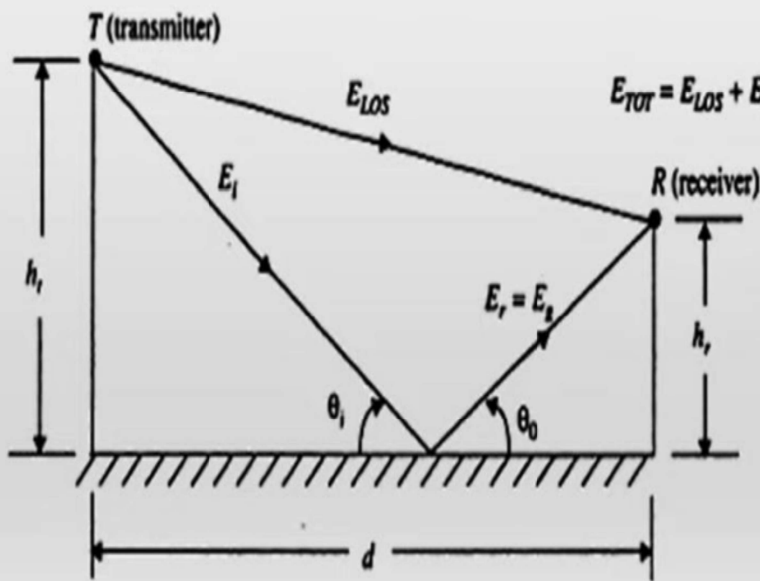


1. Free space propagation model is in accurate when used alone
2. This model is found reasonably accurate when compared with the FSPL Model
3. 2 ray model assumes both LOS and Reflected Signal for modelling the path loss



Assumptions:

1. Height of the antenna are larger than the wavelength of propagating wave. i.e., $h_t, h_r \gg \lambda$
2. Height of the antenna are lesser than the T-R Separation. i.e., $h_t, h_r \ll d$
3. Reflected waves are considered as gracing incidence waves
4. Lack of curvature of earth

Legend:

1. h_t - Height of Tx.Antenna
2. h_r - Height of Rx.Antenna
3. E_{LOS} - E Field of LOS Signal
4. E_g - E Field of Ref. Signal
5. d - (T - R) Separation

Parameters to be Estimated:

1. E Field of Both Rays E_{LOS} & E_g
2. Path Difference (Δ)
3. Phase Difference (θ_Δ)
4. Time delay (τ_d)

General Equation for Plane wave in free space is given by:

$$E(Z, t) = E_o \cdot e^{-\alpha Z} \cdot \text{Cos}(\omega t - \beta Z) \cdot \hat{a}_x$$

Let us rewrite the above equation with our own variables:

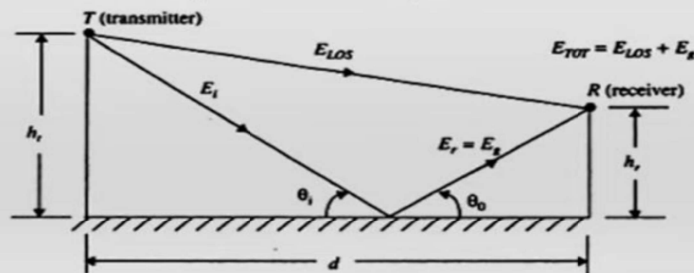
$$E(d, t) = \frac{E_o d_o}{d} \cdot \text{Cos} \left(\omega_c \left(t - \frac{d}{c} \right) \right)$$

$$\text{Cos} \left(\omega_c t - \omega_c \frac{d}{c} \right)$$

$$\text{Cos} \left(\omega_c t - 2\pi f_c \frac{d}{c} \right)$$

$$\text{Cos} \left(\omega_c t - \frac{2\pi d}{\lambda} \right)$$

$$\text{Cos}(\omega_c t - \beta d)$$



Calculating E Field (E_{tot}):

Let us define the E field eqn. for LOS Signal and Reflected Signal:

$$E_{los}(d', t) = \frac{E_o d_o}{d'} \cdot \text{Cos} \left(\omega_c \left(t - \frac{d'}{c} \right) \right) \quad \text{--- (i)}$$

$$E_g(d'', t) = \Gamma \frac{E_o d_o}{d''} \cdot \text{Cos} \left(\omega_c \left(t - \frac{d''}{c} \right) \right) \quad \text{--- (ii)}$$

