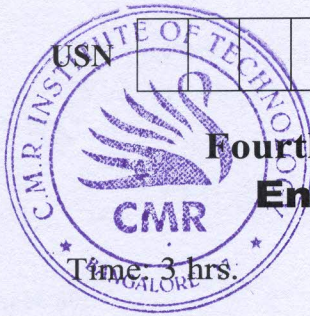


CBCS SCHEME

18EC44



Fourth Semester B.E. Degree Examination, June/July 2024 Engineering Statistics and Linear Algebra

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Derive mean, variance and characteristic function for uniformly distributed random variable. (10 Marks)
- b. The cdf for random variable z is $F_z(z) = \begin{cases} 1 - \exp(-2z^{3/2}) & z \geq 0 \\ 0 & \text{otherwise} \end{cases}$
- Evaluate $P(0.5 < z \leq 0.9)$ (04 Marks)
- c. It is given that $E[X] = 2$ and $E[X^2] = 6$.
- (i) Find standard deviation of X .
- (ii) If $Y = 6X^2 + 2X - 13$. Find mean of Y . (06 Marks)

OR

- 2 a. Given the data in the following table:
- | K | 1 | 2 | 3 | 4 | 5 |
|----------|------|------|------|------|------|
| x_K | 2.1 | 3.2 | 4.8 | 5.4 | 6.9 |
| $p(x_K)$ | 0.21 | 0.18 | 0.20 | 0.22 | 0.19 |
- (i) Plot pdf and cdf of discrete random variable X .
- (ii) Write expression for $f_x(x)$ and $F_x(x)$ using unit delta functions and unit step functions. (08 Marks)
- b. The random variable X is uniformly distributed between 0 and 2. $Y = 3x^3$. What is the pdf of X ? (06 Marks)
- c. The random variable X is uniformly distributed between 0 and 5. The event B is $B = \{X > 3.7\}$. What are $f_{X/B}(x)$, $\mu_{X/B}$ and $\sigma_{X/B}^2$? (06 Marks)

Module-2

- 3 a. A bivariate Pdf is given as $f_{XY}(x, y) = 0.2\delta(x) \delta(y) + 0.3\delta(x-1) \delta(y) + 0.3\delta(x) \delta(y-1) + C\delta(x-1) \delta(y-1)$
- i) What is the value of the constant C ?
- ii) What are the Pdfs for X and Y ?
- iii) What is $F_{XY}(x, y)$ when $(0 < x < 1)$ and $(0 < y < 1)$?
- iv) What are $F_{XY}(x, \alpha)$ and $F_{XY}(\alpha, y)$?
- v) Are X and Y independent? (08 Marks)
- b. The mean and variance of random variable X are -1 and 2. The mean and variance of random variable Y are 3 and 4. The correlation coefficient $\rho_{XY} = 0.5$. What are the covariance $COV[XY]$ and the correlation $E[XY]$. (05 Marks)
- c. Write a short note on Chi-square random variable and students random variable. (07 Marks)

OR

- 4 a. X is a random variable; $\mu_X = 4$ and $\sigma_X = 5$, Y is a random variable, $\mu_Y = 6$ and $\sigma_Y = 7$. The correlation coefficient is 0.2. If $U = 3X + 2Y$. What are $\text{var}[u]$, $\text{cov}[uX]$ and $\text{cov}[uY]$? (08 Marks)

Let 'X' and 'Y' be exponentially distributed random variable with $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

- b. Obtain the characteristic function and Pdf of $W = X + Y$. (06 Marks)
- c. The Random variables X_i have same mean of $m_x = 4$ and variance of $\sigma_x^2 = 1.5$. For $w = \sum_{i=1}^{150} X_i$, determine m_w and σ_w^2 . Also for $w = \frac{1}{150} \sum_{i=1}^{150} X_i$, determine m_y and $\sigma^2 y$. Comment on the result. (06 Marks)

Module-3

- 5 a. Define the following: (04 Marks)
- Random processes
 - Stationary processes.
- b. Write the properties of Autocorrelation function. (06 Marks)
- c. Show that the random process $X(t) = A \cos(\omega_c t + \theta)$ is wide sense stationary. 'θ' is uniformly distributed in the range $-\pi$ to π . (10 Marks)

OR

- 6 a. For the random process $X(t) = A \cos(\omega_c t + \theta)$, A and ω_c are constants. θ is a random variable, uniformly distributed between $\pm \pi$. Show that this process is ergodic. (08 Marks)
- b. Determine the power spectral density of the random process $X(t) = A \cos(\omega_c t + \theta)$ and plot the same. Here θ is random variable uniformly distributed over 0 to 2π . Hence obtain average power of X(t). If the frequency becomes zero, $X(t) = A$ i.e. a d.c. signal, then obtain power spectral density and autocorrelation function. (08 Marks)
- c. A wide sense stationary random process X(t) is applied to a LTI system with impulse response $h(t) = ae^{-at}u(t)$. Find the mean value of the output Y(t) of the system if $E[X(t)] = 6$ and 'a' = 2. (04 Marks)

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Module-4

- 7 a. Write the complete solution as $x_p +$ multiples of s in the null space.
 $x + 3y + 3z = 1$
 $2x + 6y + 9z = 5$
 $-x - 3y + 3z = 5$ (06 Marks)
- b. Find bases for the four subspaces associated with $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$. (04 Marks)
- c. Find orthogonal vector A, B and orthonormal vector q_1 q_2 from a, b using Gram Schmidt process. Factorize into $A = QR$. $a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. (10 Marks)

OR

- 8 a. Reduce A to echlon form. Which combination of rows of A produce zero row? What is the left Null space?

$$A = \begin{bmatrix} 1 & 2 & b_1 \\ 3 & 4 & b_2 \\ 4 & 6 & b_3 \end{bmatrix}$$

(04 Marks)

- b. Project the vector b onto the line through a . Check that e is perpendicular to a .

$$b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(08 Marks)

- c. In order to fit best straight line through four points passing through $b = 0, 8, 8, 20$ at $t = 0, 1, 3, 4$. Set up and solve normal equations $A^T A \hat{x} = A^T b$.

(08 Marks)

Module-5

- 9 a. Mention the properties of determinants.

(08 Marks)

- b. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$, show that matrix A is positive definite matrix.

(06 Marks)

- c. Find the eigen values of $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$.

(06 Marks)

OR

- 10 a. Diagonalize the following matrix, if possible $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

(10 Marks)

- b. Find a singular value of decomposition of, $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$.

(10 Marks)
