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Internal Assesment Test - I

Sub:	Microwaves and Antennas							Code:	21EC62
Date:	3/6/2024	Duration:	90 mins	Max Marks:	50	Sem:	6th	Branch:	ECE
Answer Any FIVE FULL Questions									

		Marks	OBE	
			CO	RBT
1.	With the help of a suitable diagram, derive the expression for the instantaneous voltage and current on a two wire transmission line.	[10]	CO1	L2
2.	A transmission line has the following primary constants per km of the line: R=8Ω/m, G=0.1μ℧/m, L=3.5mH/m, C=9nF/m. Calculate Zo, α, β, v_p and γ at ω=5000 rad/sec.	[10]	CO1	L2
3.	Define and derive an expression for reflection coefficient and standing wave ratio when the transmission line is terminated by load impedance (Z_l).	[10]	CO1	L3
4.	A load impedance of Z_l=60-j80 Ω is required to be matched to a 50 Ω coaxial line, by using short circuited stub of length `l’ located at a distance `d’ from the load. The wavelength of operation is 1m. Design the single stub impedance matching system using Smith Chart.	[10]	CO1	L3
5.	Give detailed construction of the Smith chart with suitable diagrams.	[10]	CO1	L2,L3
6.	The characteristic impedance of a uniform transmission line is 2040Ω at a frequency of 800Hz. At this frequency the propagation constant was found to be $0.054\angle 87.9^0$. Determine the values of the line constants R, L, G and C.	[10]	CO1	L3

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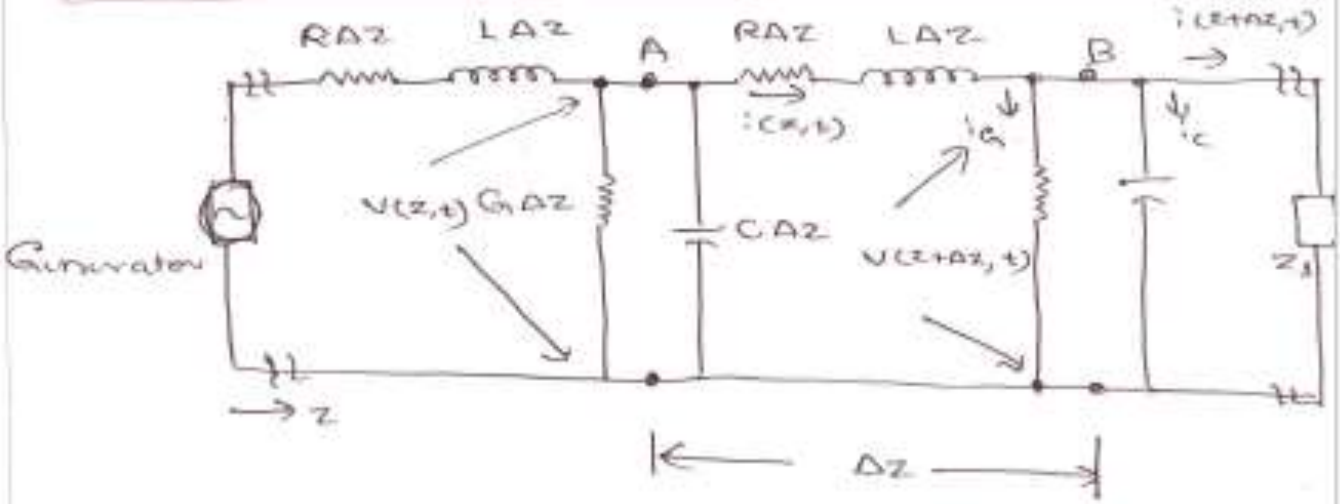
1.

