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Internal Assesment Test III – July - 2024

Sub:	Power System Analysis - II						Code:	21EE62
Date:	31/7/2024	Duration:	90 Min	Max Marks:	50	Sem: 6	Section:	A & B

Note: Answer any **FIVE FULL** Questions & Sketch Neat Figures Wherever Necessary.

		Mark	OBE CO	OBE RBT
1(a)	Explain Runge Kutta Method for the solution of Swing Equation for transient stability analysis.	[5]	CO5	L2

Introduction to the Swing Equation: The swing equation is a fundamental equation used in power system stability studies, particularly in transient stability analysis. It describes the dynamic behavior of a synchronous machine (generator) during disturbances. The swing equation is a second-order nonlinear differential equation that relates the rotor angle δ (the angle between the rotor magnetic field and the stator magnetic field) to the electrical power and mechanical power acting on the generator.

The swing equation is given by:

$$\frac{d^2\delta}{dt^2} = \frac{P_m - P_e}{M}$$

Where:

- δ is the rotor angle in radians.
- P_m is the mechanical power input to the generator.
- P_e is the electrical power output of the generator.
- M is the inertia constant of the machine.

For transient stability analysis, the equation is often expressed in terms of the angular velocity $\omega = \frac{d\delta}{dt}$:

$$M\frac{d\omega}{dt} = P_m - P_e(\delta)$$

This leads to a system of first-order differential equations:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_m - P_e(\delta)}{M}$$

Runge-Kutta Method: The Runge-Kutta method is a powerful numerical technique for solving ordinary differential equations (ODEs). The most commonly used version is the 4th-order Runge-Kutta method, which provides a good balance between accuracy and computational effort.

For the swing equation, the 4th-order Runge-Kutta method can be used to numerically solve the system of differential equations over small time steps h. The method estimates the value of δ and ω at the next time step by considering the weighted average of four estimates (slopes) of the derivative.

Let t_n be the current time, δ_n and ω_n be the current values of the rotor angle and angular velocity, respectively. The next values δ_{n+1} and ω_{n+1} are computed as follows:

1. **Compute intermediate slopes:**

$$k_1^{\delta} = h \cdot \omega_n$$

$$k_1^{\omega} = h \cdot \frac{P_m - P_e(\delta_n)}{M}$$

$$k_2^{\delta} = h \cdot \left(\omega_n + \frac{k_1^{\omega}}{2}\right)$$

$$k_2^{\omega} = h \cdot \frac{P_m - P_e\left(\delta_n + \frac{k_1^{\delta}}{2}\right)}{M}$$

$$k_3^{\delta} = h \cdot \left(\omega_n + \frac{k_2^{\omega}}{2}\right)$$

$$k_3^{\omega} = h \cdot \frac{P_m - P_e\left(\delta_n + \frac{k_2^{\delta}}{2}\right)}{M}$$

$$k_4^{\delta} = h \cdot (\omega_n + k_3^{\omega})$$

$$k_4^{\omega} = h \cdot \frac{P_m - P_e\left(\delta_n + k_3^{\delta}\right)}{M}$$

2. **Update δ and ω using weighted average:**

$$\delta_{n+1} = \delta_n + \frac{1}{6}(k_1^{\delta} + 2k_2^{\delta} + 2k_3^{\delta} + k_4^{\delta})$$

$$\omega_{n+1} = \omega_n + \frac{1}{6}(k_1^{\omega} + 2k_2^{\omega} + 2k_3^{\omega} + k_4^{\omega})$$

3. **Advance the time step:**

$$t_{n+1} = t_n + h$$

Summary:

- The Runge-Kutta method allows accurate numerical integration of the swing equation by iteratively computing the rotor angle and angular velocity at successive time steps.
- This method is particularly useful in transient stability analysis where precise modeling of the generator's response to disturbances is crucial.
- By applying the Runge-Kutta method, power system engineers can predict whether a power system will remain stable or lose synchronism following a disturbance, such as a fault or sudden change in load.

This process is repeated until the desired time span for the simulation is covered. The results can be analyzed to determine the stability of the power system under transient conditions.

1(b) Explain the algorithm for short circuit analysis using Bus Impedance Matrix. [5] CO5 L2

In order to apply the four steps of Algorithm for Short Circuit Computation developed earlier to large systems, it is necessary to evolve a systematic general algorithm so that a digital computer can be used.

