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Internal Assessment Test – I June 2024

Sub:	Discrete Mathematical Structures							Code:	BCS405A
Date:	03/06/2024	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	AIML/AIDS

Question 1 is compulsory and Answer any 6 from the remaining questions.

		Marks	OBE CO	OBE RBT
1	Define tautology, contradiction, contingency. Determine whether the following compound statement is a tautology or not. $\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$	[8]	CO1	L3
2	Verify the principle of duality for the following logical equivalence $[\sim(p \wedge q) \rightarrow \{\sim p \vee (\sim p \vee q)\}] \Leftrightarrow \sim p \vee q$.	[7]	CO1	L3
3	Prove that for all integers $'k'$ and $'l'$, if k and l both odd, then $k + l$ is an even and kl is an odd by direct proof.	[7]	CO1	L3
4	Write the following argument in symbolic form and then establish the validity: If A gets the Supervisor's position and works hard, then he will get a raise. If he gets a raise, then he will buy a car. He has not purchased a car. Therefore, he did not get the Supervisor's position or he did not work hard.	[7]	CO1	L3

5	Check the validity of following argument. If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles than it has two equal angles. A certain triangle ABC does not have two equal angles $\therefore$ The triangle ABC does not have two usual sides	[7]	CO1	L3
6	Consider the following open statement on set of all real numbers as universe: $p(x): x \geq 0;\ q(x): x^2 > 0; r(x): x^2 - 3x - 4 = 0; s(x): x^2 - 3 > 0$ Then find truth value of $a)\ \exists x[p(x) \wedge q(x)]$ $b)\ \forall x[p(x) \to q(x)]$ $c)\ \forall x[q(x) \to s(x)]$ $d)\ \forall x[r(x) \vee s(x)]$	[7]	CO1	L3
7	Prove by mathematical induction that for every positive integer n, 5 divides $n^5 - n$	[7]	CO2	L3
8	For the Fibonacci numbers $F_0, F_1, F_2, \dots \dots \dots$, prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$.	[7]	CO2	L3

Internal Assessment Test - 1

1) **Tautology**: The compound proposition which has TRUE for all possible truth value is known as Tautology

Contradiction: is a compound proposition which has FALSE for all its possible truth value is known as contradiction

Contingency: is a compound proposition which has ~~to~~ either TRUE or FALSE for its possible truth value.

$\{(p \vee q) \to r\} \leftrightarrow \{\neg r \to (p \vee q)\}$

p	q	r	(A) $(p \vee q)$	$\sim r$	(B) $A \to r$	(C) $\sim r \to A$	$B \Leftrightarrow C$
T	T	T	T	F	T	T	T
T	T	F	T	T	F	F	T
T	F	T	T	F	T	T	T
T	F	F	T	T	F	F	T
F	T	T	T	F	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	F	T	T	T
F	F	F	F	T	T	T	T

∴ The compound statement is totology.

2) let $R = [\sim(p \wedge q) \rightarrow \{\sim p \vee (\sim p \vee q)\}] \Leftrightarrow \sim p \vee q$

$LHS = [\sim(p \wedge q) \rightarrow \{\sim p \vee (\sim p \vee q)\}]$.

(WKT $p \rightarrow q = \sim p \vee q$)

$LHS = \sim(\sim(p \wedge q)) \vee \{\sim p \vee (\sim p \vee q)\}]$.

$R = (p \wedge q) \vee \{\sim p \vee (\sim p \vee q)\}] \Leftrightarrow \sim p \vee q$

$R^D = (p \vee q) \wedge \{\sim p \wedge (\sim p \wedge q)\}\} \Leftrightarrow \sim p \wedge q$

$LHS = (p \vee q) \wedge \{(\sim p \wedge \sim p) \wedge q\}$ (associative law).

$= (p \vee q) \wedge \{(\sim p \wedge q)\}$ (idempotent law).