Transmission Line Equations and Solutio



Elementary section of a transmission line

By kirchoff's Voltage law, the Summation of the Voltage drops around the central loop is given by

$$v(z,t) = R\Delta z\, i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z+\Delta z, t)$$

$$= R\Delta z\, i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z \quad (1)$$

Rearranging this equation, dividing it by Δz, and then omitting the argument (z,t) which is understood, we get

$$\boxed{-\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t}} \qquad (2)$$

Using Kirchoff's current law, the summation of the currents at point B can be expressed as

$$i(z,t) = G\,\Delta z\, v(z+\Delta z, t) + C\,\Delta z\,\frac{\partial v(z+\Delta z, t)}{\partial t} + i(z+\Delta z, t)$$

$$= G\,\Delta z\left[v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z\right] + C\,\Delta z\,\frac{\partial}{\partial t}\left[v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z\right] + \cancel{i(z,t)} + \frac{\partial i(z,t)}{\partial z}\Delta z \qquad (3)$$

$= \cancel{G\Delta z\, v(z,t) + G}$

$$0 = G\,v(z,t) + G\,\frac{\partial v(z,t)}{\partial z}\Delta z + C\,\frac{\partial}{\partial t}v(z,t) + C\,\frac{\partial}{\partial t}\left[\frac{\partial v(z,t)}{\partial z}\Delta z\right] + \frac{\partial i(z,t)}{\partial z}$$

$$\Rightarrow \quad \boxed{-\frac{\partial i}{\partial z} = Gv + C\frac{\partial v}{\partial t}} \qquad (4)$$

Differentiating Eq. ② w.r.t z we get

$$-\frac{\partial^2 v}{\partial z^2} = R\frac{\partial i}{\partial z} + L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right) \quad (5)$$

Differentiating Eq. ④ w.r.t. t we get

$$-\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right) = G\frac{\partial v}{\partial t} + C\frac{\partial^2 v}{\partial t^2} \quad (6)$$

Substituting equations ④ & ⑥ in Eq. ⑤ we get

$$-\frac{\partial^2 v}{\partial z^2} = R\left(-Gv - C\frac{\partial v}{\partial t}\right) + L\left(-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right)$$

$$= -RGv - RC\frac{\partial v}{\partial t} - LG\frac{\partial v}{\partial t} - LC\frac{\partial^2 v}{\partial t^2}$$

$$\boxed{\frac{\partial^2 v}{\partial z^2} = RGv + (RC+LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}} \quad (6)$$

~~Also~~ ~~By~~ Differentiating Eq. ② w.r.t t we get

$$-\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial z}\right) = R\frac{\partial i}{\partial t} + L\frac{\partial^2 i}{\partial t^2} \quad (7)$$

Differentiating Eq. 4 w.r.t z we get

$$-\frac{\partial^2 i}{\partial z^2} = G\frac{\partial v}{\partial z} + C\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial t}\right) \quad (8)$$

Substituting Eqs (2), (4) in Eq. (8) we get

$$-\frac{\partial^2 i}{\partial z^2} = G\left(-Ri - L\frac{\partial i}{\partial t}\right) + C\left[-R\frac{\partial i}{\partial t} - L\frac{\partial^2 i}{\partial t^2}\right]$$

$$= -RGi - LG\frac{\partial i}{\partial t} - RC\frac{\partial i}{\partial t} - LC\frac{\partial^2 i}{\partial t^2}$$

$$\therefore \boxed{\frac{\partial^2 i}{\partial z^2} = RGi + (LG+RC)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}} \quad (9)$$

The voltage and current on the line are the functions of both position z and time t.

The instantaneous line voltage and current can be expressed as

$$v(z,t) = \text{Re}\, V(z)\, e^{j\omega t} \quad (10)$$

$$i(z,t) = \text{Re}\, I(z)\, e^{j\omega t} \quad (11)$$

where Re stands for "real part of".

