \begin{bmatrix} -j6.5 & j2.5 & 0 \\ j2.5 & -j12.5 & j10 \\ 0 & j10 & -j12.0 \end{bmatrix}$$

2

For the network shown in Fig. Determine the incidence matrices A', A, K, B', B, C and C' and verify the followings: (i) $A_b K^T = U$ (ii) $B_l = A_l K^T$ (iii) $C_b = -B_l^T$ Take node 1 as reference and 1, 2 and 5 as tree branches

[10] CO1 L3



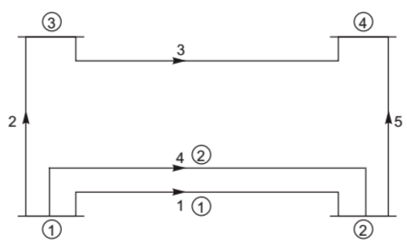


Fig. 1.25

Sol. Number of elements, $e = 5$

Number of branches, $b = n - 1 = 4 - 1 = 3$

Number of links, $l = e - n + 1 = 5 - 4 + 1 = 2$

Element-node incidence matrix (A′)

$A' =$

e \ n	①	②	③	④
1	1	– 1	0	0
2	1	0	– 1	0
3	0	0	1	– 1
4	1	– 1	0	0
5	0	1	0	– 1

Bus-incidence matrix (A)

$A =$

e \ n	②	③	④
1	– 1	0	0
2	0	– 1	0
3	0	1	– 1
4	– 1	0	0
5	1	0	– 1

(∵ ① is reference node)

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

Primitive impedance matrix $[z]$

$$z = \begin{array}{c} e/e \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix} \end{array}$$

Primitive admittance matrix $y = [z]^{-1}$

$$y = [z]^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

To calculate z^{-1} there are two methods:

Method 1:

$$y = [z]^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

Step 5. Interchange columns 3 and 4

$$y' = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ -0.81 & 2.6 & 0 & 0.33 & 0 \\ -2.6 & 0.33 & 0 & 5 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Step 6. Interchange rows 3 and 4

$$y = [z^{-1}] = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ 0.81 & 2.6 & 0 & 0.33 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -2.6 & 0.33 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Limitations: This method is applicable if only the matrices A_2 and A_3 are having the elements zero's only. Otherwise it is not applicable.

Step 1. Interchange rows 3 and 4

$$y' = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

Step 2. Interchange columns 3 and 4

$$y'' = \left[\begin{array}{ccc|cc} 0.2 & 0.05 & 0.1 & 0 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0 & 0.25 & 0 & 0 \\ \hline 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{array}\right]^{-1} = \left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right]$$

Step 3. Above matrix can divide into 4 submatrices

$$A_1 = \begin{bmatrix} 0.2 & 0.05 & 0.1 \\ 0.05 & 0.4 & 0 \\ 0.1 & 0 & 0.25 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$$

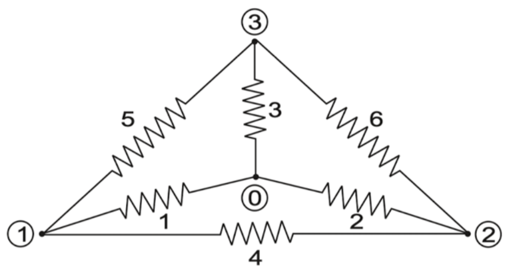
$$A_1^{-1} = \begin{bmatrix} 6.5 & -0.81 & -2.6 \\ -0.81 & 2.6 & 0.33 \\ -2.6 & 0.33 & 5.0 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

Step 4.

$$y'' = \left[\begin{array}{c|c} A_1^{-1} & A_2 \\ \hline A_3 & A_4^{-1} \end{array}\right]$$

$$y'' = \begin{bmatrix} 6.5 & -0.81 & -2.6 & 0 & 0 \\ -0.81 & 2.6 & 0.33 & 0 & 0 \\ -2.6 & 0.33 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

3	For the given network shown in Fig. draw the graph and tree. Write the cut-set and loop schedule. 	[10]	CO2	L3

Sol. Graph of Fig. 1.29 is shown in Fig. 1.30(*a*)

