CMR
INSTITUTE OF
TECHNOLOGY

USN

Internal Assesment Test-I									
Sub:	Electromagnetic Theory							Code:	BEC401
Date:	05/06/2024	Duration:	90 mins	Max Marks:	50	Sem:	4th	Branch:	ECE(A,B,C,D)
Answer any **FIVE FULL** Questions									

	Marks	OBE CO	OBE RBT
1. a) Give the rectangular coordinates of the point C(4.4, -115°, 2). Give the spherical coordinates of the point D(-3, 2, 1).	[06]	CO1	L3

(a) $x = \rho\cos\phi = 4.4\cos(-115^\circ) = -1.860$

$y = \rho\sin\phi = 4.4\sin(-115^\circ) = -3.99$

$z = 2$.

$\therefore C(x = -1.860,\ y = -3.99,\ z = 2)$

Soln.
(a) $x = -3,\ y = 2,\ z = 1$.

$\therefore r = \sqrt{x^2+y^2+z^2} = \sqrt{9+4+1} = \sqrt{14} = 3.74$.

$\theta = \cos^{-1}\left(\frac{z}{3.74}\right) = \cos^{-1}\left(\frac{1}{3.74}\right) = 74.5^\circ$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{-3}\right) = 33.69^\circ$

Given point is in 2nd quadrant.

$\therefore \phi_{final} = 180^\circ - |\phi_{calculated}| = 180^\circ - 33.69^\circ = 146.30$

	Marks	CO	RBT
1. b) Transform the vector **B** = (y $\mathbf{a_x}$ – x $\mathbf{a_y}$ + z $\mathbf{a_z}$ **)** into cylindrical coordinates.	[04]	CO1	L3

$B_\rho = \vec{B}\cdot\hat{a}_\rho = y(\hat{a}_x\cdot\hat{a}_\rho) - x(\hat{a}_y\cdot\hat{a}_\rho)$

$= y\cos\phi - x\sin\phi = \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi$

$= 0$

$B_\phi = \vec{B}\cdot\hat{a}_\phi = y(\hat{a}_x\cdot\hat{a}_\phi) - x(\hat{a}_y\cdot\hat{a}_\phi)$

$= -y\sin\phi - x\cos\phi$

$= -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$.

Thus, $\vec{B} = -\rho\,\hat{a}_\phi + z\hat{a}_z$.

[05] CO1 L1

2. a) State and explain Coulomb's law in vector form.

Coulomb stated that the force between two very small objects separated in a vacuum or free space by a distance, which is large compared to their size, is proportional to the charge on each and inversely proportional to the square of the distance between them, or

$$F = k\frac{Q_1 Q_2}{R^2}$$

where Q_1 and Q_2 are the positive or negative quantities of charge, R is the separation, and k is a proportionality constant. If the International System of Units[1] (SI) is used, Q is measured in coulombs (C), R is in meters (m), and the force should be newtons (N). This will be achieved if the constant of proportionality k is written as

$$k = \frac{1}{4\pi\epsilon_0}$$

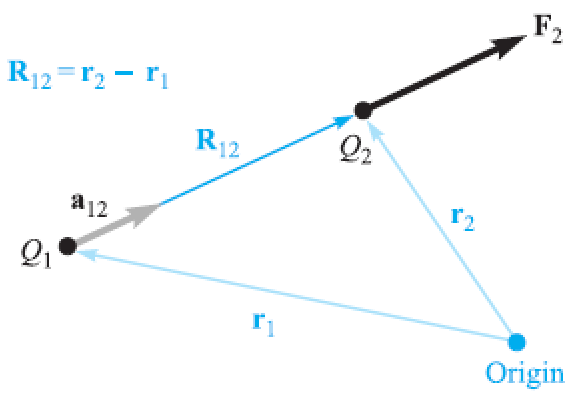
The new constant ϵ_0 is called the *permittivity of free space* and has magnitude, measured in farads per meter (F/m),

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi}10^{-9} \text{ F/m}} \quad (1)$$

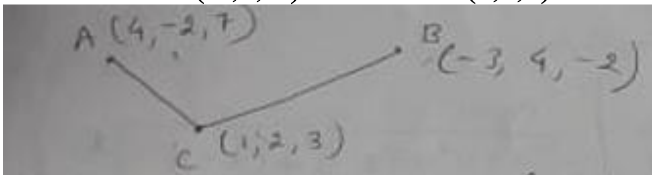
The quantity ϵ_0 is not dimensionless, for Coulomb's law shows that it has the label $C^2/N \cdot m^2$. We will later define the farad and show that it has the dimensions $C^2/N \cdot m$; we have anticipated this definition by using the unit F/m in equation (1).