The factors V(z) and I(z) are complex quantities of the sinusoidal functions of position z on the line and are known as phasor

If we substitute $j\omega$ for $\frac{\partial}{\partial t}$ in equations (2), (4), (6) and (9) and divide each equation by $e^{j\omega t}$, the transmission line equations in phasor form of the frequency domain become

$$\frac{dV}{dz} = -(R + j\omega L) I$$
$$= -ZI \qquad (12)$$

$$\frac{dI}{dz} = -(G + j\omega C) V$$
$$= -YV \qquad (13)$$

$$\frac{d^2V}{dz^2} e^{j\omega t} = RG V e^{j\omega t} + (RC + LG) j\omega V e^{j\omega t}$$
$$+ LC (j\omega)^2 V e^{j\omega t}$$

$$\frac{d^2V}{dz^2} = RG V + (RC + LG) j\omega V$$
$$+ LC (j\omega)^2 V$$
$$= \left[RG + RCj\omega + LGj\omega + LC(j\omega)^2\right] V$$
$$= \left[R(G + j\omega C) + j\omega L(G + j\omega C)\right] V$$
$$= (R + j\omega L)(G + j\omega C) V$$
$$= ZYV$$

$$\frac{d^2V}{dz^2} = \gamma^2 V \quad (14)$$

$$\frac{d^2I}{dz^2} = \gamma^2 I \quad (15)$$

in which the following substitutions have been made:

$$Z = R + j\omega L \quad \text{(ohms per unit length)}$$

$$Y = G + j\omega C \quad \text{(mhos per unit length)} \quad (16)$$

$$\gamma = \sqrt{ZY} = \alpha + j\beta \quad \text{(propagation constant)}$$

α is the attenuation constant in nepers per unit length

β is the phase constant in radians per unit length

Solutions to Transmission Line Equations

The one possible solution for Eq. (14) is

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$= V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z} \quad (17)$$

The factors V_+ and V_- represents complex quantities

2.

Problems

1) A transmission line has the following parameters:

$R = 2\ \Omega/m \quad G = 0.5\ \text{mmho}/m \quad f = 1\ GHz$

$L = 8\ nH/m \quad C = 0.23\ pF/m$

Calculate : (a) the characteristic impedance ;

(b) the propagation constant.

(a)

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{2 + j\, 2\pi \times 1\times10^{9} \times 8 \times 10^{-9}}{0.5 \times 10^{-3} + j\, 2\pi \times 1 \times 10^{9} \times 0.23 \times 10^{-12}}}$$

$$= \sqrt{\frac{50.3 \angle 87.72^\circ}{1.529 \times 10^{-3} \angle 70.91^\circ}}$$

$$= 181.37 \angle 8.40^\circ$$

$$= 179.42 + j\,26.49$$

(b)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(50.3 \angle 87.72^\circ)(1.529 \times 10^{-3} \angle 70.91^\circ)}$$

$$= 0.2774 \angle 79.31^\circ$$

$$= 0.051 + j\,0.272$$

3.

Reflection Coefficient and Transmission Coefficient

Reflection Coefficient

⇒ We know that, in the equations of the solutions of transmission-line equations, the travelling wave along the line contains two components: one travelling in the positive z direction and the other travelling in the negative z direction.

⇒ If the load impedance is equal to the line characteristic impedance, the reflected ~~travelling~~ wave does not exist.

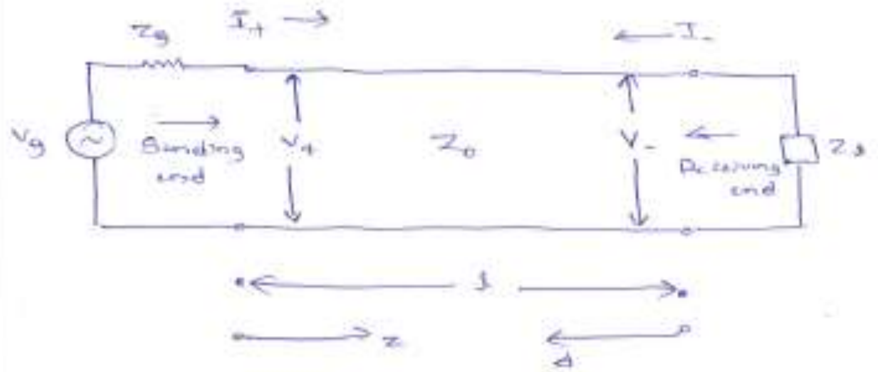


Fig: Transmission line terminated in a load impedance

The incident voltage and current waves travelling along the transmission line are given by