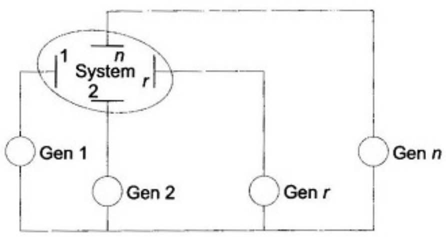


Fig. 9.20 *n*-bus system under steady load

Consider an n-bus system shown schematically in Fig. 9.20 operating at steady load. The first step towards Short Circuit Computation is to obtain prefault voltages at all buses and currents in all lines through a load flow study. Let us indicate the prefault bus voltage vector as

$$V^0_{BUS} = \begin{bmatrix} V_1^0 \\ V_2^0 \\ \vdots \\ V_n^0 \end{bmatrix} \quad (9.18)$$

Let us assume that the rth bus is faulted through a fault impedance Z^f. The postfault bus voltage vector will be given by

$$V^f_{BUS} = V^0_{BUS} + \Delta V \quad (9.19)$$

where

- ΔV is the vector of changes in bus voltages caused by the fault.

As step 2, we drawn the passive Thevenin network of the system with generators replaced by transient/subtransient reactances with their emfs shorted

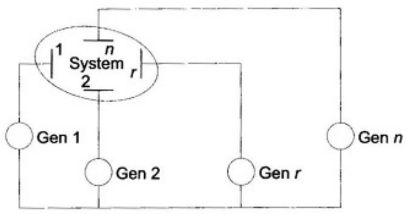


Fig. 9.20 *n*-bus system under steady load

As per step 3 we now excite the passive Thevenin network with $-V^\circ_r$ in series with Z^f as in Fig. 9.21. The vector ΔV comprises the bus voltages of this network.

Now

$$\Delta V = Z_{BUS} J^f \quad (9.20)$$

Where

$$Z_{BUS} = \begin{bmatrix} Z_{11} & \cdots & Z_{1n} \\ \vdots & & \vdots \\ Z_{n1} & \cdots & Z_{nn} \end{bmatrix} = \text{bus impedance matrix of the passive Thevenin network} \quad (9.21)$$

J^f = bus current injection vector

Since the network is injected with current $-I^f$ only at the rth bus, we have

$$J^f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_r^f = -I^f \\ \vdots \\ 0 \end{bmatrix} \quad (9.22)$$

Substituting Eq. (9.22) in Eq. (9.20), we have for the rth bus

$$\Delta V_r = -Z_{rr} I^f$$

By step 4, the voltage at the rth bus under fault is

$$V_r^f = V^0{}_r + \Delta V^0{}_r = V^0{}_r - Z_{rr} I^f \quad (9.23)$$

However, this voltage must equal

$$V_r^f = Z^f I^f \tag{9.24}$$

We have from Eqs. (9.23) and (9.24)

$$Z^f I^f = V_r^0 - Z_{rr} I^f$$

or
$$I^f = \frac{V_r^0}{Z_{rr} + Z^f} \tag{9.25}$$

At the ith bus (from Eqs (9.20) and (9.22))

$$\Delta V_i = - Z_{ir} I^f$$

$$\therefore \quad V_i^f = V_i^0 - Z_{ir} I^f, \; i = 1, 2, ..., n \tag{9.26}$$

substituting for I^f from Eq. (9.25), we have

$$V_i^f = V_i^0 - \frac{Z_{ir}}{Z_{rr} + Z^f} V_r^0 \tag{9.27}$$

For i = r in Eq. (9.27)

$$V_r^f = \frac{Z^f}{Z_{rr} + Z^f} V_r^0 \tag{9.28}$$

In the above relationship V_i^0's, the prefault bus voltages are assumed to be known from a load flow study. Z_{BUS} matrix of the short-circuit study network of Fig. 9.21 can be obtained by the inversion of its Y_{BUS} matrix or the Z_{BUS} building algorithm. It should be observed here that the SC study network of Fig. 9.21 is different from the corresponding load flow study network by the fact that the shunt branches corresponding to the generator reactances do not appear in the load flow study network. Further, in formulating the SC study network, the load impedances are ignored, these being very much larger than the impedances of lines and generators. Of course synchronous motors must be included in Z_{BUS} formulation for the SC study.

Postfault currents in lines are given by

$$I^f_{ij} = Y_{ij} (V^f_i - V^f_j) \tag{9.29}$$

For calculation of postfault generator current, examine Figs. 9.22(a) and (b). From the load flow study (Fig. 9.22(a))

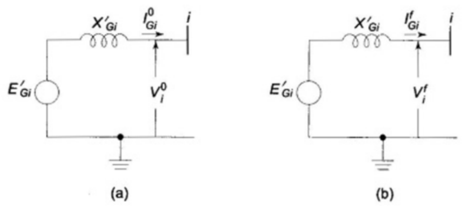


Fig. 9.22

Prefault generator output = $P_{Gi} + jQ_{Gi}$

$$\therefore \quad I^0_{Gi} = \frac{P_{Gi} - jQ_{Gi}}{V_i^0}; \text{ (prefault generator output} = P_{Gi} + jQ_{Gi}) \tag{9.30}$$

$$E'_{Gi} = V_i + jX'_{Gi} I^0_{Gi} \tag{9.31}$$

From the SC study, V^f_i is obtained, It then follows from Fig. 9.22(b) that

$$I^f_{Gi} = \frac{E'_{Gi} - V_i^f}{jX'_{Gi}} \tag{9.32}$$

2	Derive the generalize algorithm for finding Z_{bus}	[10]	CO5	L2

- Added between an old bus and reference bus.
- Added between two old buses.
- Added between new bus to reference bus.
- Added between new bus and existing bus