Coulomb's law is now

$$\boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}} \quad (2)$$



2\. b) Let a point charge Q1 = 25 nC be located at A(4,-2,7) and a charge Q2 = 60 nC be located at B(-3,4,-2). Find **E** at C(1,2,3). [05] CO1 L3



$$\vec{R_1} = (1-4)\,\hat{a}_x + (2+2)\,\hat{a}_y + (3-7\,\hat{a}_z)$$

$$= (-3\hat{a}_x + 4\hat{a}_y - 4\hat{a}_z)$$

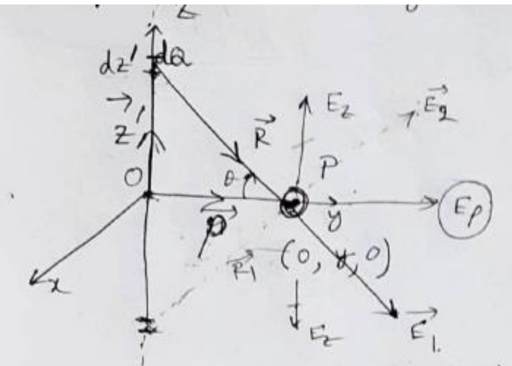
$$\vec{R_2} = (4\hat{a}_x - 2\hat{a}_y + 5\hat{a}_z)$$

$$|\vec{R_1}| = \sqrt{41} \quad \text{and} \quad |\vec{R_2}| = \sqrt{45}$$

$$\vec{E} = \frac{10^{-9}}{4\pi\epsilon_0}\left[\frac{25\times(-3\hat{a}_x+4\hat{a}_y-4\hat{a}_z)}{(41)^{3/2}} + \frac{60\times(4\hat{a}_x-2\hat{a}_y+5\hat{a}_z)}{(45)^{3/2}}\right]$$

$$= 4.58\,\hat{a}_x - 0.15\,\hat{a}_y + 5.51\,\hat{a}_z$$

3\. Define line charge density. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution. [10] CO1 L2



$\frac{z'}{\rho} = \tan\theta$

$z' = \rho \tan\theta$

ρ_L C/m

Find $\vec{E}$ at $P(0, y, 0)$ because of the line charge along z-axis.

We consider incremental charge

$dQ = \rho_L \, dz'$ . . . (1) [charge on length dz']

$\vec{R} = \vec{\rho} - \vec{z}'$ $\quad (\vec{z}' + \vec{R} = \vec{\rho})$

$\vec{R} = (\rho \hat{a}_\rho - z' \hat{a}_z)$ . . (2) $\quad |\vec{R}| = \sqrt{\rho^2 + z'^2}$

∴ The field at P, because of dQ,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 |\vec{R}|^2} \hat{a}_R = \frac{\rho_L \, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)} \cdot \frac{(\rho \hat{a}_\rho - z' \hat{a}_z)}{(\rho^2 + z'^2)^{1/2}}$$

$[\because |\vec{R}| = \sqrt{\rho^2 + z'^2}]$

$$= \frac{\rho_L \, dz' \cdot (\rho \hat{a}_\rho - z' \hat{a}_z)}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

We know from the symmetry of the problem - (3)

$$\vec{dE} = \frac{\rho_L \, dz' \rho \hat{a}_\rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$\therefore$ The field at P,

$$\vec{E} = \int d\vec{E} = \int_{z' \to -\infty}^{\infty} \frac{\rho_L \, dz' \rho \, \hat{a}_\rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2 + z'^2)^{3/2}} \hat{a}_\rho$$

Note
$\tan\theta = \frac{z'}{\rho}$

$z' = \rho \tan\theta$ | $z' \to \infty, \ \theta = \pi/2$

$dz' = \rho \sec^2\theta \, d\theta$ | $z' \to -\infty, \ \theta = -\pi/2$

$$\therefore \vec{E} = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2 + \rho^2 \tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2)^{3/2} (1 + \tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \cancel{\rho}}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\cancel{\rho} \sec^2\theta \, d\theta}{\cancel{\rho^3}\,\rho \, (\sec^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{\sec^3\theta} \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} \, d\theta \, \hat{a}_\rho$$