$$V = V_+ e^{-\gamma z} + V_- e^{+\gamma z} \quad (1)$$

$$I = \frac{V_+}{Z_0} e^{-\gamma z} - \frac{V_-}{Z_0} e^{\gamma z} \quad (2)$$

The voltage and current at the load end are

$$V_l = V_+ e^{-\gamma l} + V_- e^{\gamma l} \quad (3)$$

$$I_l = \frac{1}{Z_0}\left(V_+ e^{-\gamma l} - V_- e^{\gamma l}\right) \quad (4)$$

$$Z_l = \frac{V_l}{I_l} = Z_0 \left[\frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma l} - V_- e^{\gamma l}}\right] \quad (5)$$

Reflection coefficient = $\dfrac{\text{reflected voltage or current}}{\text{incident voltage or current}}$

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{-I_{ref}}{I_{inc}} \quad (6)$$

$$\Gamma = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}$$

Solving eq (5) for $\frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}$ we get

$$\frac{Z_l}{Z_0} = \frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma l} - V_- e^{\gamma l}}$$

$$= \frac{1 + \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}}{1 - \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}}$$

$$= \frac{1 + \Gamma_l}{1 - \Gamma_l}$$

$$\Rightarrow Z_l (1-\Gamma) = Z_0 (1+\Gamma)$$

$$Z_l - Z_l\Gamma = Z_0 + Z_0\Gamma$$

$$Z_l - Z_0 = Z_l\Gamma + Z_0\Gamma = (Z_l + Z_0)\Gamma$$

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0}$$

$$\Gamma_l = |\Gamma_l| e^{j\theta_l}$$

$|\Gamma_l|$ is the magnitude, $|\Gamma_l| \leq 1$

θ_l is the phase angle between incident and reflected voltages at the receiving end. Usually called the phase angle of the reflection coefficient.

Standing Wave Ratio

> The ratio of the maximum of the standing-wave pattern to the minimum is defined as the standing-wave ratio, designated by ρ.

i.e.,

Standing-wave ratio

$$= \frac{\text{maximum voltage or current}}{\text{minimum voltage or current}}$$

$$\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|}$$

The standing-wave ratio results from the fact that the two travelling-wave components of the equation for a voltage standing wave add in phase at some points and subtract at other points.

The distance between two successive maxima or minima is $\lambda/2$.

The standing-wave ratio cannot be defined on a lossy line because the standing-wave pattern changes markedly from one position to another. (i.e., V_{max} and V_{min} change in V_{max}/V_{min}).

SWR is defined only for lossless line.

$$\rho = \frac{V_{max}}{V_{min}} = \frac{V_+ e^{-\alpha z}(1+|\Gamma|)}{V_+ e^{-\alpha z}(1-|\Gamma|)}$$

$$= \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{\rho - 1}{\rho + 1}$$

$$|\Gamma| \leq 1$$

$$\rho \geq 1$$

Problem: Standing wave Ratio

4.

4] $Z_L = 60 - j80\,\Omega$

$Z_0 = 50\,\Omega$

Normalized load impedence $(\bar{z}_L) = \dfrac{Z_L}{Z_0}$

$= \dfrac{60-j80}{50} = 1.2 - j1.6$

$\bar{y}_l = \dfrac{1}{\bar{z}_L} = \dfrac{1}{1.2-j1.6} = \underline{\underline{0.3+j0.4}}$.

