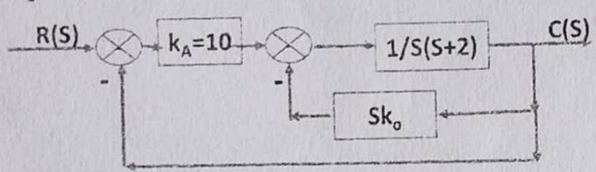
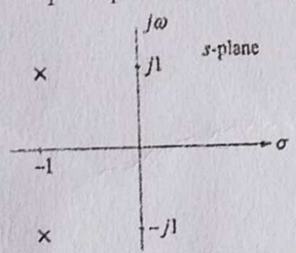
CMR
INSTITUTE OF
TECHNOLOGY

USN

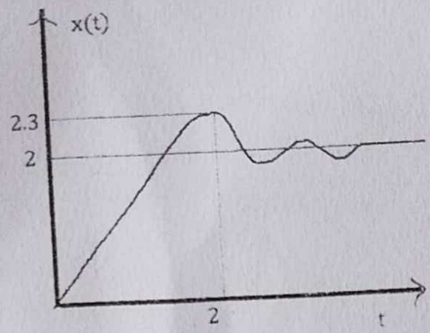
Internal Assessment Test – II

Sub:		Control Systems				Code:	BEC403
Date:	11/ 07/ 2024	Duration: 90 mins	Max Marks:	50	Sem: 4^{th}	Branch:	ECE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1.	Make use of the response curve of second order underdamped system to define and derive the expression for (i) Peak time (ii) Peak overshoot (ii) Rise time	[10]	CO2	L2
2.	Explain the necessary and sufficient conditions specified under Routh-Hurwitz stability criteria.	[04]	CO2	L3
	Using Routh-Hurwitz stability criteria, determine the stability of the control system having the Characteristic equation, $F(s)= s^4+3s^3+3s^2+2s+6=1$. How many poles lie on the right half of the s-plane	[06]		
3.	In the system shown below, determine the derivative feedback constant ko, which will increase the damping factor of the system to 0.6. What is the steady state error to unit ramp input with this setting of the derivative feedback constant? 	[10]	CO2	L3
4	The below figure shows the closed loop poles of a negative feedback control system. Find the unit-step response of the system 	[10]	CO2	L3

PTO…

5	a) The unit-step response of a control system is shown below. Find the transfer function of the prototype system used to model a second-order system  b) Determine the value of K for a control system with open loop transfer function of K(S+5)/S(S+2)(S+10) which produces 30% steady state error with unit ramp input.	[06] [04]	CO2	L3
6	Derive the total output expression for second order Under-damped system	[10]	CO2	L3
7.	The response of a servo mechanism is c (t) =1+0.2 e^{-60t}-1.2 e^{-10t}.when subjected to a unit step input. Obtain an expression for closed loop transfer function. Determine the time domain specifications.	[10]	CO2	L3

CI

CCI

HOD

Control Systems

IAT 2 Solutions

By Dr. Sushmitha Alamuru.

(1)

Peak time $t_p = \frac{\pi}{\omega_d}$

Peak overshoot $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

Rise time $t_r = \frac{\pi - \theta}{\omega_d}$

(2)

$$s^4 + 3s^3 + 3s^2 + 2s + 6 = 1$$

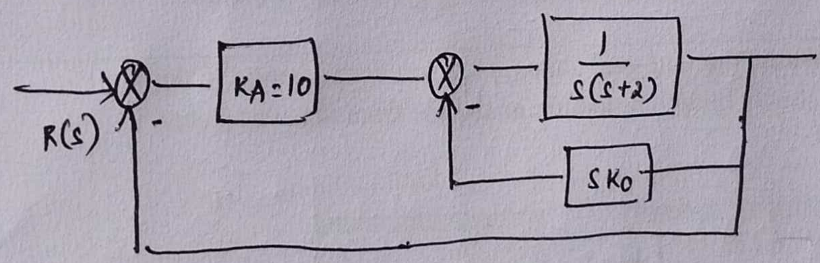
$$s^4 + 3s^3 + 3s^2 + 2s + 5 = 0$$

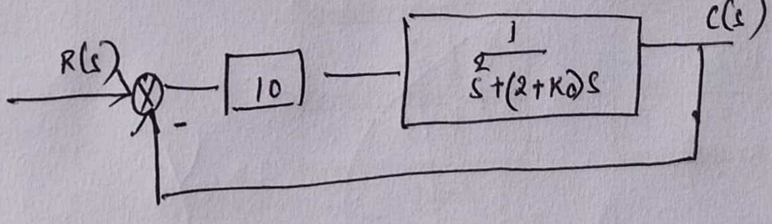
s^4	1	3	5
s^3	3	2	
s^2	2.33	5	0
s^1	-4.4294	0	
s^0	5	0	