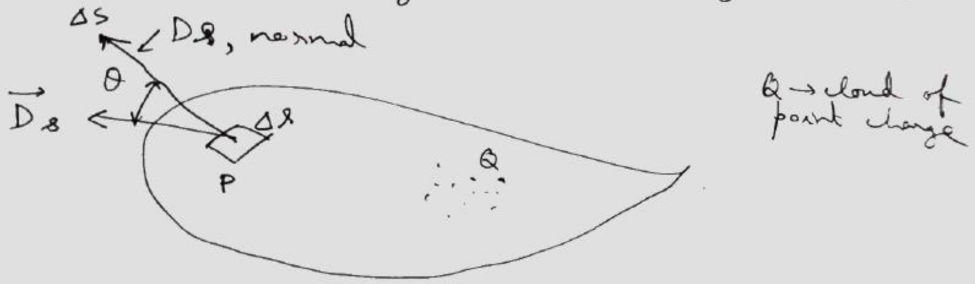
$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} [\sin\theta]_{-\pi/2}^{\pi/2} \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} [1 + 1] \hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho$$

4. a) State and explain the mathematical form of Gauss's law. [05] CO2 L1

2. (a) Derive Mathematical form of Gauss's law, and appl...

Gauss's law : The electric flux passing through any closed surface is equal to the total charge enclosed by the surface



$D_s \rightarrow$ flux density at the surface
↓
varies from one point to another on the surface

$\Delta S \rightarrow$ a small portion of the surface.

$\vec{\Delta S} \rightarrow$ mag. and direction

direction normal at that point.

outward +ve for any surface.

At any pt. P consider an incremental surface ΔS.

$\vec{D}_s$ makes an angle θ with $\Delta \vec{S}$.

Then flux crossing $\Delta \vec{S}$ is then,

$\Delta\psi$ = flux crossing ΔS

$= \vec{D}_s \cdot \vec{\Delta S}$.

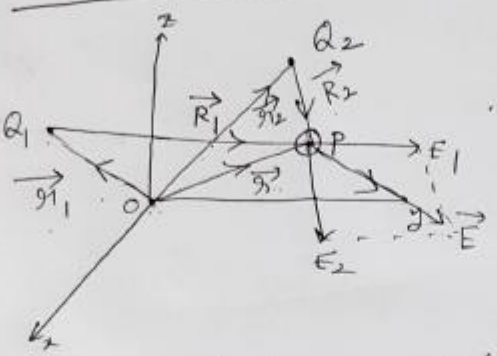
∴ Total flux passing through entire closed surface,

$$\psi = \int d\psi = \oint_{\text{closed surface}} \vec{D}_s \cdot d\vec{S} = \text{charge enclosed} = Q$$

This type of closed surface is a //

4. b) Derive the expression for electric field intensity at a point due to N number of point charges. [05] CO1 L2

Electric Field Intensity due to n p. charges



$$\therefore \boxed{\vec{E} = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0|\vec{r}-\vec{r_m}|^2}\hat{a}_m \ \text{V/m}}$$

Intensity $\vec{E_1}$ at P due to charge Q_1,

$\vec{E_1} = \frac{Q_1}{4\pi\epsilon_0|\vec{R_1}|^2}\hat{a_1}$ where, $\hat{a_1} \to$ unit vector along $\vec{R_1}$.

$\vec{r_1} + \vec{R_1} = \vec{r}$ or $\vec{R_1} = (\vec{r} - \vec{r_1})$

$$\therefore \vec{E_1} = \frac{Q_1}{4\pi\epsilon_0|\vec{r}-\vec{r_1}|^2} \cdot \frac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|} = \frac{Q_1}{4\pi\epsilon_0|\vec{r}-\vec{r_1}|^2} \cdot \hat{a_1} \ \text{V/m}$$

Now, from ~~od~~ diagram, $\vec{r_2} + \vec{R_2} = \vec{r}$

or $\vec{R_2} = \vec{r} - \vec{r_2}$

$\therefore$ Field at P due to charge Q_2

$$\vec{E_2} = \frac{Q_2}{4\pi\epsilon_0|\vec{r}-\vec{r_2}|^2} \cdot \hat{a_2} \ \text{V/m}$$

