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Internal Assessment Test – III Aug 2024

Sub:	Discrete Mathematical Structures							Code:	BCS405A
Date:	02/08/2024	Duration:	90 mins	Max Marks:	50	Sem:	IV	Branch:	**AIML (A and B sec)**

Question 1 is compulsory and answer any 6 from the remaining questions.

		Marks	OBE	
			CO	RBT
1	In how many ways the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs?	[8]	CO4	L3
2	Define Derangement. There are eight letters to eight different people to be placed in eight different envelopes. Find the number of ways of doing this so that at least one letter gets to the right person.	[7]	CO4	L3
3	Find the rook polynomial for the 3×3 board using expansion formula.	[7]	CO4	L3
4	Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}, for\ n \geq 2\ given\ a_1 = 5, a_2 = 3$.	[7]	CO4	L3

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5	Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3x - 5 & if \ \ x > 0 \\ -3x + 1 & if \ \ x \leq 0 \end{cases}$. Find $f^{-1}(0), f^{-1}(1), f^{-1}(3), f^{-1}(6)\ and\ f^{-1}([-5,5])$.	[7]	CO3	L3
6	Let f and g be functions from R to R defined by $f(x) = ax + b\ and\ g(x) = 1 - x + x^2.\ If\ (gof)(x) = 9x^2 - 9x + 3$, determine a and b.	[7]	CO3	L3
7	Sate Pigeon-hole Principle. Prove that if any number from 1 to 8 are chosen then two of them will have their sum as 9.	[7]	CO3	L3
8	Sate and prove Lagrange's theorem.	[7]	CO5	L2

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1). The English alphabets can be arranged in 26! ways. $\therefore |S| = 26!$

Let A_1, A_2, A_3, A_4 be the sets where the pattern CAR, DOG, PUN and BYTE occur respectively.

$|A_1| = 24!$ $|A_2| = 24!$ $|A_3| = 24!$ $|A_4| = 23!$

$|A_1 \cap A_2| = |A_1 \cap A_3| = |A_2 \cap A_3| = 22!$

$|A_1 \cap A_4| = |A_2 \cap A_4| = |A_3 \cap A_4| = 21!$

$|A_1 \cap A_2 \cap A_3| = 20!$

$|A_1 \cap A_2 \cap A_4| = |A_2 \cap A_3 \cap A_4| = |A_1 \cap A_3 \cap A_4| = 19!$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 17!$

$\therefore$ The no of way the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs is given by $|\overline{A_1 \cap A_2 \cap A_3 \cap A_4}|$

$$|\overline{A_1 \cap A_2 \cap A_3 \cap A_4}| = |S| - (|A_1| + |A_2| + |A_3| + |A_4|) + (|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3| + |A_1 \cap A_4| + |A_2 \cap A_4| + |A_3 \cap A_4|) - (|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + |A_2 \cap A_3 \cap A_4| + |A_1 \cap A_3 \cap A_4|) + (|A_1 \cap A_2 \cap A_3 \cap A_4|)$$

$$= 26! - (3(24!) + 23!) + (3(22!) + 3(21!)) - (20! + 3(19!)) + 17!$$

$$= 4.014 \times 10^{26} \text{ ways}$$

2) **Derangement**:- It is permutation of n distinct objects where none of the n objects are in its natural place is called derangement.

eight letters to be delived to 8 different pepple. This no of ways the 8 letters are delivered to the 8 different people is given by $8!$ ways

~~The no. of ways in which the 8 letters are delivered~~ The no. of dearrangements of the 8 letters is d_8.

$\therefore$ The No. of ways of doing this so that at least one letter gets to the right person.

$= 8! - d_8$.

$= 8! - (8! \times e^{-1})$

$= 8! (1 - e^{-1})$

$= 25487.10093$.

3] .

1	2	3
4	5 $\circledast$	6
7	8	9

In the 3 × 3 board lets mark 5 as $\circledast$

$\boxed{1}$ $\boxed{3}$

$\boxed{7}$ $\boxed{9}$

D.

1	2	3
4		6
7	8	9

E.

$r_1 = 4$.

$r_2 : (1,9), (3,7)$

$r_2 = 2$.

$r_3 = 0$.

$r_4 = 0$.

$r(D, x) = 1 + 4x + 2x^2$.

$r_1 = 8$.

$r_2 : (1,6)\ (1,8)\ (1,9)$
$(2,4)(2,6)(2,7)(2,9)$
$(3,4)\ (3,7)(3,8)$
$(4,8)\ (4,9)$
$(6,7)\ (6,8)$

$r_2 = 14$.

$r_3 : (1,6,8)\ (2,4,9)\ (2,6,7)$
$(3,4,8)\ (\cancel{1,2,9})$

$r_3 = 4$.

$r_4 = 0$.

$r(E, x) = 1 + 8x + 14x^2 + 4x^3$.

According to rook polynomial.

$r(C,E) = x\,(r(D,x)) + r(E,x)$.

$= x\,(1+4x+2x^2) + 1+8x+14x^2+4x^3$.

$= x+4x^2+2x^3+1+8x+14x^2+4x^3$

$$\boxed{r(C,E) = 1+9x+18x^2+6x^3}$$

4) $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$.

Given $a_1 = 5$, $a_2 = 3$.