An Ideal receiver just adds up all the received signal so the total E Field Eqn. is Given by:

$$|E_{tot}| = |E_{los}| + |E_g|$$

$$|E_{tot}| = \frac{E_o d_o}{d'} \cdot \text{Cos} \left(\omega_c \left(t - \frac{d'}{c} \right) \right) + \Gamma \frac{E_o d_o}{d''} \cdot \text{Cos} \left(\omega_c \left(t - \frac{d''}{c} \right) \right) \quad \text{--- (iii)}$$

We know that from the laws of reflection in dielectrics, the reflection coefficient is (-1)

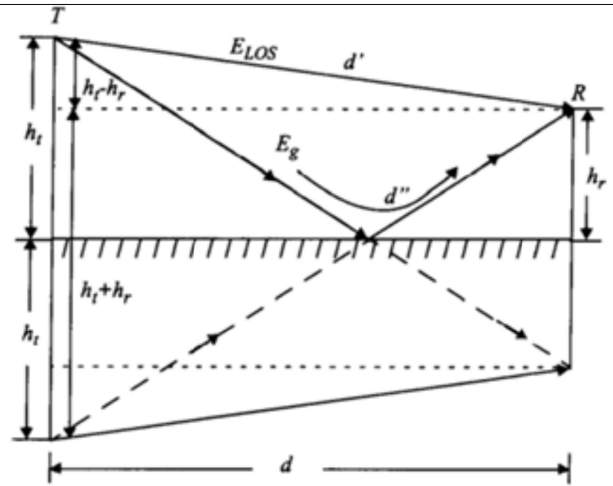
$$|E_{tot}| = \frac{E_o d_o}{d'} \cdot \text{Cos} \left(\omega_c \left(t - \frac{d'}{c} \right) \right) + (-1) \frac{E_o d_o}{d''} \cdot \text{Cos} \left(\omega_c \left(t - \frac{d''}{c} \right) \right) \quad \text{--- (iv)}$$

Reflection
Coefficient

Using the *method of images*, Δ , between the line-of-sight and the ground reflected paths can be expressed as

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$\Delta = d'' - d' \approx \frac{2h_t h_r}{d}$$



The combined E-field becomes

$$E_{\text{TOT}}(d, t) = E_{\text{LOS}}(d', t) + E_g(d'', t) = \frac{E_0 d_0}{d'} \exp\left(j\omega_c \left(t - \frac{d'}{c}\right)\right) - \frac{E_0 d_0}{d''} \exp\left(j\omega_c \left(t - \frac{d''}{c}\right)\right)$$

At some time, say at $t = d''/c$

$$E_{\text{TOT}}\left(d, t = \frac{d''}{c}\right) = \frac{E_0 d_0}{d'} \exp\left(j \frac{\omega_c \Delta}{c}\right) - \frac{E_0 d_0}{d''} \quad - \quad (V)$$

Also note that, when $d \gg h_t + h_r$,

$$\left| \frac{E_0 d_0}{d'} \right| \approx \left| \frac{E_0 d_0}{d''} \right| \approx \left| \frac{E_0 d_0}{d} \right|$$

- 3 A mobile is located 5 km away from the base station and uses a vertical $\lambda/4$ monopole antenna with a gain of 2.55 dB, to receive cellular signal. The E field at 1 km away from the transmitter is measured to be 10–3 v/m, the carrier frequency is 900 MHz A] Find the length and the effective aperture of the receiving antenna. B] Find the received power at the mobile using 2 ray ground reflection model assuming height of 50 m and receiving antenna is 1.5m above the ground. 5+5

Given:

T-R separation distance = 5 km

E-field at a distance of 1 km = 10^{-3} V/m

Frequency of operation, $f = 900$ MHz

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = 0.333 \text{ m.}$$

Length of the antenna, $L = \lambda/4 = 0.333/4 = 0.0833 \text{ m} = 8.33 \text{ cm.}$

Gain of antenna = 1.8 = 2.55 dB.