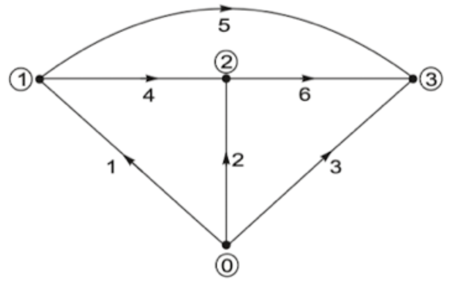


Fig. 1.30(a)

Tree of Fig. 1.30(*a*) is shown in Fig. 1.30(*b*)

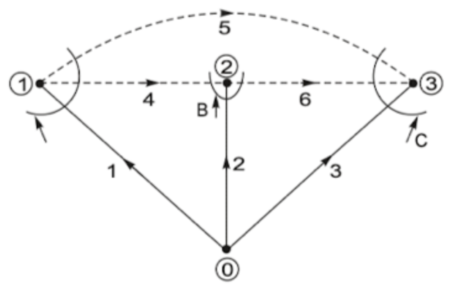


Fig. 1.30(b)

4

Obtain the Primitive impedance and admittance matrix for the elements whose data is given below

[10] CO2 L2

Table 1.3

Element	*Self impedance in p.u.*		*Mutual impedance in p.u.*	
	Bus Code $p - q$	*Impedance* Z_{pq-pq}	*Bus Code* $r - s$	*Impedance* Z_{pq-rs}
1	1 – 2 (1)	0.2	—	—
2	1 – 3	0.4	1–2 (1)	0.05
3	3 – 4	0.5	—	—
4	1 – 2 (2)	0.25	1–2 (1)	0.1
5	2 – 4	0.2	—	—

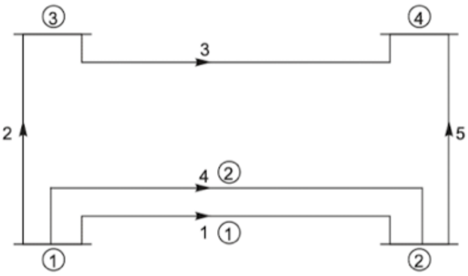


Fig. 1.25

Sol. Number of elements, $e = 5$

Number of branches, $b = n - 1 = 4 - 1 = 3$

Number of links, $l = e - n + 1 = 5 - 4 + 1 = 2$

Element-node incidence matrix (A′)

e \ n	①	②	③	④
1	1	– 1	0	0
2	1	0	– 1	0
3	0	0	1	– 1
4	1	– 1	0	0
5	0	1	0	– 1

A′ = (above)

Bus-incidence matrix (A)

e \ n	②	③	④
1	– 1	0	0
2	0	– 1	0
3	0	1	– 1
4	– 1	0	0
5	1	0	– 1

A = (above) (∵ ① is reference node)

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

Primitive impedance matrix $[z]$

e / e	1	2	3	4	5
1	0.2	0.05	0	0.1	0
2	0.05	0.4	0	0	0
3	0	0	0.5	0	0
4	0.1	0	0	0.25	0
5	0	0	0	0	0.2

$$z = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}$$

Primitive admittance matrix $y = [z]^{-1}$

$$y = [z]^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

To calculate z^{-1} there are two methods:

Method 1:

$$y = [z]^{-1} = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

Step 1. Interchange rows 3 and 4

$$y' = \begin{bmatrix} 0.2 & 0.05 & 0 & 0.1 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{bmatrix}^{-1}$$

Step 2. Interchange columns 3 and 4

$$y'' = \left[\begin{array}{ccc:cc} 0.2 & 0.05 & 0.1 & 0 & 0 \\ 0.05 & 0.4 & 0 & 0 & 0 \\ 0.1 & 0 & 0.25 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.2 \end{array}\right]^{-1} = \left[\begin{array}{c:c} A_1 & A_2 \\ \hdashline A_3 & A_4 \end{array}\right]$$