Modification of an existing bus impedance matrix

Let us denote the orginal $\mathbf{Z}_{bus}$ of a system with n- number of independent buses as $\mathbf{Z}_{orig}$. When a branch of impedance $\mathbf{Z}_b$ is added to the system the $\mathbf{Z}_{orig}$ gets modified. The branch impedance $\mathbf{Z}_b$ can be added to the original system in the following four different ways.

Case 1 : *Adding a branch of impedance Z_b from a new bus-p to the reference bus.*

Case 2 : *Adding a branch of impedance Z_b from a new bus-p to an existing bus-q.*

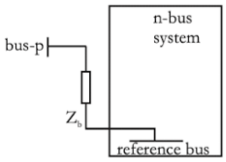
Case 3 : *Adding a branch of impedance Z_b from an existing bus-q to the reference bus.*

Case 4 : *Adding a branch of impedance Z_b between two existing buses h and q.*

The modification of $\mathbf{Z}_{orig}$ for the above four cases have been presented here without proof

Case 1 : Adding Z_b from a new bus-p to the reference bus.

Consider a n-bus system as shown in fig 1.9. Let us add a bus-p through an impedance Z_b to the reference bus. The addition of a bus will increase the order of the bus impedance.



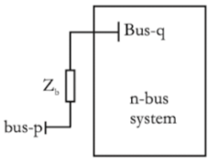
$$Z_{bus,new} = \left[\begin{array}{cccc|c} & & & & 0 \\ & & Z_{orig} & & 0 \\ & & & & \vdots \\ & & & & 0 \\ \hline 0 & 0 & \ldots\ldots & 0 & Z_b \end{array}\right] \quad \ldots\ldots(1.78)$$

Fig 1.9 : *Adding a new bus through an impedance to reference bus.*

In this case the elements of $(n+1)^{th}$ column and row are all zeros except the diagonal. The diagonal element is the added branch impedance Z_b. The elements of original Z_{bus} matrix are not altered. The new bus impedance matrix will be as shown in equ (1.78).

Case 2 : Adding Z_b from a new bus-p to an existing bus-q.

Consider a n-bus system as shown in fig 1.10. in which a new bus-p is added through an impedance Z_b to an existing bus-q. The addition of a bus will increase the order of the bus impedance matrix by one.



$$Z_{bus,new} = \left[\begin{array}{cccc|c} & & & & Z_{1q} \\ & & Z_{orig} & & Z_{2q} \\ & & & & \vdots \\ & & & & Z_{nq} \\ \hline Z_{q1} & Z_{q2} & \ldots\ldots & Z_{qn} & Z_{qq}+Z_b \end{array}\right] \quad \ldots\ldots(1.79)$$

Fig 1.10 : *Adding a new bus through an impedance to an existing bus.*

In this elements of $(n+1)^{th}$ column are the elements of q^{th} column and elements of $(n+1)^{th}$ row are the elements q^{th} row. The diagonal element is given by sum of Z_{qq} and Z_b. The elements of original Z_{bus} matrix are not altered. The new bus impedance matrix will be as shown in equ (1.79).

Case 3 : Adding Z_b from an existing bus-q to the reference bus

Consider a n-bus system shown in fig 1.11 in which an impedance Z_b is added from an existing bus-q to the reference bus. Let us consider as if the impedance Z_b is connected from a new bus-p and existing bus-q. Now it will be an addition as that of case-2. The new bus impedance matrix order $(n+1)$ can be formed as that of case-2. Then we can short-circuit the bus-q to reference bus. This is equivalent to eliminating $(n+1)^{th}$ bus (i.e., bus-p in this case) and so the bus impedance matrix has to be modified by eliminating $(n+1)^{th}$ row and $(n+1)^{th}$ column. The reduced bus impedance matric can be formed by a procedure similar to that of bus elimination in bus admittance matrix, developed in section 1.5. This reduced bus impedance matrix is the actual new bus impedance matrix. Every element of actual new bus impedance matrix can be determined using the equation (1.80).