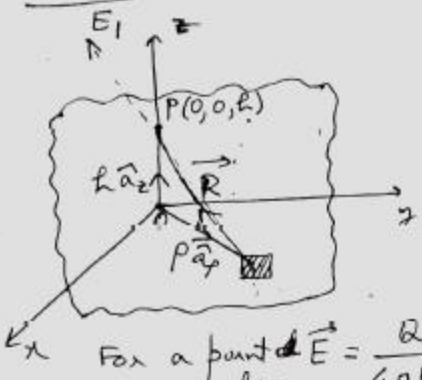
Similarly for the n-th charge, $\vec{E_n} = \frac{Q_n}{4\pi\epsilon_0|\vec{r}-\vec{r_n}|^2}\hat{a}_n$

$\therefore$ The total field $\vec{E}$,

$$\vec{E} = \vec{E_1} + \vec{E_2} + \dots \vec{E_n} = \frac{Q_1}{4\pi\epsilon_0|\vec{r}-\vec{r_1}|^2}\hat{a_1} + \frac{Q_2}{4\pi\epsilon_0|\vec{r}-\vec{r_2}|^2}\hat{a_2} + \dots \frac{Q_n}{4\pi\epsilon_0|\vec{r}-\vec{r_n}|^2}\hat{a_n}$$

5. Define surface charge density. Obtain an expression of electric field intensity due to an infinite sheet of charge with uniform surface charge distribution ρ_s C/m^2. Assume the charge is placed over $x - y$ plane. [10] CO1 L2

Electric field intensity due to an infinite surface charge ~~along~~ on x-y plane



Surface charge density,

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta Q}{\Delta s} \quad C/m^2$$

For a point charge, $\vec{E} = \dfrac{Q}{4\pi\epsilon_0 |\vec{R}|^2}\, \hat{a}_R$

1) $dQ = ??$
2) $\vec{R} = ??$ $\;\rightarrow\;$ $\vec{\rho} + \vec{R} = h\hat{a}_z$
$\vec{R} = h\hat{a}_z - \rho\hat{a}_\rho$
3) $\hat{a}_R = ??$
4) ~~Variable~~ Symmetry of problem
5) Integration.

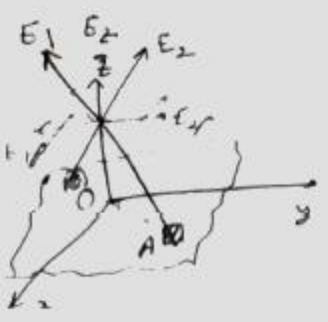
uniform charge density $\rho_s\ C/m^2$

$$dQ = \rho_s \cdot |d\vec{A}| = \rho_s \cdot (\rho\, d\rho\, d\phi)$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}}$$

$\therefore$ The intensity at P due to the chosen infinitesimal surface area,

$$\vec{dE} = \frac{\rho_s \cdot (\rho\, d\rho\, d\phi)}{4\pi\epsilon_0 (h^2+\rho^2)} \frac{(h\hat{a}_z - \rho\hat{a}_\rho)}{\sqrt{h^2+\rho^2}} \quad \text{(1)}$$



From symmetry of the problem,

$$\vec{dE} = \frac{dQ}{4\pi\epsilon_0(\rho^2+h^2)} \cdot \frac{h\,\hat{a}_z}{\sqrt{\rho^2+h^2}}$$

$$\therefore \vec{dE} = \frac{\rho_s.(\rho\,d\rho\,d\phi)(h.\hat{a}_z)}{4\pi\epsilon_0(\rho^2+h^2)^{3/2}} \cdot (4)$$

$\therefore$ The total electric field intensity at P,

$$\vec{E} = \iint \vec{dE} = \frac{\rho_s.h}{4\pi\epsilon_0}\int_{\rho=0}^{\infty}\int_{\phi=0}^{2\pi}\frac{\rho.d\rho.d\phi}{(\rho^2+h^2)^{3/2}}\hat{a}_z$$

$$= \frac{\rho_s.h}{4\pi\epsilon_0}\int_{\rho=0}^{\infty}\frac{\rho\,d\rho}{(\rho^2+h^2)^{3/2}}\int_{\phi=0}^{2\pi}d\phi\ \hat{a}_z$$

$$= \frac{\rho_s.h}{2\epsilon_0}\int_{\rho=0}^{\infty}\frac{\rho\,d\rho}{(\rho^2+h^2)^{3/2}}\hat{a}_z$$