$a_n - 3a_{n-1} + 2a_{n-2} = 0$ for $n \geq 2$.

$C_n = 1 \quad C_{n-1} = -3 \quad C_{n-2} = 2$.

according to formula $C_n k^2 + C_{n-1}k + C_{n-2} = 0$.

$k^2 - 3k + 2 = 0$.

$k^2 - 2k - k + 2 = 0$

$k(k-2) - 1(k-2) = 0$

$(k-2)(k-1) = 0$

$k = 2, 1$

General solⁿ is given by

$a_n = A\,2^n + B\,1^n$.

Given $a_1 = 5$, $a_2 = 3$

$5 = A\,2^1 + B\,1^1$

$5 = 2A + B$

$B = 5 - 2A$.

$3 = A\,2^2 + B\,1^2$.

$3 = 4A + B$.

$B = 3 - 4A$.

$5-2A = 3-4A.$

$4A-2A = 3-5.$

$2A = -2.$

$A = -1$

$B = 5-2(-1)$

$B = 5+2$

$B = 7.$

$$\boxed{\therefore a_n = -2^n + 7}$$

5). $f(x) = \begin{cases} 3x-5 & \text{if } x > 0. \\ -3x+1 & \text{if } x \leq 0. \end{cases}$

$f^{-1}(0)$

$f(x) = 0.$

$3x-5 = 0.$

$x = \frac{5}{3} > 0.$

$-3x+1 = 0.$

$x = \frac{1}{3} \nleq 0.$

$f^{-1}(0) = \{5/3\}.$

$f^{-1}(1)$

$f(x) = 1$

$3x-5 = 1$

$x = 2.$

$-3x+1 = 1$

$x = 0.$

$f^{-1}(1) = \{2, 0\}.$

$f^{-1}(3)$

$f(x) = 3$

$3x-5 = 3.$

$3x = 8$

$x = \frac{8}{3}.$

$-3x+1 = 3.$

$x = -\frac{2}{3}$

$f^{-1}(3) = \{8/3, -2/3\}.$

$f^{-1}(6)$

$f(x) = 6.$

$3x-5 = 6.$

$3x = 11$

$x = \frac{11}{3}$

$-3x+1 = 6$

$-3x = 5$

$x = \frac{-5}{3}$

$f^{-1}(6) = \{\frac{11}{3}, \frac{-5}{3}\}.$

$f^{-1}([-5,5])$

$-5 \leq f(x) \leq 5.$ $\quad$ $-5 \leq -3x+1 \leq 5.$

$-5 \leq 3x-5 \leq 5.$ $\quad$ $-6 \leq -3x \leq 4.$ $\quad$ -4/3<=x<=2

$0 \leq 3x \leq 10.$

$0 \leq x \leq \frac{10}{3}$

Here $x \neq 0$ so ignore

$f^{-1}([-5,5]) = \{-4/3, \frac{10}{3}\}.$

6]. $f(x) = ax + b.$

$g(x) = 1 - x + x^2.$

$(g \circ f)(x) = 9x^2 - 9x + 3 \quad - (1).$

$(g \circ f)(x) = g\{f(x)\}.$

$= g\{ax+b\}.$

$= 1 - (ax+b) + (ax+b)^2.$

$= 1 - ax - b + a^2x^2 + b^2 + 2abx.$

$(g \circ f)(x) = a^2x^2 - (a - 2ab)x + (1 - b + b^2). \quad - (2)$

comparing eq (1) & (2).

$a^2 = 9.$ $\quad$ $(a - 2ab) = 9.$

$\boxed{a = \pm 3.}$

when $a = 3.$ $\quad$ when $a = -3.$

$3 - 6b = 9$ $\quad$ $-3 + 6b = 9.$

$-6b = 6.$ $\quad$ $6b = 12$

$\underline{b = -1}$ $\quad$ $\underline{b = 2.}$

7). If m pegions are there and n pegion holes are there such that $m > n$ then ~~at least~~ $p+1$ (or) more pegions will exist in at least one pegion hole.

where. $P = \left\lfloor \frac{m-1}{n} \right\rfloor$

Let us create sets which ~~onto~~ contains pair of numbers from 1 to 8 that will have sum 9.

$A_1 = \{1, 8\}$

$A_2 = \{2, 7\}$.

$A_3 = \{3, 6\}$

$A_4 = \{4, 5\}$.

The pairs in the set A_1, A_2, A_3, and A_4. have sum 9. If we take three ~~nos.~~ numbers together they will not add up to 9

$\therefore$ If we select two numbers from 1 to 8 then. they will have their sum as 9.

8). If G is a finite group and H is a subgroup of G then the order of H will divide G.

<u>Proof</u> :-

If G is a finite set then H is finite.

~~Hae~~ Since H is a subgroup of ~~H~~ G then its closets are also finite

letus consider the right closets of H.

$Ha_1, Ha_2, Ha_3 \cdots Ha_r$.

According to right decomposition

$G = Ha_1 \cup Ha_2 \cup Ha_3 \cdots \cup Ha_r$

$O(G) = O(Ha_1) + O(Ha_2) + \cdots + O(Ha_r)$.

wkt. $O(Ha_1) = O(Ha_2) = \cdots = O(Ha_r) = O(H)$

$O(G) = r\,O(H)$.

$\therefore$ H divides G

<u>Hence proved</u>.