$\underline{\underline{\lambda = 1m}}$

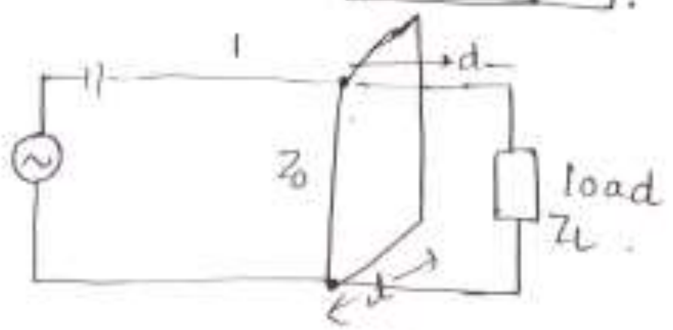
distance ~~length~~ of the stub $= 0.175\lambda - 0.066\lambda$

$d = 0.109\lambda = \underline{0.109m}$.

length of the stub $= 0.25\lambda - 0.348\lambda$

$= 0.098\lambda$

$\boxed{l = 0.098\,m}$.



$d = B' \sim A'$

$= 0.175\lambda - 0.066\lambda$

$= 0.109\lambda = 0.109m \; [\lambda = 1m]$

$l = C' \sim D'$

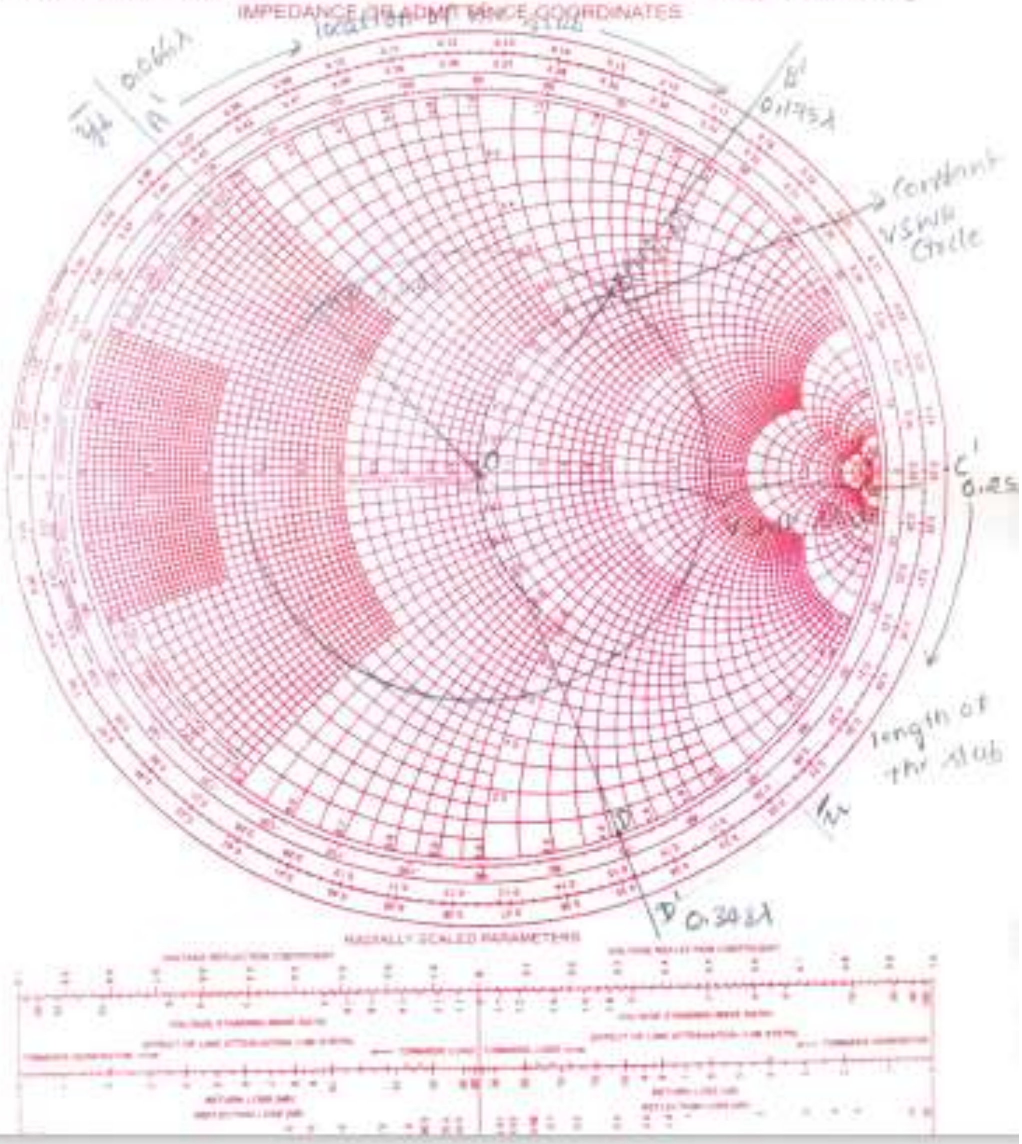
$= 0.25\lambda - 0.348\lambda$

$= 0.098\lambda = 0.098m$

SMITH CHART

NAME Nikhitha-K-N	TITLE	DWG. No. / DATE 3/6/24

IMPEDANCE OR ADMITTANCE COORDINATES



5.

Smith Chart

The Smith Chart is a plot of the normalized impedance or admittance with angle and magnitude of a generalized complex reflection co-efficient.

Normalized impedance,

$$\bar{z} = \frac{Z}{Z_0} \qquad (1)$$

At load end,

$$\bar{z}_l = \frac{Z_l}{Z_0} = \frac{1+\Gamma_l}{1-\Gamma_l} = \frac{R_l + jX_l}{Z_0} \qquad (2)$$

$$= r + jx$$

$$\Gamma_l = \frac{\bar{z}_l - 1}{\bar{z}_l + 1} = \Gamma_r + j\Gamma_i \qquad (3)$$

Substituting Eq. (3) into Eq (2) we get

$$r + jx = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i} = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

$$= \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} \times \frac{(1-\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) + j\Gamma_i}$$

$$= \frac{(1+j\Gamma_i)^2 - \Gamma_r^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{1 - \Gamma_i^2 + 2j\Gamma_i - \Gamma_r^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \qquad (4)$$