No of sign changes = 2

No of closed loop poles on right half of s plane = 2

(3)





$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + (2+K_o)s + 10}$$

Compare with $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$,

e_{ss} (unit ramp) $= \frac{1}{K_v}$

$K_v = \lim_{s \to 0} s \cdot G(s)H(s)$

$G(s)H(s) = \frac{10}{s^2 + (2+K_o)s}$

$= \frac{10}{s(s + (2+K_o))}$

Given $\zeta = 0.6$

$\omega_n^2 = 10$

$\omega_n = \sqrt{10} = 3.162$ rad/sec

$2 + K_o = 2\zeta\omega_n$

$\boxed{K_o = 1.7944}$

$$K_v = \lim_{s \to 0} s \cdot G(s)H(s)$$

$$= \lim_{s \to 0} \cancel{s} \cdot \frac{10}{\cancel{s}\,(s + (2 + K_0))}$$

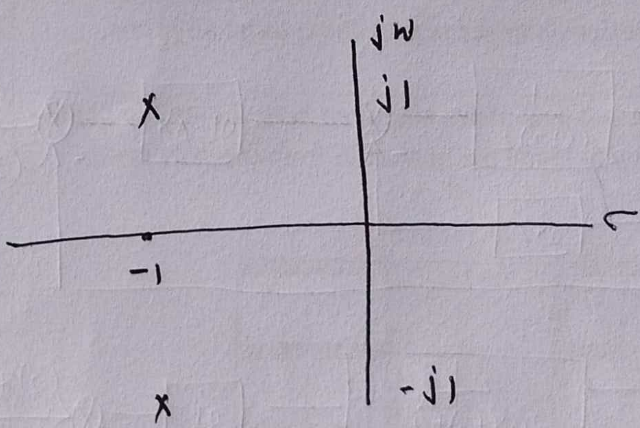
$$= \lim_{s \to 0} \left[\frac{10}{s + (2 + 1.7944)} \right] \qquad K_0 = 1.7944$$

$$= \lim_{s \to 0} \left(\frac{10}{s + 3.7944} \right)$$

$$= \frac{10}{3.7944} = 2.635$$

$$\boxed{e_{ss} = \frac{1}{K_v} = \frac{1}{2.635} = 0.379}$$

(4)



The closed loop poles are $-1 + j1$ and $-1 - j1$

so the denominator of closed loop T.F is

$$(s + 1 - j1)(s + 1 + j1)$$

The C.E is $(s + 1 - j1)(s + 1 + j1) = 0$

$$(s+1)^2 + 1 = 0 \Rightarrow s^2 + s + 2 = 0$$

From the given graph

$$t_p = \frac{\pi}{\omega_d} = 2 \text{ sec.}$$

$$\frac{\pi}{\omega_n \cdot \sqrt{1-\zeta^2}} = 2$$

Peak over shoot $M_p = \dfrac{c(t_p) - c(t_\alpha)}{c(t_\alpha)}$

$$c(t)\Big|_{\text{at } t = t_p} = 2.3 \quad = \frac{2.3 - 2}{2} = \frac{0.3}{2} = 0.15$$

% peak overshoot $= 0.15 \times 100$

$= 15$

$$e^{-\pi\zeta / \sqrt{1-\zeta^2}} = 0.15$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} = \ln(0.15)$$

$$\frac{-\pi\zeta}{\sqrt{1-\zeta^2}} = -1.8971$$

Square on both sides

$$\frac{\pi^2\zeta^2}{1-\zeta^2} = 3.599$$

$$\pi^2\zeta^2 = 3.599 - 3.599\zeta^2$$

$$9.8596\zeta^2 + 3.599\zeta^2 = 3.599$$

$$13.4586\zeta^2 = 3.599$$

$\omega_n^2 = 2$

$\omega_n = \sqrt{2} = 1.414$ rad/sec

$2\zeta\omega_n = 1$

$\zeta = \frac{1}{2\times\omega_n} = \frac{1}{2\times 1.414} = \frac{1}{2.828} = 0.35$

$\boxed{\zeta = 0.35}$

As $\zeta < 1 \longrightarrow$ the given system is underdamped.

