CBCS SCHEME

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15MATDIP31

Third Semester B.E. Degree Examination, June/July 2024
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Express $\frac{3+4i}{3-4i}$ in the form $x + iy$. **(06 Marks)**

b. Express $\sqrt{3} + i$ in the polar form and hence find their modulus and amplitudes. **(05 Marks)**

c. Find the sine of the angle between $\vec{a} = 2i - 2j + k$ and $\vec{b} = i - 2j + 2k$. **(05 Marks)**

OR

2 a. Simplify : $\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta + i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 + (\cos 5\theta + i\sin 5\theta)^{-4}}$. **(06 Marks)**

b. If $\vec{a} = i + 2j - 3k$ and $\vec{b} = 3i - j + 2k$, then show that $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are orthogonal. **(05 Marks)**

c. Find the value of λ, so that the vectors $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are co-planar. **(05 Marks)**

Module-2

3 a. Obtain the n^{th} derivative of $\sin(ax + b)$. **(05 Marks)**

b. Find the pedal equation of $r^n = a^n \cos n\theta$. **(05 Marks)**

c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$. **(06 Marks)**

OR

4 a. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **(05 Marks)**

b. If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(05 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ **(06 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi/6} \sin^6 3x\, dx$ using Reduction formula. **(06 Marks)**

b. Evaluate $\int_0^1 x^6 \sqrt{1 - x^2}\, dx$ – using Reduction formula. **(05 Marks)**

c. Evaluate $\int_1^2 \int_0^{2-y} xy\, dx\, dy$. **(05 Marks)**

OR

6 a. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^7 x \, dx$. **(06 Marks)**

b. Evaluate $\int_0^{\pi} x \cos^6 x \, dx$. **(05 Marks)**

c. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x + y + z) \, dz \, dx \, dy$. **(05 Marks)**

Module-4

7 a. A particle moves along a curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time variable. Determine the magnitudes of velocity and acceleration at $t = 0$. **(05 Marks)**

b. If $\phi = \log(x^2 + y^2 + z^2)$, find the magnitude of the grad ϕ at (1, 2, 3). **(05 Marks)**

c. If $\vec{R} = x\hat{i} + y\hat{i} + z\hat{k}$ find $\nabla \cdot \vec{R}$ and $\nabla \times \vec{R}$. **(06 Marks)**

OR

8 a. Find a unit vector normal to the surface $xy^3z^2 = 4$ at (-1, -1, 2). **(05 Marks)**

b. Find the value of 'a' if the vector $(ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal. **(05 Marks)**

c. Show that grad $(x^3 + y^3 + z^3 - 3xyz)$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. **(05 Marks)**

b. Solve $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. **(05 Marks)**

c. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. **(06 Marks)**

OR

10 a. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(05 Marks)**

b. Solve $x^2ydx - (x^3 + y^3)dy = 0$ **(05 Marks)**

c. Solve $y(x + y)dx + (x + 2y - 1)dy = 0$ **(06 Marks)**

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