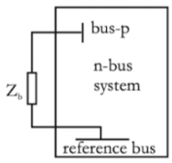


Fig 1.11 : *Adding an impedance between existing bus and reference.*

$$\mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)}Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \quad \ldots\ldots(1.80)$$

Note: 1. $Z_{jk,act}$ is the impedance corresponding to row-j and column-k of actual new bus impedance matrix.

2. Z_{jk}, $Z_{(n+1)k}$, $Z_{j(n+1)}$, $Z_{(n+1)(n+1)}$ are impedance of new bus impedance matrix of order (n + 1).

3. Since bus impedance matix is symmetrical

$$Z_{jk,act} = Z_{kj,act}$$

Case 4 : Adding Z_b between two existing buses h and q

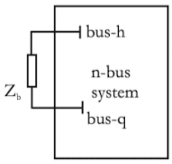


Fig 1.12 : *Adding an impedance between bus-h and bus-q.*

Consider a n-bus system shown in fig 1.12, in which an impedance Z_b is added between two existing buses h and q.

In this case the bus impedance matrix is formed as shown in equ(1.81). Here the elements of $(n + 1)^{th}$ column is the difference between the elements of column-h and column-q. The elements of $(n + 1)^{th}$ row is the difference between the elements of row-h and row-q. The diagonal element is given by equ(1.82).

$$Z_{bus,new} = \left[\begin{array}{c|c} & Z_{1h} - Z_{1q} \\ Z_{orig} & Z_{2h} - Z_{2q} \\ & \vdots \\ & Z_{nh} - Z_{nq} \\ \hline Z_{h1} - Z_{q1} \; Z_{h2} - Z_{q2} \; ......Z_{hn} - Z_{qn} & Z_{(n+1)(n+1)} \end{array}\right] \quad ....(1.81)$$

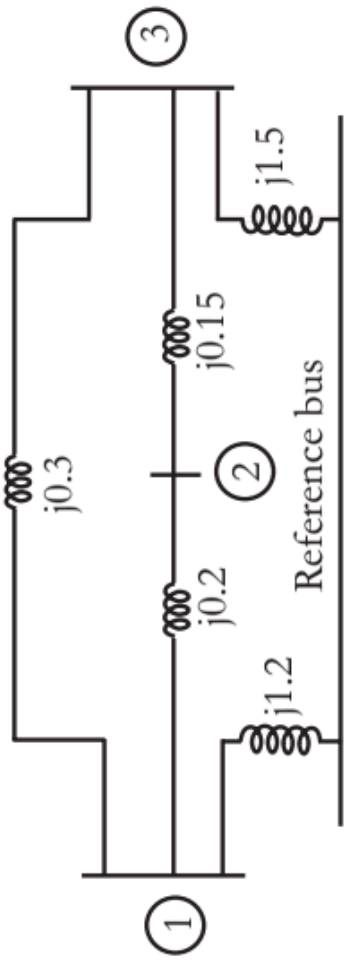
$$Z_{(n+1)(n+1)} = Z_b + Z_{hh} + Z_{qq} - 2\,Z_{hq} \quad ....(1.82)$$

Since the modification does not involve addition of new bus, the order of new bus impedance matrix has to be reduced to nxn by eliminating the $(n + 1)^{th}$ column and $(n + 1)^{th}$ row.

This reduced bus impedance matrix is the actual new bus impedance matrix.Every element of this actual new bus impedance matrix can be determined using equ (1.80) which is also given below for reference.

$$\mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}}$$

3 Determine Z_{bus} for system whose reactance diagram is shown in fig where the impedance is given in p.u. preserve all the three nodes. [10] CO5 L3



Step 1: Consider the branch with impedance j1.2 p.u. connected between bus-1 and reference as shown in fig 1.17.2. The system shown in fig 1.17.2. has a single bus and so the order of bus impedance matrix is on, as shown in below.

Fig 1.17.1

$$\mathbf{Z}_{bus} = [j1.2]$$

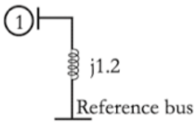


Fig 1.17.2

Step 2: Connect bus-2 to bus-1 through an impedance j0.2 as shown in fig 1.17.3. This is case-2 modification and so the order of bus impedance matrix increases by one. In the new bus impedance matrix, the elements of 1[st] column are copied as elements of 2[nd] column and the elements of 1[st] row are copied as elements of 2[nd] row.The diagonal elements is given by $Z_{11} + Z_b$ where $Z_b = j0.2$.

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.2 + j0.2 \end{bmatrix} = \begin{bmatrix} j1.2 & j1.2 \\ j1.2 & j1.4 \end{bmatrix}$$

Step 3: Connect the bus -3 to bus-2 through an impedance j0.15 as shown in fig 1.17.4. This is case-2 modification and so the order of the bus impedance matrix increases by one. In the new bus impedance matrix the elements of 2[nd] row are copied as elements of 3[rd] row. The diagonal element is given by $Z_{22} + Z_b$ where $Z_b = j0.15$.