$$r = \frac{1 - \Gamma_i^2 - \Gamma_r^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \qquad (5)$$

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2} \qquad (6)$$

From (5)

$$(1 + \Gamma_r^2 - 2\Gamma_r + \Gamma_i^2)\,r = 1 - \Gamma_i^2 - \Gamma_r^2$$

$$r + r\Gamma_r^2 - 2\Gamma_r r + \Gamma_i^2 r = 1 - \Gamma_i^2 - \Gamma_r^2$$

$$r\Gamma_r^2 - 2\Gamma_r r + \Gamma_i^2 r + \Gamma_i^2 + \Gamma_r^2 = 1 - r$$

$$(1+r)\Gamma_r^2 + (1+r)\Gamma_i^2 - 2\Gamma_r r = 1 - r$$

$\div$ by $(1+r)$
$$\Gamma_r^2 + \Gamma_i^2 - \frac{2r\Gamma_r}{1+r} = \frac{1-r}{1+r}$$

Adding $\left(\frac{r}{1+r}\right)^2$ to both sides

$$\Gamma_r^2 - \frac{2r}{1+r}\Gamma_r + \left(\frac{r}{1+r}\right)^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \left(\frac{r}{1+r}\right)^2$$

$$= \frac{1 - r^2 + r^2}{(1+r)^2}$$

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2 \qquad (7)$$

From Eq. (6), $(1 + \Gamma_r^2 - 2\Gamma_r)x + \Gamma_i^2 x = 2\Gamma_i$

$$x + \Gamma_r^2 x - 2\Gamma_r x + \Gamma_i^2 x - 2\Gamma_i = 0$$

$$\Gamma_r^2 x - 2\Gamma_r(1+x) + \Gamma_i^2 x = -x$$

$$(\Gamma_r - 1)^2 + \Gamma_i^2 = \frac{2\Gamma_i}{x}$$

$$(\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x} = 0$$

Adding $\left(\frac{1}{x}\right)^2$ to both sides

$$(\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x} + \left(\frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \quad (8)$$

Equation (7) is an equation of family of circles in which each circle has a constant resistance r.