Let, $\rho = h\tan\theta$, $d\rho = h\sec^2\theta\,d\theta$ | when $\rho=0$, $\theta=0$; when $\rho=+\infty$, $\theta=\pi/2$

$$\vec{E} = \frac{\rho_s.h}{2\epsilon_0}\int_0^{\pi/2}\frac{(h\tan\theta).h\sec^2\theta\,d\theta}{(h^2+h^2\tan^2\theta)^{3/2}}\hat{a}_z$$

$$= \frac{\rho_s.h}{2\epsilon_0}\int_0^{\pi/2}\frac{h^2\tan\theta\sec^2\theta\,d\theta}{h^3.\sec^2\theta\sec\theta}\hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0}\int_0^{\pi/2}\tan\theta.\frac{1}{\sec\theta}\,d\theta\,\hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0}\int_0^{\pi/2}\frac{\sin\theta}{\cos\theta}.\cos\theta\,d\theta\,\hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0}\int_0^{\pi/2}\sin\theta\,d\theta\,\hat{a}_z$$

$$= \frac{\rho_s}{2\epsilon_0}\left[-\cos\theta\right]_0^{\pi/2}\hat{a}_z = \frac{\rho_s}{2\epsilon_0}\hat{a}_z$$

i.e. $\boxed{\vec{E} = \frac{\rho_s}{2\epsilon_0}\hat{a}_z}$ V/m.

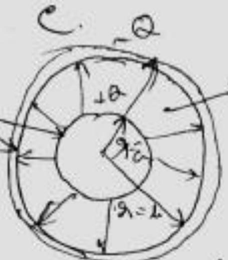
6. a) Define electric flux density. Derive the relation between electric flux density and electric field intensity. [02] CO2 L1

charge on the outer sphere depends on proportionality constant which is 1 for S.I.

- If electric flux ψ.

For Faraday's experiment $\boxed{\psi = \phi}$

ψ unit → C.



metal conducting sphere

Insulating or dielectric material

At the surface of the inner sphere, ψ C of electric flux are produced by charge Q.

charge on inner sphere = Q

" " outer " = −Q.

Surface area of inner sphere = $4\pi a^2$ m^2.

∴ Density of the flux at this surface is

$$\frac{\psi}{4\pi a^2} \text{ or } \frac{Q}{4\pi a^2} \text{ C/m}^2.$$

Electric flux density = $\vec{D}$.

Also called displacement flux density or displacement density.

The direction of D at a pt. is the direction of the flux lines at that point, and the mag. is given by the no. of flux lines crossing a surface normal to the lines divided by the surface area

$$\therefore \vec{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\,\hat{a}_r \quad \text{(inner sphere)}$$

$$\vec{D}\Big|_{r=b} = \frac{-Q}{4\pi b^2}\,\hat{a}_r \quad \text{(outer sphere).}$$

$a \le r \le b$

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r.$$

Shrink inner sphere, smaller and smaller. we reach point charge

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r.$$

Q lines of flux are symmetrically directed outward from the pt. and pass through an imaginary spherical surface of area $4\pi r^2$.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r.$$

$\therefore \boxed{D = \epsilon_0 E}$ → free space.

6. b) Find electric field intensity at P(1,1,1) caused by 4 identical 3 nC charges located at $P_1(1,1,0)$, $P_2(-1,1,0)$, $P_3(-1,-1,0)$, $P_4(1,-1,0)$. [08] CO1 L2

Find $\vec{E}$ at (1, 1, 1) caused by 4 identical 3nC charges located at P1 (1, 1, 0), P2 (−1, 1, 0), P3 (−1, −1, 0) and P4 (1, −1, 0).