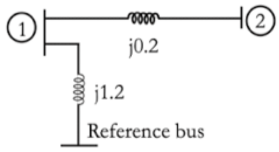


Fig 1.17.3

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.4 + j0.15 \end{bmatrix}$$

$$= \begin{bmatrix} j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 \end{bmatrix}$$

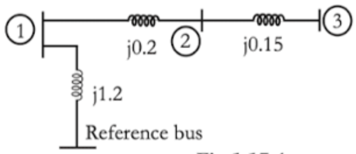


Fig 1.17.4

Step 4 : Connect the impedance j1.5 from bus-3 to reference bus as shown in fig 1.17.5. This is case-3 modification. In case-2 and then the last row and column are eliminated by node elimination techniques.

In new bus impedance matrix the elements of 3[rd] column are copied as elements of 4[th] column and the elements of 3[rd] row are copied as elements of 4[th] row. The diagonal element is given by $Z_{33} + Z_b$ where $Z_b = j1.5$.

$$\therefore \mathbf{Z}_{bus} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j1.55 + j1.5 \end{bmatrix} = \begin{bmatrix} j1.2 & j1.2 & j1.2 & j1.2 \\ j1.2 & j1.4 & j1.4 & j1.4 \\ j1.2 & j1.4 & j1.55 & j1.55 \\ j1.2 & j1.4 & j1.55 & j3.05 \end{bmatrix}$$

The actual new bus impedance matrix is obtained by eliminating the 4[th] row and 4[th] column. The element $\mathbf{Z}_{jk}$ of the actual new bus impedance matrix is given by,

$$\therefore \mathbf{Z}_{jk,act} = \mathbf{Z}_{jk} - \frac{Z_{j(n+1)} Z_{(n+1)k}}{Z_{(n+1)(n+1)}} \quad \text{where } n = 3 \text{ ; } j = 1, 2, 3 \text{ and } k = 1, 2, 3$$

$$Z_{11,act} = Z_{11} - \frac{Z_{14}Z_{41}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.2}{j3.05} = j0.728$$

$$Z_{12,act} = Z_{12} - \frac{Z_{14}Z_{42}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.4}{j3.05} = j0.649$$

$$Z_{13,act} = Z_{13} - \frac{Z_{14}Z_{43}}{Z_{44}} = j1.2 - \frac{j1.2 \times j1.55}{j3.05} = j0.590$$

$$Z_{21,act} = Z_{12,act} = j0.649$$

$$Z_{22,act} = Z_{22} - \frac{Z_{24}Z_{42}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.4}{j3.05} = j0.757$$

$$Z_{23,act} = Z_{23} - \frac{Z_{24}Z_{43}}{Z_{44}} = j1.4 - \frac{j1.4 \times j1.55}{j3.05} = j0.689$$

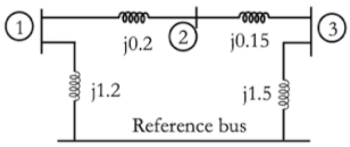


Fig 1.17.5

$$Z_{31,act} = Z_{13,act} = j0.590$$

$$Z_{32,act} = Z_{23,act} = j0.689$$

$$Z_{33,act} = Z_{33} - \frac{Z_{34}Z_{43}}{Z_{44}} = j1.55 - \frac{j1.55 \times j1.55}{j3.05} = j0.762$$

$$\therefore \ \mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 \\ j0.649 & j0.757 & j0.689 \\ j0.590 & j0.689 & j0.762 \end{bmatrix}$$

Step 5: Connect the impedance j0.3 between bus-1 and bus-3 as shown in fig 1.17.6. This is case-4 modification.

In new bus impedance matrix, the elements of 4th column are obtained by substracting the elements of 3rd column from 1st column and the elements of 4th row are obtained by substracting the elements of 3rd row from 1st row.The diagonal element Z_{44} is given by the following equation.

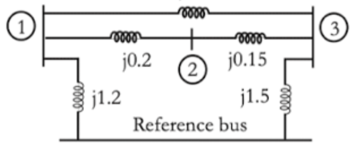


Fig 1.17.6

$$Z_{44} = Z_b + Z_{11} + Z_{33} - 2Z_{13}$$

where $Z_b = j0.3$

$$\therefore \ Z_{44} = j0.3 + j0.728 + j0.762 - 2(j0.59) = j0.61$$

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.728 - j0.59 \\ j0.649 & j0.757 & j0.689 & j0.649 - j0.689 \\ j0.59 & j0.689 & j0.762 & j0.59 - j0.762 \\ j0.728 - j0.59 & j0.649 - j0.689 & j0.59 - j0.762 & j0.61 \end{bmatrix}$$

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.728 & j0.649 & j0.590 & j0.138 \\ j0.649 & j0.757 & j0.689 & -0.04 \\ j0.59 & j0.689 & j0.762 & -j0.172 \\ j0.138 & -0.04 & -j0.172 & j0.61 \end{bmatrix}$$

Since this modification does not add a new bus, the 4th row and column has to be eliminated using node elimination technique, to determine the actual new bus impedance matrix. The element Z^{jk} of actual new bus impedance matrix is given by.