Step 3. Above matrix can divide into 4 submatrices

$$A_1 = \begin{bmatrix} 0.2 & 0.05 & 0.1 \\ 0.05 & 0.4 & 0 \\ 0.1 & 0 & 0.25 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$A_1^{-1} = \begin{bmatrix} 6.5 & -0.81 & -2.6 \\ -0.81 & 2.6 & 0.33 \\ -2.6 & 0.33 & 5.0 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

Step 4.

$$y'' = \left[\begin{array}{c:c} A_1^{-1} & A_2 \\ \hdashline A_3 & A_4^{-1} \end{array}\right]$$

$$y'' = \begin{bmatrix} 6.5 & -0.81 & -2.6 & 0 & 0 \\ -0.81 & 2.6 & 0.33 & 0 & 0 \\ -2.6 & 0.33 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Step 5. Interchange columns 3 and 4

$$y' = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ -0.81 & 2.6 & 0 & 0.33 & 0 \\ -2.6 & 0.33 & 0 & 5 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Step 6. Interchange rows 3 and 4

$$y = [z^{-1}] = \begin{bmatrix} 6.5 & -0.81 & 0 & -2.6 & 0 \\ 0.81 & 2.6 & 0 & 0.33 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ -2.6 & 0.33 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Limitations: This method is applicable if only the matrices A_2 and A_3 are having the elements zero's only. Otherwise it is not applicable.

<table>
<tr><td>5(a)</td><td>Develop the relation between I_{bus}, V_{bus} and Y_{bus} by assuming no mutual coupling between transmission lines of a three-bus system

The bus impedance matrix Z_{Bus} and bus admittance matrix Y_{Bus} can be determined by using the bus incidence matrix 'A' to related variable parameters of the primitive network quantities of the inter-connected network.
From the primitive network equation (1.8)
$$i + j = [y]\, v \qquad \ldots(1.9)$$
Pre-multiplying the both sides with A^T
$$A^T i + A^T j = A^T [y]\, v \qquad \ldots(1.10)$$
According to Kirchhoff's current law, the algebraic sum of currents meets at any node is equal to zero.
i.e., sum of currents meeting at a node $= A^T i = 0$ …(1.11)
Similarly $A^T j$ = sum of current sources of element incidence at a node and is equal to impressed bus current. It is a column vector
$$A^T j = I_{Bus} \qquad \ldots(1.12)$$
Substituting the equations (1.11) and (1.12) in equation (1.10), we get
$$I_{Bus} = A^T [y]\, v \qquad \ldots(1.13)$$
Power into the network $= [I^*_{Bus}]^T V_{Bus}$
= the sum of powers in the primitive network, *i.e.*, $[j^*]^T v$
Therefore, $$[I^*_{Bus}]^T V_{Bus} = [j^*]^T v \qquad \ldots(1.14)$$
Taking conjugate transpose of equation (1.12), it is modified as
$$[A^T]^{*T} [j^*]^T = [I^*_{Bus}]^T \qquad \ldots(1.15)$$
But A is a real matrix so $A^* = A$,
From matrix property $[A^T]^T = A$
Applying these two conditions in equation (1.15), we get
$$[I^*_{Bus}]^T = A[j^*]^T \qquad \ldots(1.16)$$
Substituting the equation (1.16) in equation (1.14) and simplify
$$A[j^*]^T V_{Bus} = [j^*]^T v$$
$$A\, V_{Bus} = v \qquad \ldots(1.17)$$
Substituting the 'v' from equation (1.17) in equation (1.13)
$$\therefore \quad I_{Bus} = A^T [y]\, A\, V_{Bus} \qquad \ldots(1.18)$$
Bus frame reference of admittance form as
$$I_{Bus} = [Y_{Bus}]\, V_{Bus}$$
Equating the equations (1.1) and (1.18), we get
$$[Y_{Bus}] = A^T [y]\, A \qquad \ldots(1.19)$$
And Z_{Bus} can be determined by
$$[Z_{Bus}] = [Y_{Bus}]^{-1} = \{A^T [y]\, A\}^{-1} \qquad \ldots(1.20)$$</td><td>[5]</td><td>CO1</td><td>L2</td></tr>
<tr><td>5(b)</td><td>What is a primitive network? Obtain the impedance and admittance form of the primitive network.</td><td>[5]</td><td>CO1</td><td>L2</td></tr>
</table>