$= [p \wedge (\sim p \wedge q)] \vee [q \wedge (\sim p \wedge q)]$ (distributive law)

$= [(p \wedge \sim p) \wedge q] \vee [q \wedge (\sim p \wedge q)]$ (associative law).

$= (T_t \wedge q) \vee (q \wedge (q \wedge \sim p)]$ (Inverse & Commutative law).

$= (q) \vee ((q \wedge q) \wedge \sim p)$ (identity & associative law)

$= q \vee (q \wedge \sim p)$ (idempotent law)

$= (q \vee q) \wedge \sim p$ (associative)

$= q \wedge \sim p$ (idempotent law)

$= \sim p \wedge q$ (commutative law).

$=$ RHS

∴ principle of duality is proved.

3) Let,

P = K is odd number. & L is odd number.

q = K+L is even.

r = KL is odd.

Symbolically: $P \rightarrow (q \wedge r)$

Direct proof:

Hypothesis :- $K = 2m+1$ $\left.\begin{matrix} \\ \end{matrix}\right\}$ as K & L is odd number.

$L = 2n+1$ (m belongs to Z → integer & n also belongs to Z).

∴ P is true.

Analysis :- ① K + L is even

$$= (2m+1) + (2n+1)$$

$$= 2m + 2n + 2$$

$$= 2\underbrace{(m+n+1)}_{x}$$

$$= 2x$$ (where $x = m+n+1$ & x belongs to Z).

∴ K+L is even

② $KL = (2m+1)(2n+1)$

$$= 4mn + 2m + 2n + 1$$

$$= 2(2mn + m + n) + 1$$

$$= 2y + 1$$ [where $y = 2mn+m+n$ y belong to Z]

∴ KL is odd

Conclution :- P is true

$(q \wedge r)$ is also true

∴ $P \rightarrow (q \wedge r)$ is true.

4) P: A gets the supervisor's position.
q: A works hard.
r: A will get a raise
s: A will buy a new car.

Symbolically:

$$\left.\begin{array}{c}(p\wedge q)\rightarrow r\\ r\rightarrow s\\ \sim s\end{array}\right\}\text{LHS (primises)}$$

$$\therefore \sim p \vee \sim q \quad \}\text{RHS (conclusion)}.$$

$p \rightarrow q$
$\sim q$
$\sim p$

~~LHS~~
$=[(p\wedge q)\rightarrow r]\wedge[r\rightarrow s]\wedge(\sim s)$
$=[p\wedge q\rightarrow s]\wedge(\sim s)$ (rule of syllogysm)
$=\sim(p\wedge q)$ (Rule of Deny)
$=\sim p\vee\sim q$ (Demorgan's law)
$=$ RHS.

$\therefore$ The argument is valid.

5) P: ~~A~~ triangle has two Equal sides.
q: triangle is isoceles.
r: triangle has two Equal angles
a → ~~the~~ be the triangle.

Symbolically:-

$p(a)\rightarrow q(a)$
$q(a)\rightarrow r(a)$
$\sim r(a)$

$\therefore \sim P(a)$

$[P(a) \rightarrow q(a)] \wedge [q(a) \rightarrow r(a)] \wedge (\sim r(a)) \Rightarrow \sim P(a)$

$= [P(a) \rightarrow r(a)] \wedge (\sim r(a))$ (Rule of syllogysm)

$= \sim [P(a)]$ (Deny)

$= \sim P(a) =$ RHS

$\therefore$ The argument is valid.

6) $P(x) : x \geq 0$

$q(x) : x^2 \geq 0$

$r(x) : x^2 - 3x - 4 = 0$

$s(x) : x^2 - 3 > 0$

a) $\exists x [P(x) \wedge q(x)]$

for $x = 1$

$P(1) : 1 \geq 0$

$q(1) : 1 \geq 0$

$\therefore$ Truth value is True (1)

b) $\forall x [P(x) \rightarrow q(x)]$

for all value of x $P(x) : x \geq 0$ is true also

$q(x) : x^2 \geq 0$ is true.