4	What are the transmission loss Coefficients? Derive an expression for transmission loss as a function of plant generation for a two-plant system. **Transmission Loss Coefficients (B-Coefficients)** Transmission losses in a power system occur due to the resistance of transmission lines. These losses are a function of the power flowing through the lines and can be represented mathematically using transmission loss coefficients, also known as **B-Coefficients**.	[10]	CO4	L2

Expression for Transmission Loss in a Two-Plant System

For a power system with two generating plants, the transmission loss P_L can be expressed as a quadratic function of the power outputs of the plants P_1 and P_2. The general expression for transmission loss is:

$$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

Where:

- P_1 is the power generated by Plant 1.
- P_2 is the power generated by Plant 2.
- B_{11}, B_{22}, and B_{12} are the transmission loss coefficients for the two-plant system.

Derivation of Transmission Loss Expression

1. **Total Power Generation:**

   Let P_D be the total power demand. The sum of power generated by both plants must meet the demand plus the transmission losses:

   $$P_1 + P_2 = P_D + P_L$$

2. **Transmission Loss Function:**

   The total transmission loss in the syste ↓ an be represented using the B-Coefficients as mentioned earlier:

   $$P_L = B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2$$

3. **Power Balance Equation:**

   Substituting the transmission loss equation into the power balance equation:

   $$P_1 + P_2 = P_D + (B_{11}P_1^2 + B_{22}P_2^2 + 2B_{12}P_1P_2)$$

4. **Solving for Power Generation:**

   This equation can be used to determine the power outputs P_1 and P_2 of the two plants, given the power demand P_D and the B-Coefficients.

Special Case: Symmetrical System

If the two generating plants are similar (i.e., the transmission loss coefficients are symmetrical, meaning $B_{11} = B_{22}$ and $B_{12} = B_{21}$), the expression simplifies, and further analytical solutions can be derived depending on the given coefficients and demand.

5 (a)	Derive an expression for economic load schedule for an n plant system neglecting the transmission losses	[5]	CO4	L2

Economic load scheduling aims to distribute the total power demand among multiple generating plants in a way that minimizes the total fuel cost while meeting the power demand. When transmission losses are neglected, the problem simplifies significantly.

Step-by-Step Derivation

1. **Objective Function:**

   The total fuel cost C for an n-plant system can be expressed as the sum of the individual costs of operating each plant:

   $$C = \sum_{i=1}^{n} C_i(P_i)$$

   where:

   - $C_i(P_i)$ is the fuel cost function of the i-th plant.
   - P_i is the power generated by the i-th plant.

   The fuel cost function $C_i(P_i)$ is typically a quadratic function of the power output:

   $$C_i(P_i) = a_i + b_iP_i + c_iP_i^2$$

   where a_i, b_i, and c_i are cost coefficients for the i-th plant.

2. **Power Balance Constraint:**

   The total power generated by all the plants must equal the total power demand P_D:

$$\sum_{i=1}^{n} P_i = P_D$$

3. **Lagrange Multiplier Method:**

   To minimize the total cost subject to the power balance constraint, we use the method of Lagrange multipliers. The Lagrangian function $\mathcal{L}$ is defined as:

$$\mathcal{L} = \sum_{i=1}^{n} C_i(P_i) + \lambda \left(\sum_{i=1}^{n} P_i - P_D \right)$$

   where λ is the Lagrange multiplier.

4. **Optimality Condition:**

   To find the minimum cost, take the derivative of the Lagrangian with respect to P_i and set it to zero:

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{dC_i(P_i)}{dP_i} + \lambda = 0$$

   Simplifying, we get:

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{dC_i(P_i)}{dP_i} + \lambda = 0$$

   Simplifying, we get:

$$\frac{dC_i(P_i)}{dP_i} = -\lambda$$

   Since $\frac{dC_i(P_i)}{dP_i}$ is the incremental cost of generation at plant i, denoted as IC_i, we can write:

$$IC_i = b_i + 2c_i P_i = \lambda$$

   This equation implies that the incremental cost of generation for all plants should be equal to the Lagrange multiplier λ for optimal economic dispatch.