(b) Since $d \gg \sqrt{h_t h_r}$, the electric field is given by

$$E_R(d) \approx \frac{2E_0 d_0 2\pi h_t h_r}{d \lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

$$= \frac{2 \times 10^{-3} \times 1 \times 10^3}{5 \times 10^3} \left[\frac{2\pi(50)(1.5)}{0.333(5 \times 10^3)} \right]$$

$$= 113.1 \times 10^{-6} \text{ V/m.}$$

$P_r(d) = E^2 / \eta * A_e$ [$A_e = \lambda^2 / 4\pi$]

The received power at a distance d can be obtained using equation (3.15)

$$P_r(d) = \frac{(113.1 \times 10^{-6})^2}{377} \left[\frac{1.8(0.333)^2}{4\pi} \right]$$

$$P_r(d = 5 \text{ km}) = 5.4 \times 10^{-13} \text{ W} = -122.68 \text{ dBW or } -92.68 \text{ dBm.}$$

4 Explain (a) Delay Spread and Coherence Bandwidth (b) Doppler spread and Coherence Time (c) Angular spread and Coherence Distance -7
Find the Fraunhofer distance for an antenna with maximum dimension of 1m and operating frequency of 900MHz. If antenna have unity gain, calculate the path loss -3

a) Delay Spread

- The delay spread is the amount of time that elapses between the echo (typically the first line-of-sight component and the last arriving) path.
- It is also referred to as the Multipath Intensity Profile, or power delay profile as it measures the multipath richness of channel.
- It specifies the duration of the channel impulse response $h(\tau, t)$.

Delay spread can be quantified through different metrics, although the most common one is the root mean square, **the mean delay of the channel** is :

$$\mu_\tau = \frac{\int_0^\infty \Delta\tau A_\tau(\Delta\tau) d(\Delta\tau)}{\int_0^\infty A_\tau(\Delta\tau) d(\Delta\tau)}$$

And **the rms delay spread** is given by :

$$\tau_{\text{rms}} = \sqrt{\frac{\int_0^\infty (\Delta\tau - \mu_\tau)^2 A_\tau(\Delta\tau) d(\Delta\tau)}{\int_0^\infty A_\tau(\Delta\tau) d(\Delta\tau)}}$$

The delay spread can be found by channel autocorrelation function $A(\Delta\tau, 0)$ by setting $\Delta t = 0$.

b) Coherence bandwidth B_c -

- Bandwidth over which the channel transfer function remains virtually constant.
- Channel is considered relatively constant over the transmit bandwidth – happens if the transmission $BW < \text{'coherence' } BW$ (i.e. B_c of the channel).
- A signal sees a narrowband channel if the bit duration \gg inter-arrival time of reflected waves - ISI is small.

c) Doppler Spread

- In channels where transmitter and receiver move relative to each other the signal frequency is shifted depending on the velocity and as a result the spectrum of the narrowband signal

transmitted widens.

- This so-called Doppler effect can be observed on passing cars, moving stars and wireless communications.

d) Coherence Time

- It gives the time period over which the channel is significantly correlated i.e. the time duration over which two received signals have a strong potential for amplitude correlation.
- Coherence time is actually a statistical measure of the time duration over which the channel impulse response is essentially invariant.

Mathematically

$$|t_1 - t_2| \leq T_c \Rightarrow \mathbf{h}(t_1) \approx \mathbf{h}(t_2)$$

$$|t_1 - t_2| > T_c \Rightarrow \mathbf{h}(t_1) \text{ and } \mathbf{h}(t_2) \text{ are uncorrelated}$$

Coherence time and doppler spread inversely related

$$T_c \approx \frac{1}{f_D}$$

e) Angular spread

- It refers to the statistical distribution of the angle of the arriving energy.
- A large θ_{rms} implies that channel energy is coming in from many directions, whereas a small θ_{rms} implies that the received channel energy is more focused.
- A large angular spread generally occurs when there is a lot of local scattering, and this results in more statistical diversity in the channel

f) Coherence Distance

- The coherence distance is the spatial distance over which the channel does not change appreciably.
- The dual of angular spread is coherence distance.
- As the angular spread increases, the coherence distance decreases, and vice versa.
- Mathematically: For Rayleigh fading which has a uniform angular spread

$$D_c \approx \frac{.2\lambda}{\theta_{rms}}$$

Part 2 Solution:-

Given, an antenna with maximum dimension of 1metre with operating frequency = 900 MHz. So, $\lambda = c/f = 3 \times 10^8 / (900 \times 10^6) = 0.33$ Far field distance = $2 \cdot D^2 / \lambda = 6m$.