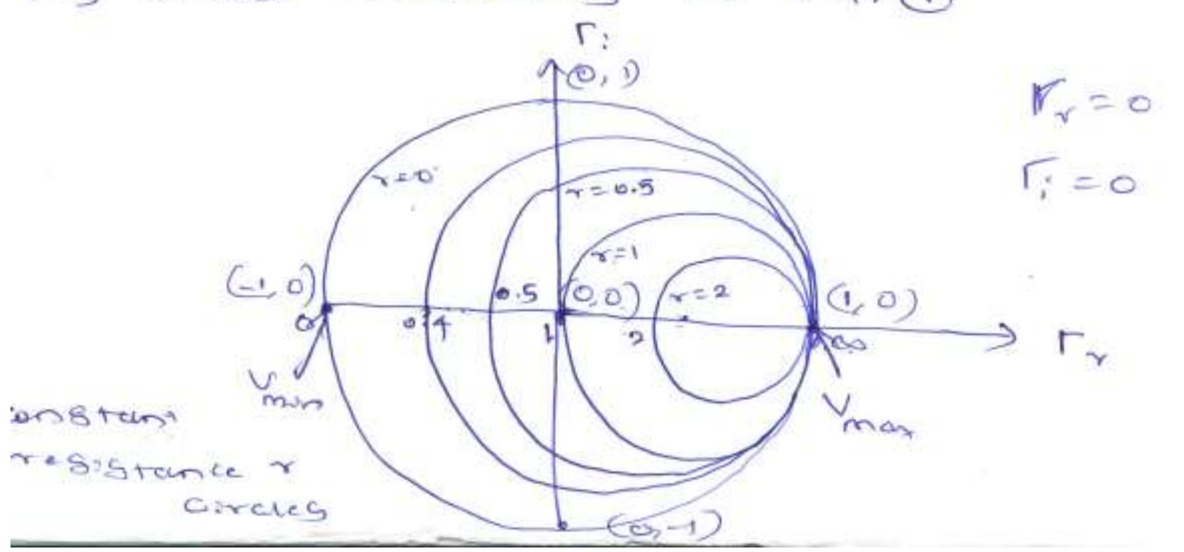
Radius of any circle $= \frac{1}{1+r}$

Center of any circle $= \left(\frac{r}{1+r}, 0\right)$

along the real axis

where r varies from 0 to ∞.

All constant resistance circles are plotted in Fig below according to Eq. (7)

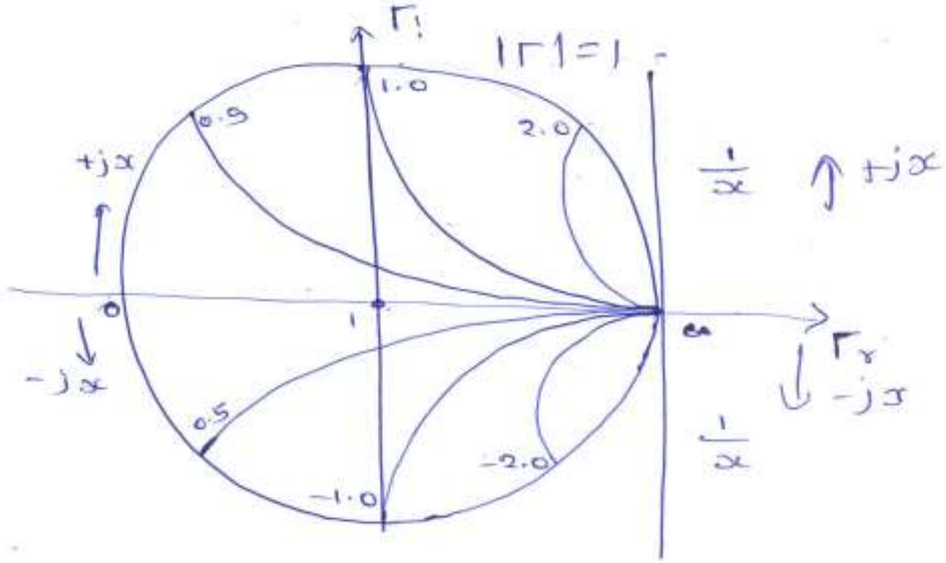


→ Equation (8) also describes a family of circles, but each of these circles specifies a constant reactance x.

