CBCS SCHEME

USN

17MATDIP31

Third Semester B.E. Degree Examination, June/July 2024
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the sine of the angle between $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(08 Marks)**

b. Express the complex number $\dfrac{(1+i)(1+3i)}{1+5i}$ in the form $a + ib$. **(06 Marks)**

c. Find the modulus and amplitude of $\dfrac{(1+i)^2}{3+i}$. **(06 Marks)**

OR

2 a. Show that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cdot\cos^n\left(\dfrac{\theta}{2}\right)\cdot\cos\left(\dfrac{n\theta}{2}\right)$. **(08 Marks)**

b. If $\vec{a} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{b} = 8\hat{i} - 4\hat{j} + \hat{k}$, then prove that $\vec{a}$ is perpendicular to $\vec{b}$. Also find $|\vec{a} \times \vec{b}|$. **(06 Marks)**

c. Determine λ such that $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{j} + 3\hat{k}$ are coplanar. **(06 Marks)**

Module-2

3 a. If $y = e^{a\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$. **(08 Marks)**

b. Find the angle between the curves $r = \dfrac{a}{1+\cos\theta}$ and $r = \dfrac{b}{1-\cos\theta}$. **(06 Marks)**

c. If $u = \log\left(\dfrac{x^4 + y^4}{x+y}\right)$ then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3$. **(06 Marks)**

OR

4 a. Using Maclaurin's series expand $\sin x$ upto the term containing x^5. **(08 Marks)**

b. Find the pedal equation of the curve $r^m \cos m\theta = a^m$. **(06 Marks)**

c. If $u = x + y + z$, $v = y + z$, $w = z$ then find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$. **(06 Marks)**

Module-3

5 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x\,dx$, $(n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^a \dfrac{x^7}{\sqrt{a^2 - x^2}}\,dx$ **(06 Marks)**

c. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$ **(06 Marks)**

OR

6 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x dx$, $(n > 0)$. **(08 Marks)**

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\, dx$ **(06 Marks)**

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)\,dx\,dy\,dz$ **(06 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\,\hat{k}$. Find the components of velocity and acceleration at $t = \frac{\pi}{8}$ along $\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + \hat{k}$. **(08 Marks)**

b. Find divergence and curl of the vector $\vec{F} = (xyz + y^2z)\hat{i} + (3x^2 + y^2z)\hat{j} + (xz^2 - y^2z)\hat{k}$. **(06 Marks)**

c. Find the directional derivative of $\phi = x^2yz^3$ at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(06 Marks)**

OR

8 a. Find the angle between the tangents to the curve $x = t^2$, $y = t^3$, $z = t^4$ at $t = 2$ and $t = 3$. **(08 Marks)**

b. Find $\text{curl}(\text{curl}\vec{A})$ where $\vec{A} = xy\hat{i} + y^2z\hat{j} + z^2y\hat{k}$. **(06 Marks)**

c. Find the constants a, b, c such that the vector field $(\sin y + az)\hat{i} + (bx\cos y + z)\hat{j} + (x + cy)\hat{k}$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve : $(x^2 - y^2)dx - xy\,dy = 0$. **(08 Marks)**

b. Solve : $(1 + y^2)dx = (\tan^{-1}y - x)dy$. **(06 Marks)**

c. Solve : $(x^2 + y^2 + 1)dx + 2xy\,dy = 0$. **(06 Marks)**

OR

10 a. Solve : $x^2y\,dx - (x^3 + y^3)dy = 0$. **(08 Marks)**

b. Solve : $\left\{y\left(1 + \frac{1}{x}\right) + \cos y\right\}dx + (x + \log x - x\sin y)dy = 0$. **(06 Marks)**

c. Solve : $(x+1)\frac{dy}{dx} - ye^{3x}(x+1)^2\frac{dy}{dx} + \frac{y}{x} = 1$. **(06 Marks)**

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