CMR Institute of Technology
Department of ECE
21EC62 – Microwave Theory & Antennas
2nd IAT Scheme & Solutions – July 2024

INTERNAL ASSESSMENT TEST – II

Sub	MICROWAVE THEORY & ANTENNAS							Code	21EC62
Date	08/07/2024	Duration	90 mins	Max Marks	50	Sem	VI A,B,C,D	Branch	ECE

Answer any 5 full questions

		Marks	**CO**	**RBT**
1	State and explain the properties of Scattering Matrix	10	CO2	L3
2	Explain Magic Tee and derive its Scattering Matrix	10	CO2	L3
3	With a neat diagram, explain the working of Precision Phase Shifter	10	CO3	L2
4(i)	A 20 mw signal is fed into one of the collinear port 1 of a lossless H-plane T-Junction. Calculate the power delivered through each port when other ports are terminated in matched load	5	CO3	L3
4(ii)	Discuss briefly about the coaxial connectors	5	CO3	L1
5	Derive an equation for Characteristic Impedance for Micro strip Lines	10	CO3	L3
6	Explain the following parameters with respect to Antenna: (i) Radiation Pattern (ii) Beam Area (iii) Directivity (D) (iv) Beam Efficiency	10	CO4	L1
7(i)	Derive Friis transmission formula	05	CO4	L2
7(ii)	The power received by the receiving antenna at a distance of 0.5 km over a free space at a frequency of 1 GHz is 10.8 mW. Calculate the input to the transmitting antenna if the gain of the transmitting antenna and receiving antenna is 25dB and 20dB respectively. The gain is with respect to the isotropic source.	05	CO4	L3
8	Explain briefly about the losses encountered in micro strip lines.	10	CO3	L3

Q.no	Scheme	Marks
1	Properties of Scattering Matrix: (i) Zero Diagonal elements for perfect matched network (ii) Symmetry of [S] for a reciprocal network (iii) Unitary property for a lossless junction (iv) Phase shift property	 1 4 4 1
2	Explanation of Magic tee with diagram Derivation of its Scattering Matrix	4 6
3	Working of Precision Phase Shifter Diagram	7 3
4(i)	Determination of S_{11}, P_1,P_2,P_3	5
4(ii)	Explanation of type N, BNC,TNC, APC	5
5	Derivation of Characteristic Impedance of Micro strip Lines	10
6	Antenna Parameters (i) Radiation Pattern (ii) Beam Area (iii) Directivity (iv) Beam Efficiency	 3 2 3 2
7(i)	Derivation of Friis transmission formula	5
7(ii)	Calculation of power transmitted	5
8	Dielectric Losses Ohmic Losses Radiation Losses	4 3 3

1. Properties of Scattering Matrix

6.3.1 Properties of *S*-Parameters

In general the scattering parameters are complex quantities having the following properties for different characteristics of the microwave network.

(a) Zero diagonal elements for perfect matched network

For an ideal *N*-port network with matched termination, $S_{ii} = 0$, since there is no reflection from any port. Therefore, under perfect matched conditions the diagonal elements of $[S]$ are zero.

(b) Symmetry of $[S]$ *for a reciprocal network*

A reciprocal device has the same transmission characteristics in either direction of a pair of ports and is characterised by a symmetric scattering matrix.

$$S_{ij} = S_{ji} \ (i \neq j) \qquad (6.25)$$

which results

$$[S]_t = [S] \tag{6.26}$$

This condition can be proved in the following manner. For a reciprocal network with the assumed normalisation, the impedance matrix equation is