→ The radius of any circle is $\left(\frac{1}{x}\right)$ and the center of any circle is at

$$\Gamma_r = 1 \qquad \Gamma_i = \frac{1}{x} \quad \left(\text{where } -\infty \le x \le \infty \right)$$

→ All constant reactance circles are plotted in Fig. below according to Eq. (8)



constant reactance x circles

6.

In a certain microwave transmission line, the characteristic impedance was measured to be $210\angle 10^\circ\ \Omega$ and the propagation constant $0.2\angle 78^\circ$. Determine the primary constants of the line, if the frequency of operation is 1GHz.

Given $Z_0 = 210\angle 10^\circ\ \Omega$

$\gamma = 0.2\angle 78^\circ$

$f = 1\text{GHz}$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0\gamma = R+j\omega L$$

$$\frac{\gamma}{Z_0} = G+j\omega C$$

$\therefore\ R+j\omega L = 210\angle 10^\circ \times 0.2\angle 78^\circ$

$= 42\angle 88^\circ$

$= 1.465 + j\,41.97$

$R = 1.465\ \Omega/m$

$\omega L = 41.97$

$L = \frac{41.97}{\omega} = \frac{41.97}{2\pi f} = \frac{41.97}{2\pi \times 1\times 10^{9}}$

$= 6.67\times 10^{-9}$

$= 6.67\ nH/m$

$G + j\omega C = \frac{0.2\angle 78^\circ}{210\angle 10^\circ}$

$= 9.52\times 10^{-4}\angle 68^\circ$

$= 3.56\times 10^{-4} + j\,8.82\times 10^{-4}$

$G = 3.56\times 10^{-4}$ mho/m

$\omega C = 8.82\times 10^{-4}$

$C = \frac{8.82\times 10^{-4}}{\omega} = \frac{8.82\times 10^{-4}}{2\pi f}$

$= \frac{8.82\times 10^{-4}}{2\pi\times 1\times 10^{9}} = 1.4\times 10^{-13}$ F/m

Scheme of Evaluation

IAT1 MWA 21EC62

1. Diagram of the line transmission line – 2M

Applying KVL, KCL and getting

$\frac{\partial v}{\partial z}, \frac{\partial i}{\partial z}$ – 4M

Introducing phasors – 2M

Getting $V(z) = V_1 e^{-\gamma z} + V_2 e^{\gamma z}$

$I(z) = \frac{1}{Z_0}(V_1 e^{-\gamma z} - V_2 e^{\gamma z})$ 2M

---

2. $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$ 4M

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$ 2M

α, β $\gamma = \alpha + j\beta$ 2M

v_p 2M

---

3. Definition of reflection coefficient and standing wave ratio – 2M

Deriving $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ – 4M

Deriving $VSWR(\rho) = \frac{1+|\Gamma|}{1-|\Gamma|}$ – 4M

---

4. Finding y_L [illegible] 3M

Plotting Z_L on Smith chart 3M

Finding distance of the stub (d) – 4M

Finding length of the stub 3M

---

5. Arriving at

$$r = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

$$x = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

– 5M

Constant resistance circle – $2\frac{1}{2}$M

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$

Constant reactance circle – $2\frac{1}{2}$M

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

---

6. $Z_0 = \sqrt{\frac{Z}{Y}}$; $\gamma = \sqrt{ZY}$

$\frac{\gamma}{Z_0} = G + j\omega C$; $Z_0\gamma = R + j\omega L$

4M

R —
L —
G —
C —

6M