5. **Economic Dispatch Equation:**

   The economic dispatch condition for the i-th plant is:

$$b_i + 2c_i P_i = \lambda$$

   Since λ is the same for all plants, we can equate the incremental costs of different plants:

$$b_1 + 2c_1 P_1 = b_2 + 2c_2 P_2 = \ldots = b_n + 2c_n P_n$$

6. **Solving for Power Outputs:**

   To solve for the individual power outputs P_i, express P_i in terms of λ:

$$P_i = \frac{\lambda - b_i}{2c_i}$$

   Substituting this into the power balance equation:

$$\sum_{i=1}^{n} \left(\frac{\lambda - b_i}{2c_i} \right) = P_D$$

   Solve this equation for λ, and then substitute back to find each P_i.

7. **Summary Expression:**

   The final expression for the power generated by each plant is:

$$P_i = \frac{\lambda - b_i}{2c_i}$$

   where λ is determined by:

$$\lambda = \frac{\sum_{i=1}^{n} \left(\frac{b_i}{2c_i} \right) + P_D}{\sum_{i=1}^{n} \left(\frac{1}{2c_i} \right)}$$

Q. No.	Question	Marks	CO	RBT
5 (b)	Explain the steps involved in solving power system stability solution of swing equation using point by point method	[5]	CO5	L2

Consider the swing equation of a power system.

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad .....(5.85)$$

We know that, $M = \frac{H}{\pi f}$ .....(5.86)

$$P_e = P_{max} \sin \delta \quad .....(5.87)$$

$$P_a = P_m - P_e = P_m - P_{max} \sin \delta \quad .....(5.88)$$

$$\therefore \; M \frac{d^2\delta}{dt} = P_a$$

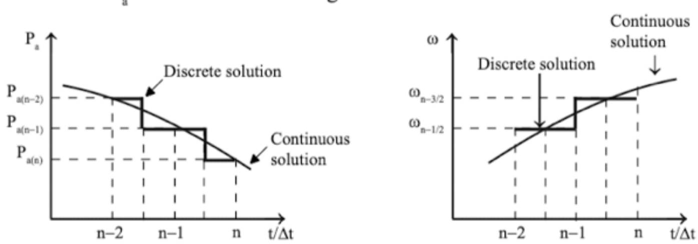
$$\frac{d^2\delta}{dt} = \frac{P_a}{M} \quad .....(5.89)$$

The equation (5.89) in a nonlinear equation. During transient state the δ is a function of time, t and so it can be denoted as δ(t). In point-by-point method, the solution of δ(t) is obtained by dividing the time into small equal values of Δt. (i.e., the entire time (range) of interest is divided into number of small equal interval Δt).

The accelerating power and the change in speed are also continuous function of time. They are discretized as follows,

1. The accelerating power P_a computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered.
2. The angular velocity is assumed constant throughout any interval. This constant value is the value corresponding to the midpoint of concerned interval.

The discretization of P_a and ω are shown in fig 5.13.



Note : *The n represents discretized time instant and it pertains to the end of the interval.*

Fig 5.13 : *Discretization of P_a and ω.*

For each discrete interval the values of P_a, ω and δ are calculated as shown below.

The solution starts from the initial condition values, which corresponds to a stable operating point. Let δ_0 be the angle corresponding to initial operating point.

Let us assume that, the values ω and δ for (n −1)[th] interval are known

δ_{n-1} = The value of δ at the end of $(n-1)^{th}$ interval

$\omega_{n-1/2}$ = The value of ω at the end of $(n-1)^{th}$ interval

$P_{a(n-1)}$ = Value of P_a at the end of $(n-1)^{th}$ interval.

Now from equ(5.89) we get,

$$P_{a(n-1)} = P_m - P_{max} \sin \delta_{n-1} \quad .....(5.90)$$

The equ (5.89) can be written in the modified form as shown in equ(5.91)

$$\frac{d\omega}{dt} = \frac{P_a}{M} \quad .....(5.91)$$

where, $\omega = \frac{d\delta}{dt}$ and $\frac{d\omega}{dt} = \frac{d^2\delta}{dt^2}$

For small changes in δ, the equation (5.91) can be linearized as shown below

$$\frac{\Delta\omega}{\Delta t} = \frac{P_a}{M}$$

$$\therefore\ \Delta\omega = \frac{\Delta t}{M} P_a \qquad .....(5.92)$$

Let $\omega_{n-3/2}$ = The value of ω at the end of n^{th} interval.