$$[V] = [Z]\,[I] = [Z]\,([a] - [b]) = [a] + [b]$$

or,

$$([Z] + [U])\,[b] = ([Z] - [U])\,[a]$$

or,

$$[b] = ([Z] + [U])^{-1}\,([Z] - [U])\,[a] \tag{6.27}$$

where $[U]$ is the unit matrix. The S-matrix equation for the network is

$$[b] = [S]\,[a] \tag{6.28}$$

Comparing Eqs 6.27 and 6.28, we have

$$[S] = ([Z] + [U])^{-1}\,([Z] - [U]) \tag{6.29}$$

Let

$$[R] = [Z] - [U],\ [Q] = [Z] + [U] \tag{6.30}$$

For reciprocal network, the Z-matrix is symmetric. Hence

$$[R]\,[Q] = [Q]\,[R]$$

or,

$$[Q]^{-1}\,[R]\,[Q]\,[Q]^{-1} = [Q]^{-1}\,[Q]\,[R]\,[Q]^{-1}$$

or,

$$[Q]^{-1}\,[R] = S = [R]\,[Q]^{-1} \tag{6.31}$$

Now the transpose of $[S]$ is

$$[S]_t = ([Z] - [U])_t\,([Z] + [U])_t^{-1} \tag{6.32}$$

Since the Z-matrix is symmetrical

$$([Z] - [U])_t = [Z] - [U] \tag{6.33}$$

$$([Z] + [U])_t = [Z] + [U] \tag{6.34}$$

Therefore,

$$\begin{aligned} [S]_t &= ([Z] - [U])\,([Z] + [U])^{-1} \\ &= [R]\,[Q]^{-1} = [S] \end{aligned} \tag{6.35}$$

Thus it is proved that $[S]_t = [S]$, for a symmetrical junction.

(c) Unitary property for a lossless junction

For any lossless network the sum of the products of each term of any one row or of any column of the S-matrix multiplied by its complex conjugate is unity.

For a lossless n-port device, the total power leaving N-ports must be equal to the total power input to these ports, so that

$$\sum_{n=1}^{N} |b_n|^2 = \sum_{n=1}^{N} |a_n|^2$$

or,
$$\sum_{n=1}^{N} \left| \sum_{i=1}^{n} S_{ni}\, a_i \right|^2 = \sum_{n=1}^{N} |a_n|^2 \tag{6.36}$$

If only ith port is excited and all other ports are matched terminated, all $a_n = 0$, except a_i, so that,

$$\sum_{n=1}^{N} |S_{ni}\, a_i|^2 = \sum_{n=1}^{N} |a_i|^2 \tag{6.37}$$

$$\sum_{n=1}^{N} |S_{ni}|^2 = 1 = \sum_{n=1}^{N} S_{ni}\, S_{ni}^* \tag{6.38}$$

Therefore, for a lossless junction

$$\sum_{n=1}^{N} S_{ni} \cdot S_{ni}^* = 1 \tag{6.39}$$

If all $a_n = 0$, except a_i and a_k,

$$\sum_{n=1}^{N} S_{nk} \cdot S_{ni}^* = 0 \; ; \; i \neq k \tag{6.40}$$

In matrix notation, these relations can be expressed as

$$[S^*]\,[S]_t = [U]$$

or,
$$[S^*] = [S]_t^{-1} \tag{6.41}$$

Here $[U]$ is the identity matrix or unit matrix. A matrix $[S]$ for lossless network which satisfies the above three conditions 6.39 – 6.41 is called a *unitary matrix.*

(d) Phase shift property

Complex S-parameters of a network are defined with respect to the positions of the port or reference planes. For a two-port network with unprimed reference planes 1 and 2 as shown in Fig. 6.2, the S-parameters have definite complex values

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{6.42}$$

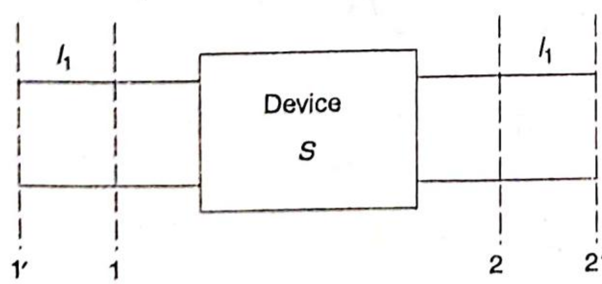


Fig. 6.2 *Phase shift property of S*

If the reference planes 1 and 2 are shifted outward to 1′ and 2′ by electrical phase shifts $\phi_1 = \beta_1 l_1$ and $\phi_2 = \beta_2 l_2$, respectively, then the new wave variables are $a_1 e^{j\phi_1}$, $b_1 e^{-j\phi_1}$, $a_2 e^{j\phi_2}$, $b_2 e^{-j\phi_2}$. The new S-matrix S' is then given by