<table>
<tr><td></td><td>
Consider the network having two nodes *‘a’* and *‘b’* is shown in Fig. 1.10, the equations of primitive network in impedance form can be written as

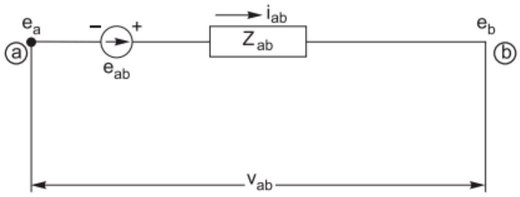


Fig. 1.10

$$v_{ab} = e_a - e_b$$
$$e_a + e_{ab} - z_{ab} i_{ab} = e_b$$
$$e_a - e_b + e_{ab} = z_{ab} i_{ab}$$
$$v_{ab} + e_{ab} = z_{ab} i_{ab}$$
$$v + e = [z]\, i \qquad \ldots(1.7)$$

where v_{ab} = voltage across element ‘$a - b$’
e_{ab} = voltage source in series with element ‘$a - b$’
i_{ab} = current through the element ‘$a - b$’
z_{ab} = impedance matrix of element ‘$a - b$’.

Consider the network shown in Fig. 1.11, the equations of primitive network in admittance form can be written as

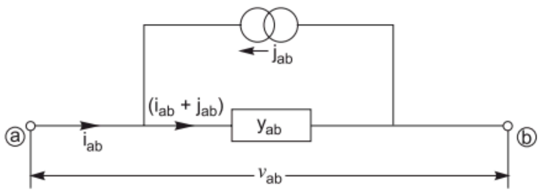


Fig. 1.11

$$i_{ab} + j_{ab} = y_{ab} v_{ab}$$
$$i + j = [y]\, v \qquad \ldots(1.8)$$

where j_{ab} = current source between nodes $a - b$
y_{ab} = admittance matrix of branch $a - b$.
</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td>
Define Directed Graph, Node, Edge, Tree, Twig, Primitive Admittance Matrix, Primitive Impedance Matrix, Branch Path Incidence Matrix, Augmented Loop Matrix and Augmented Cut set Matrix.

Element of a graph: Each network element is replaced by a line segment or an arc while constructing a graph for a network. Each line segmer or arc is called an element. Each potential source is replaced by a short circuit. Each current source is replaced by an open circuit.

Node or vertex (n): The terminal of an element is called a node or a vertex.

Edge (e): An element of a graph is called an edge.

Degree: The number of edges connected to a vertex or node is called its degree.

Rank: The rank of a graph is n-1 where n is the number of nodes in the graph.

Path: A path is defined as a subgraph of connected elements such that not more than two elements are connected to any one node. If there is a path between every pair of node then the graph is said to be connected. Alternatively, a graph is said to be connected if there exists a least one path between every pair of nodes.

Twigs : These are the branches of a tree.

Links : The elements that are not on a tree or elements of the co-tree are called links or chords. The number of links is represented by ‘l’ Number of links, l = e – n + 1. where e =number of elements n =number of nodes
</td><td>[10]</td><td>CO1</td><td>L1</td></tr>
</table>

- **Graph:** An element is said to be incident on a node, if the node is a terminal of the element. Nodes can be incident to one or more elements. The network can thus be represented by an interconnection of elements. The actual interconnections of the elements give a graph.
- **Subgraph:** Any subset of elements of the graph is called a subgraph. A subgraph is said to be proper if it consists of strictly less than all the elements and nodes of the graph.
- **Oriented Graph:** If each line segment of a graph is assigned with a direction, it is called as a oriented graph.
- **Connected Graph:** When there exits at least one path between every pair of nodes, then the graph is called as a connected graph.
- **Planar graph:** A graph is said to be planar, if it can be drawn without-out cross over of edges. Otherwise, it is called nonplanar.
- **Closed path or loop:** The set of elements traversed starting from one node and returning to the same node form a closed path or loop.

******** ALL THE BEST ********

Signature of Paper Setter(s) Signature of CCI HOD – EEE