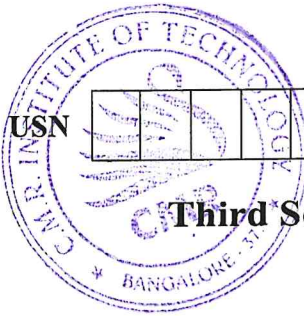


CBCS SCHEME



15MATDIP31

Third Semester B.E. Degree Examination, June/July 2024

Additional Mathematics - I

Time: 3 hrs.

Max. Marks : 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Express $\frac{3+4i}{3-4i}$ in the form $x + iy$. (06 Marks)
- b. Express $\sqrt{3} + i$ in the polar form and hence find their modulus and amplitudes. (05 Marks)
- c. Find the sine of the angle between $\vec{a} = 2i - 2j + k$ and $\vec{b} = i - 2j + 2k$. (05 Marks)

OR

- 2 a. Simplify :
$$\frac{(\cos 3\theta + i \sin 3\theta)^4 (\cos 4\theta + i \sin 4\theta)^5}{(\cos 4\theta + i \sin 4\theta)^3 + (\cos 5\theta + i \sin 5\theta)^{-4}}$$
 (06 Marks)
- b. If $\vec{a} = i + 2j - 3k$ and $\vec{b} = 3i - j + 2k$, then show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are orthogonal. (05 Marks)
- c. Find the value of λ , so that the vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are co-planar. (05 Marks)

Module-2

- 3 a. Obtain the n^{th} derivative of $\sin(ax + b)$. (05 Marks)
- b. Find the pedal equation of $r^n = a^n \cos n\theta$. (05 Marks)
- c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$. (06 Marks)

OR

- 4 a. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$. (05 Marks)
- b. If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (05 Marks)
- c. If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$ (06 Marks)

Module-3

- 5 a. Evaluate $\int_0^{\pi/6} \sin^6 3x \, dx$ using Reduction formula. (06 Marks)
- b. Evaluate $\int_0^1 x^6 \sqrt{1-x^2} \, dx$ - using Reduction formula. (05 Marks)
- c. Evaluate $\int_1^2 \int_0^{2-y} xy \, dx \, dy$. (05 Marks)

OR

- 6 a. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^7 x \, dx$. (06 Marks)
- b. Evaluate $\int_0^{\pi} x \cos^6 x \, dx$. (05 Marks)
- c. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z) \, dz \, dx \, dy$. (05 Marks)

Module-4

- 7 a. A particle moves along a curve $x = e^t$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time variable. Determine the magnitudes of velocity and acceleration at $t = 0$. (05 Marks)
- b. If $\phi = \log(x^2 + y^2 + z^2)$, find the magnitude of the grad ϕ at $(1, 2, 3)$. (05 Marks)
- c. If $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ find $\nabla \cdot \vec{R}$ and $\nabla \times \vec{R}$. (06 Marks)

OR

- 8 a. Find a unit vector normal to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$. (05 Marks)
- b. Find the value of 'a' if the vector $(ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal. (05 Marks)
- c. Show that grad $(x^3 + y^3 + z^3 - 3xyz)$ is irrotational. (06 Marks)

Module-5

- 9 a. Solve $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$. (05 Marks)
- b. Solve $(y^3 - 3x^2y) \, dx + (3xy^2 - x^3) \, dy = 0$. (05 Marks)
- c. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. (06 Marks)

OR

- 10 a. Solve $\frac{dy}{dx} + y \cot x = \cos x$. (05 Marks)
- b. Solve $x^2y \, dx - (x^3 + y^3) \, dy = 0$. (05 Marks)
- c. Solve $y(x + y) \, dx + (x + 2y - 1) \, dy = 0$. (06 Marks)