For calculating n^{th} interval value of ω, put, $\Delta\omega = \omega_{n-1/2} - \omega_{n-3/2}$ and $P_a = P_{a(n-1)}$ in equ(5.92).

$$\therefore\ \omega_{n-1/2} - \omega_{n-3/2} = \frac{\Delta t}{M} P_{a(n-1)}$$

$$\therefore\ \omega_{n-3/2} = \omega_{n-1/2} - \frac{\Delta t}{M} P_{a(n-1)} \qquad .....(5.93)$$

For small changes in δ, we can write

$$\omega = \frac{\Delta\delta}{\Delta t} \qquad .....(5.94)$$

To solve for change in δ in $(n-1)^{th}$ interval put $\Delta\delta = \Delta\delta_{n-1}$ and $\omega = \omega_{n-3/2}$ in equ(5.94)

$$\therefore \Delta\delta_{n-1} = \Delta t\ \omega_{n-3/2} \qquad .....(5.95)$$

Similarly for change in δ in n^{th} interval put $\Delta\delta = \Delta\delta_n$ and $\omega = \omega_{n-1/2}$ in equ(5.94)

$$\therefore \Delta\delta_n = \Delta t\ \omega_{n-1/2} \qquad .....(5.96)$$

Let δ_n = The value of δ at the end of n^{th} interval.

$$\text{Now, } \delta_n = \delta_{n-1} + \Delta\delta_n \qquad .....(5.97)$$

The above process of computation is repeated to obtain $P_{a(n)}$, $\Delta\delta_{(n+1)}$ and $\delta_{(n+1)}$. The solution of δ(t) is thus obtained in discrete form over the desired length of time. The normal desired length of time is 0.5 sec.

The continuous form of solution is obtained by drawing a smooth curve through discrete values as shown in fig 5.14.

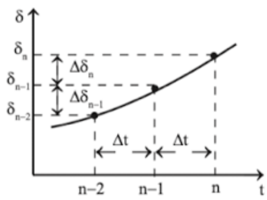


***Fig 5.14 :** Solution of swing equation by point-by-point method*

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6 Incremental fuel costs in rupees per MWh for a plant consisting of Two units are, $dC_1 / dP_{G1} = 0.20\ P_{G1} + 40.00$ and $dC_2 / dP_{G2} = 0.25PG2 + 30.00$. Assume that both the units are always operating, and total load varies from 40MW to 250MW and the maximum and minimum loads on each unit are to be 125 and 20 MW respectively. How will the load be shared between the two units as the system load varies over the full range. **[10] CO4 L3**

Given:

- Unit 1: $\frac{dC_1}{dP_{G1}} = 0.20P_{G1} + 40.00$
- Unit 2: $\frac{dC_2}{dP_{G2}} = 0.25P_{G2} + 30.00$

Condition for Economic Dispatch

For economic dispatch, the incremental costs must be equal:

$$0.20P_{G1} + 40.00 = 0.25P_{G2} + 30.00$$

Solving for the Load Sharing

Rearrange the equation to express one of the power outputs in terms of the other:

$$0.20P_{G1} - 0.25P_{G2} = -10.00$$

Multiply through by 20 to simplify:

$$4P_{G1} - 5P_{G2} = -200$$

$$5P_{G2} = 4P_{G1} + 200$$

$$P_{G2} = \frac{4}{5}P_{G1} + 40$$

This equation relates the power outputs of t wo units.

This equation relates the power outputs of the two units.

Load Sharing Over the Full Range

Now, let's calculate how the load is shared between the two units for the minimum and maximum total system loads.

1. **Minimum Load (40 MW):**

   Let's assume the total load P_D is 40 MW.

   $P_D = P_{G1} + P_{G2} = 40\,\text{MW}$

   Substitute the expression for P_{G2}:

$$P_{G1} + \left(\frac{4}{5}P_{G1} + 40\right) = 40$$

$$P_{G1} + \frac{4}{5}P_{G1} = 0$$

$$\frac{9}{5}P_{G1} = 0 \quad \Rightarrow \quad P_{G1} = 0$$

   From $P_{G2} = \frac{4}{5}P_{G1} + 40$:

$$P_{G2} = 40\,\text{MW}$$

   At the minimum load of 40 MW, all of the load is carried by Unit 2, and Unit 1 does not carry any load.

2. **Maximum Load (250 MW):**

   $P_D = P_{G1} + P_{G2} = 250\,\text{MW}$

   Again, substituting the expression for P_{G2}:

$$P_{G1} + \left(\frac{4}{5}P_{G1} + 40\right) = 250$$

$$P_{G1} + \frac{4}{5}P_{G1} = 210$$

$$\frac{9}{5}P_{G1} = 210 \quad \Rightarrow \quad P_{G1} = \frac{210 \times 5}{9} \approx 116.67\,\text{MW}$$

   Now, P_{G2} can be calculated:

$$P_{G2} = 250 - 116.67 \approx 133.33\,\text{MW}$$

   At the maximum load of 250 MW, Unit 1 generates approximately 116.67 MW, and Unit 2 generates approximately 133.33 MW.

******** ALL THE BEST ********

Signature of Paper Setter(s) | Signature of CCI | HOD – EEE