5 Explain the three statistical channel model of a broadband fading channel

Rayleigh- and Rician-fading

- Consider two independent Gaussian random variables X and Y , both with mean zero and equal variance σ^2 . Then envelope $Z = \sqrt{X^2 + Y^2}$ is Rayleigh-distributed and Z^2 is exponentially distributed.
- For variance σ^2 for both $r_I(t)$ and $r_Q(t)$ then signal envelope

$$z(t) = |r(t)| = \sqrt{r_I^2(t) + r_Q^2(t)}$$

is Rayleigh-distributed with distribution

$$p_z(z) = \frac{2z}{P_r} e^{-z^2/P_r} = \frac{z}{\sigma^2} e^{-z^2/(2\sigma^2)}, z \geq 0,$$

$$P_r = \sum_n E[\alpha_n^2] = 2\sigma^2 \quad \text{average received signal power, i.e. received power based on path loss and shadowing alone}$$

$$\text{For } z^2(t) = |r(t)|^2 \quad p_{Z^2}(x) = \frac{1}{P_r} e^{-x/P_r} = \frac{1}{2\sigma^2} e^{-x/(2\sigma^2)}, x \geq 0.$$

Thus, received signal power is exponentially distributed with mean $2\sigma^2$

When signal envelope $|r(t)|$ & signal power $|r(t)|^2$ have above distributions, channel is said to be a *Rayleigh-fading* channel. 10

Nakagami fading

- The above Rayleigh and Rician models do not always fit experimental data. A distribution that has been empirically shown to fit data well is Nakagami fading distribution

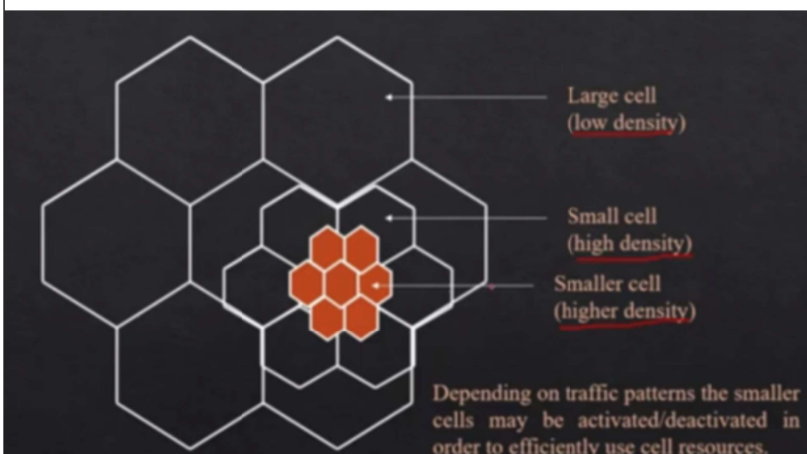
$$p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)P_r^m} \exp\left[-\frac{mz^2}{P_r}\right], \quad m \geq .5 \quad P_r = \text{average received power}$$

- Nakagami distribution is determined by its fading parameter m . For $m = 1$, we get Rayleigh fading, and for $m = \frac{(K+1)^2}{2K+1}$, we get Rician fading. For $0.5 \leq m \leq 1$, we get fading that is “worse” than Rayleigh.
- Power distribution for Nakagami fading is

$$p_{Z^2}(x) = \left(\frac{m}{P_r}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left(-\frac{mx}{P_r}\right)$$