$\therefore \forall x [P(x) \rightarrow q(x)]$ is True (1)

c) $\forall x [q(x) \rightarrow s(x)]$

$\forall x$ $q(x)$ is true but $s(x)$ not true ~~for~~ $\forall x$

$\therefore \forall [q(x) \rightarrow s(x)]$ is False (0).

d) $\forall x\ [r(x) \vee s(x)]$

$r(x)$ is false $\forall$ values of x.

$s(x)$ is also false $\forall$ ~~all~~ values of x.

↳ $s(x)$ is only false when $x = 1$

$\therefore\ \forall x [r(x) \vee s(x)]$ is False (0).

7) 5 divides $n^5 - n$

<u>Basic step</u> :- let $n = 1$

$$1^5 - 1 = 1 - 1 = 0$$

0 is ~~disv.~~ divisible by 5

$\therefore\ n = 1$ is True.

<u>Induction step</u> :- $n = k$

$$k^5 - k = 5a. \rightarrow (1).$$

Substitute $k = k+1$

i.e $= (k+1)^5 - (k+1)$

$= (5k^5 + 10k^4 + 10k^3 + 5k^2 + 5k + 5) - (k+1)$

(By binomial Expansion)

$= 5(k^5 + 2k^4 + 2k^3 + k^2 + k + 1) - (k+1)$

$= (k^5 - k) - 5\underbrace{(k^5 + 2k^4 + 2k^3 + k^2 + k + 1)}_{b}$

$= 5a - 5b \rightarrow (2)$

By comparing ① & ② the proof of mathematical induction is proved.

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8) $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$

$F_n = F_{n-1} + F_{n-2}$; $n \geq 2$ $\quad F_0 = 0$, $F_1 = 1$

$F_{k+2} = F_{k+1} + F_{k+2-2}$.

Substitute $n = 0$

$F_0 = \frac{1}{\sqrt{5}}[1-1] = 0$

$n = 1$

$F_1 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right]$

$= \underline{\underline{1}}$.

$\therefore$ for $n=0$ and $n=1$ the values are True.

$n = k$

i.e $F_k = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^k - \left(\frac{1-\sqrt{5}}{2}\right)^k\right] \rightarrow ①$

for $n = k+1$

$F_{k+1} = F_k + F_{k+1}$

$$F_{k+2} = F_{k+1} + F_k$$

$$F_{k+1} = F_{k+2} - F_k$$

$$F_{k+1} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1+\sqrt{5}}{2}\right)^{k} - \left(\frac{1-\sqrt{5}}{2}\right)^{k-1} + \left(\frac{1-\sqrt{5}}{2}\right)^{k}\right].$$

$$= \frac{1}{\sqrt{5}}\left[\left(\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left\{\frac{1+\sqrt{5}}{2} + 1\right\}\right) - \left(\left(\frac{1-\sqrt{5}}{2}\right)^{k}\left\{\frac{1-\sqrt{5}}{2} + 1\right\}\right)\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left(\frac{3+\sqrt{5}}{2}\right)\right] - \left(\left(\frac{1-\sqrt{5}}{2}\right)^{k}\left(\frac{3-\sqrt{5}}{2}\right)\right)\right]$$

$$(1+\sqrt{5})^2 = 1+2\sqrt{5}+5 = 6+2\sqrt{5} = 2(3+\sqrt{5})$$

$|||^{ly}$ for $(1-\sqrt{5})^2 = 2(3-\sqrt{5})$.

$$= \frac{1}{\sqrt{5}}\left[\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k-1}\left(\frac{1+\sqrt{5}}{2}\right)^{2}\right] - \left[\left(\frac{1-\sqrt{5}}{2}\right)^{k}\left(\frac{1-\sqrt{5}}{2}\right)^{2}\right]\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{k+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{k+1}\right] \rightarrow (2)$$

$\therefore$ by comparing (1) & (2) recursive function of Fibonacci number is proved.