Solution

$$\left.\begin{array}{l} P1\,(1,1,0) \\ P2\,(-1,1,0) \\ P3\,(-1,-1,0) \\ P4\,(1,-1,0) \end{array}\right\} \rightarrow 3\,nC \qquad \vec{E} \text{ at } P(1,1,1)$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 |\vec{R}|^2}\,\hat{a}_R \ \ V/m$$

$$3nC \underset{(1,1,0)}{\overset{P1}{\bullet}} \xrightarrow{\vec{R}_1} \odot P(1,1,1)$$

$$\vec{R}_1 = (1-1)\,\hat{a}_x + (1-1)\,\hat{a}_y + (1-0)\,\hat{a}_z = \hat{a}_z$$

$$|\vec{R}_1| = \sqrt{1^2} = 1$$

$$\hat{a}_R = \frac{\hat{a}_z}{|\hat{a}_z|} = \hat{a}_z$$

$$\vec{E}_1 = \frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\cdot\frac{\hat{a}_z}{1^2} = \frac{3\times10^{-9}\,\hat{a}_z}{4\pi\times8.854\times10^{-12}}$$

$$=$$

$$3nC \underset{(-1,1,0)}{\overset{P2}{\bullet}} \xrightarrow{\vec{R}_2} \odot \overset{(1,1,1)}{P}$$

$$\vec{R}_2 = (2\hat{a}_x + \hat{a}_z)$$

$$|\vec{R}_2| = \sqrt{5}\ , \quad \hat{a}_R = \frac{(2\hat{a}_x + \hat{a}_z)}{\sqrt{5}}$$

$$\therefore \vec{E}_2 = \left(\frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\right)\frac{1}{(\sqrt{5})^2}\cdot\frac{(2\hat{a}_x + \hat{a}_z)}{\sqrt{5}}$$

$$\underset{P_3}{(-1,-1,0)} \xrightarrow{\vec{R}_3} \odot P(1,1,1) \qquad \vec{R}_3 = 2\hat{a}_x + 2\hat{a}_y + \hat{a}_z$$

$$\therefore |\vec{R}_3| = \sqrt{2^2+2^2+1} = \sqrt{9} = 3$$

$$\therefore \vec{E}_3 = \frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\cdot\frac{1}{3^2}\cdot\boxed{\frac{(2\hat{a}_x + 2\hat{a}_y + \hat{a}_z)}{3}}$$

3nC P_4 $\quad \vec{R_4} \quad$ P

(1, −1, 0) $\longrightarrow$ (1, 1, 1)

$$\vec{R_4} = (2\hat{a}_y + \hat{a}_z)$$

$$\therefore |\vec{R_4}| = \sqrt{5}$$

$$\vec{E_4} = \left(\frac{3\times 10^{-9}}{4\pi \times 8.854\times 10^{-12}}\right)\frac{1}{(\sqrt{5})^2}\cdot\frac{(2\hat{a}_y + \hat{a}_z)}{\sqrt{5}}$$

∴ The total intensity at P

$$\vec{E} = \vec{E_1} + \vec{E_2} + \vec{E_3} + \vec{E_4}$$

$$= \frac{3\times 10^{-9}}{4\pi\times 8.854\times 10^{-12}}\left[\hat{a}_z + \frac{1}{5\sqrt{5}}(2\hat{a}_x + \hat{a}_z) + \frac{(2\hat{a}_x + 2\hat{a}_y + \hat{a}_z)}{27} + \frac{(2\hat{a}_y + \hat{a}_z)}{5\sqrt{5}}\right]$$

$$\simeq 26.96 [\qquad]$$

$$= 6.82\,\hat{a}_x + 6.82\,\hat{a}_y + 32.8\,\hat{a}_z \text{ V/m}$$

7. Four 10 nC positive charges are located on $z = 0$ plane at the corners of a square 8 cm. on a side. A fifth 10 nC charge is located at a point 8 cm. distant from other charges. Calculate the magnitude of the total force on this fifth charge in free space. [10] CO1 L3

Arrange the charges in the xy plane at locations (4,4), (4,-4), (-4,4), and (-4,-4). Then the fifth charge will be on the z axis at location $z = 4\sqrt{2}$, which puts it at 8cm distance from the other four. By symmetry, the force on the fifth charge will be z-directed, and will be four times the z component of force produced by each of the four other charges.

$$F = \frac{4}{\sqrt{2}} \times \frac{q^2}{4\pi\epsilon_0 d^2} = \frac{4}{\sqrt{2}} \times \frac{(10^{-8})^2}{4\pi(8.85\times 10^{-12})(0.08)^2} = \underline{4.0 \times 10^{-4}\ \text{N}}$$