# CBCS SCHEME

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21MATCS41



## Fourth Semester B.E. Degree Examination, June/July 2024
# Mathematical Foundations for Computing, Probability and Statistics

Time: 3 hrs. Max. Marks: 100

*Note:* 1. ***Answer any FIVE full questions, choosing ONE full question from each module.***
2. ***Provide data table book.***

## Module-1

1 a. Define tautology. Show that the compound proposition $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology for any propositions p, q, r. **(06 Marks)**

b. Prove that (i) $p \vee [p \wedge (p \vee q)] \equiv p$ (ii) $[(\neg p \vee \neg q) \to (p \wedge q \wedge r)] \equiv p \wedge q$ using the laws of logic. **(07 Marks)**

c. Prove that for all integers k and $\ell$ is k and $\ell$ are both odd, then $k + \ell$ is even and $k\ell$ is odd. **(07 Marks)**

**OR**

2 a. Define: (i) Universal quantifiers (ii) Existential quantifiers, with an example. **(06 Marks)**

b. Test the validity of the following argument.
I will become famous or I will not become a musician.
I will become a musician.
Therefore I will become famous. **(07 Marks)**

c. Suppose the universe consist of integers. Consider the following open statements:
$p(x) : x \leq 3$, $q(x) : x + 1$ is odd $r(x) : x > 0$.
Write down the truth values of:
(i) $p(2)$ (ii) $\vee q(4)$ (iii) $p(-1) \wedge q(1)$ (iv) $\sim p(3) \vee r(0)$
(v) $p(0) \to q(0)$ (vi) $p(1) \leftrightarrow \sim q(2)$ (vii) $p(4) \vee (q(1) \wedge r(2))$ **(07 Marks)**

## Module-2

3 a. Let A and B be finite sets with $|A| = m$ and $|B| = n$. Find how many one to one functions are possible from A to B. If there are 60 1 – 1 functions from A to B and $|A| = 3$, what is $|B|$? **(06 Marks)**

b. Let $A = \{1, 2, 3, 4, 6, 12\}$ and R be a relation on A defined by aRb if "a is a multiple of b". Write down the relation R, relation matrix M(R) and draw its digraph. **(07 Marks)**

c. Define: (i) Null graph (ii) Bipartite graph (iii) Euler circuit. Give an example for each. **(07 Marks)**

**OR**

4 a. Draw the Hasse diagram representing the positive divisors of 48. **(06 Marks)**

b. Consider the functions f and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1 \ \forall \ x \in R$. Find gof, fog, $f^2$. **(07 Marks)**

c. Define isomorphism of graphs. Prove that 2 graphs below are isomorphic.

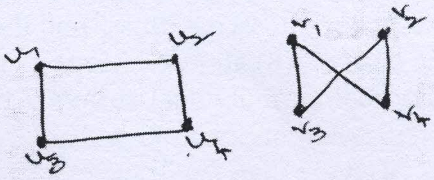


Fig.Q.4(c) **(07 Marks)**



Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

## Module-3

5 a. Find the correlation coefficient between the speed and the stopping distance and the equations of regression lines.

| Speed, x | 16 | 24 | 32 | 40 | 48 | 56 |
|---|---|---|---|---|---|---|
| Stopping distance, y | 0.39 | 0.75 | 1.23 | 1.91 | 2.77 | 3.81 |

(06 Marks)

b. Fit a best curve of the form $y = ax^b$ for the following data:

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| y | 0.5 | 2 | 4.5 | 8 | 12.5 |

(07 Marks)

c. Fit a straight line by the method of least squares.

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| y | 14 | 13 | 9 | 5 | 2 |

(07 Marks)

**OR**

6 a. The following are the percentage of marks in 2 subjects of 9 students. Find the rank correlation coefficient.

| x | 38 | 50 | 42 | 61 | 43 | 55 | 67 | 46 | 72 |
|---|---|---|---|---|---|---|---|---|---|
| y | 41 | 64 | 70 | 75 | 44 | 55 | 62 | 56 | 60 |

(06 Marks)

b. Fit a 2nd degree parabola $y = a + bx + cx^2$ for the data:

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 1 | 3 | 7 | 13 | 21 | 31 |

(07 Marks)

c. Given that $8x - 10y + 66 = 0$ and $40x - 18y = 214$ are the regression equations. Find the means of x and y and correlation coefficient. Find $\sigma_y$ if $\sigma_x = 3$. (07 Marks)

## Module-4

7 a. A random variable X has the following probability function:

| x | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| P(x) | 0.1 | K | 0.2 | 2K | 0.3 | K |

Find: (i) K (ii) $P(X < 1)$ (iii) $P(X > -1)$ (06 Marks)

b. Find the mean and standard deviation of Poisson distribution. (07 Marks)

c. The mean weight of 500 students in a school is 50 kgs and the standard deviation is 6 kgs. Assuming that the weights are normally distributed, find the expected number of students weighing (i) between 40 and 50 kg (ii) more than 60 kg. Given that $A(1.67) = 0.4525$. (07 Marks)

**OR**



8 a. Find the constant K such that

$$f(x) = \begin{cases} Kx^2, & 0 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function. Find the mean. (06 Marks)

b. When an honest coin is tossed 4 times, find the probability of getting:
(i) exactly one head (ii) atmost 3 heads (iii) at least 2 heads (07 Marks)

c. The probability that an individual suffers a bad reaction from a certain injection is 0.001. Using Poisson distribution, find the probability that out of 2000 individuals:
(i) exactly 3 (ii) more than 2 will suffer a bad reaction. (07 Marks)

## Module-5

**9** a. X and Y are independent random variables such that X takes 1, 5 with probabilities $\frac{1}{2}, \frac{1}{2}$ respectively. Y takes –4, 2, 7 with probabilities $\frac{3}{8}, \frac{3}{8}$ and $\frac{1}{4}$ respectively. Find the joint probability distribution of X and Y. Find Cov (X, Y). **(06 Marks)**

b. Find the student 't' for the following variables values in a sample of eight –4, –2, –2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero. **(07 Marks)**

c. The following are the I.Q's of a randomly chosen sample of 10 boys: 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Does this data support the hypothesis that the population mean of I.Q's is 100 at 5% level of significance? **(07 Marks)**

**OR**

**10** a. Explain the terms:
(i) Null hypothesis
(ii) Alternate hypothesis
(iii) Levels of significance
(iv) Type 1 and Type 2 errors **(06 Marks)**

b. A die is thrown 60 times and the frequency distribution for the number appearing on the face x is given by the following table:

| x | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequency | 15 | 6 | 4 | 7 | 11 | 17 |



Test the hypothesis that the die is unbiased. Use Chisquare test at 5% level of significantly. **(07 Marks)**

c. The nine items of a sample have the following values 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5 ($t_{0.05} = 2.31$). **(07 Marks)**

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