For second order underdamped control system

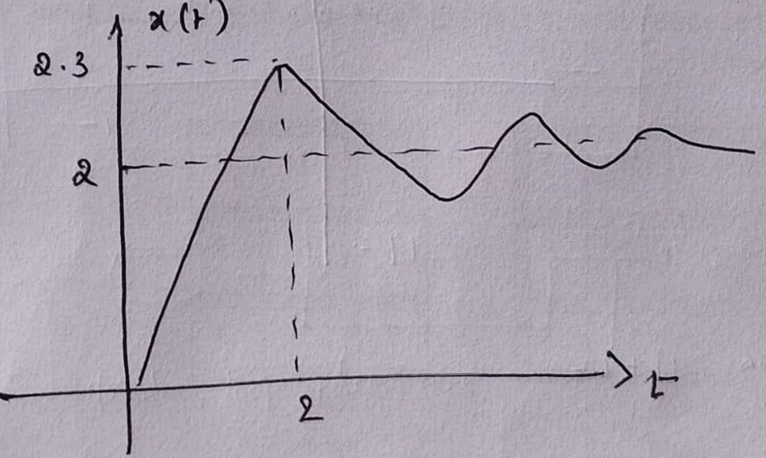
the unit step response

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta).$$

Substitute ζ, ω_n, ω_d and θ values in the above expression.

5

a)



The second order closed loop transfer function

is $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$\xi^2 = \frac{3.599}{13.4586} = 0.2674$$

$$\boxed{\xi = 0.517}$$

$$t_p = \frac{\pi}{\omega_n \cdot \sqrt{1-\xi^2}} = 2.$$

$$\therefore \quad \omega_n = \frac{3.14}{2 \times \sqrt{1-(0.517)^2}}$$

$$\boxed{\omega_n = 1.834 \text{ rad/sec.}}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{3.364}{s^2 + 1.896 s + 3.364}}$$

5 b)

$$G(s)H(s) = \frac{K(s+5)}{s(s+2)(s+10)}$$

Given For unit ramp i/p $e_{ss} = 30\% = \frac{30}{100} = 0.3$

$$e_{ss} = 0.3.$$

$$e_{ss\,\text{unit ramp}} = \frac{1}{K_v}$$

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{K(s+5)}{\cancel{s}(s+2)(s+10)}$$

$$= \frac{5K}{20}$$

$$e_{ss} = \frac{1}{K_v} = \frac{20}{5K} = 0.3$$

$$\boxed{K = \frac{20}{5 \times 0.3} = 13.33}$$

(6) second order under damped system

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta).$$

(7) The response of the servo meachanism is

$$c(t) = 1 + 0.2\, e^{-60t} - 1.2\, e^{-10t}$$

Take laplace T.F on both sides

$$c(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{(s+10)}.$$

$$c(s) = \frac{(s+60)(s+10) + 0.2 \cdot s(s+10) - 1.2s(s+60)}{s(s+60)(s+10)}$$

$$c(s) = \frac{60}{s(s^2+70s+600)}.$$

Since given is unit step $\quad r(t) = 1$, $\cancel{R}(s) = \frac{1}{s}$.

$$c(s) = \frac{60}{s^2+70s+600} \cdot \frac{1}{s}$$

$$c(s) = \frac{60}{s^2+70s+600} \cdot R(s)$$

$$\boxed{\frac{c(s)}{R(s)} = \frac{60}{s^2+70s+600}}$$

$\omega_n^2 = 600$

$\omega_n = 24.494$ rad/sec

$2\zeta\omega_n = 70$

$$\zeta = \frac{70}{2 \times 24.494}$$

$$\boxed{\zeta = 1.428}$$

As ζ is >1, over damped Control system.

Cannot find Time domain specifications.