7. Explain briefly about the cellular concepts

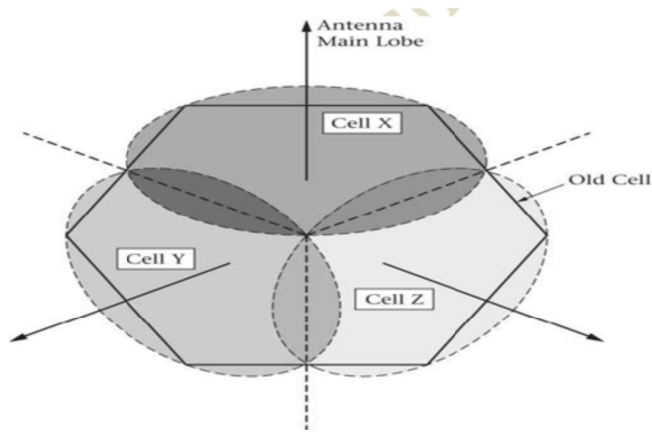
(a) Cell splitting



- It is the process of subdividing a congested cell into smaller cells. (each with its own base station and a corresponding reduction in antenna height and transmitter power).
- The increased no. of cells would increase the no. of clusters which in turn would increase the no. of channels reused and capacity.
- Each cell is divided into six new smaller cells with approximately one-quarter the area of the larger cells and use the same channel as shown in figure 3.4.
- To preserve the overall system frequency reuse plan, the transmit power of these cells must be reduced by a factor of approximately 16 or 12dB.

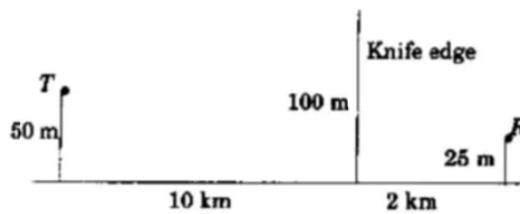
(b) Cell sectoring

Cell Sectoring - Uses directional antennas to effectively split a cell into 3 or sometimes 6 new cells.



- Reuse Factor/ frequency reuse ratio : $Q = D/R = (3N)^{1/2}$
- 3 directional antennas with 120° beamwidth to illuminate the entire area previously services by omnidirectional antenna
- **Cell Sectoring** provides co-channel interference reduction, hence S/I ratio increases.
- It does not require new cell sites and additional antennas and triangular mounting only.

6. Geometry as shown in the figure determine a) the loss due to knife edge diffraction and b) the height of the obstacle required to induce 6db diffraction loss. Assume $f=900\text{MHz}$.



$$\tan \beta = \frac{\text{opp}}{\text{adj}} = \frac{50}{10 \times 10^3 \text{ m}}$$

$$\beta = \tan^{-1} \left(\frac{50}{10 \times 10^3} \right)$$

$$\beta = 0.004999 \text{ rad}$$

$$\tan \gamma = \frac{\text{opp}}{\text{adj}} = \frac{75}{2 \times 10^3 \text{ m}}$$

$$\gamma = \tan^{-1} \left(\frac{75}{2 \times 10^3} \right)$$

$$\gamma = 0.03749 \text{ rad.}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = 0.333 \text{ m}$$

a) $G_d(\text{dB}) \rightarrow \mathcal{V}$

$$\mathcal{V} = \frac{\alpha}{\lambda} \sqrt{\frac{2d_1 d_2}{\lambda(d_1 + d_2)}}$$

$$\alpha = \beta + \gamma$$

$$\mathcal{V} = 0.04248 \sqrt{\frac{2 \times 10 \times 10^3 \times 2 \times 10^3}{0.333 (10 \times 10^3 + 2 \times 10^3)}}$$

$$\mathcal{V} = 4.24$$

$$G_d(\text{dB}) = 20 \log \left(\frac{0.225}{\mathcal{V}} \right)$$

$$= -25 \text{ dB}$$

(b) $h_{obs} \rightarrow G_a(\text{dB}) = 6 \text{ dB}$

$$\boxed{\nu = 0}$$

Triangle rule

$$\frac{25}{12 \text{ km}} = \frac{h'}{2 \text{ km}}$$

$$\frac{25}{12 \times 10^3} = \frac{h'}{2 \times 10^3}$$

$$\Rightarrow h' = \frac{25 \times 2 \times 10^3}{12 \times 10^3}$$

$$h' = 4.16 \text{ m}$$