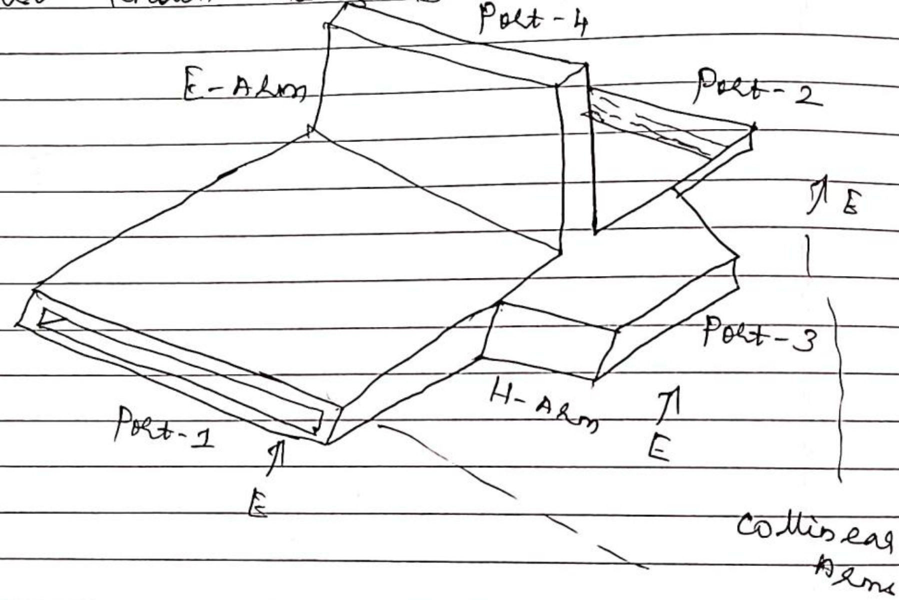
$$[S'] = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} [S] \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} \quad (6.43)$$

This property is valid for any number of ports and is called the *phase shift property* applicable to a shift of reference planes.

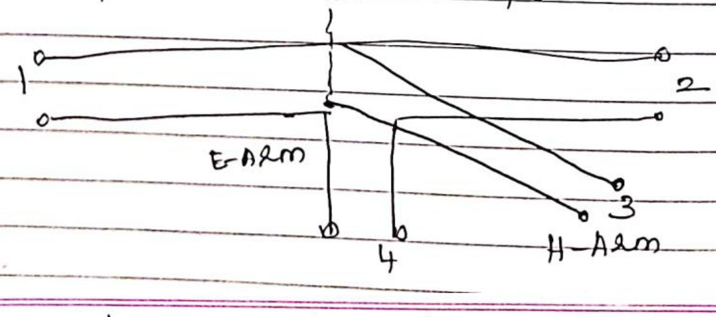
2. Magic Tee

Hybrid or Magic T

A Combination of the E-plane and H-plane tee forms a hybrid tee called Magic-T having 4 ports. This is also known as 3-dB coupler.



Arms of rectangular waveguides makes 2 arms called collinear arms (ports), (i.e) Port 1 and port 2, while port 3 is called as H-Arm or sum port or Parallel port. Port 4 is called as E-Arm or Difference port or series port.



Characteristics of E-H plane Tee

* If a signal of equal phase and magnitude is sent to port 1 and port 2, then the output at Port 4 is zero and the output at port 3 will be the additive of both the ports 1 and 2.

* If a signal is sent to port 4, E-arm then the power is divided between Port 1 and 2 equally but in opposite ~~dir~~ Phase, while there would be no output at port 3, Hence $S_{34} = 0$.

* If a signal is fed at port 3, then the power is divided between port 1 and 2 equally, while there would be no output at port 4. Hence $S_{43} = 0$.

* If a signal is fed at one of the collinear ports, then there appears no output at the other collinear port, as the E-arm produces a phase delay and the H-arm produces a phase advance. So $S_{12} = S_{21} = 0$

Properties of E-H plane Tee

Scattering matrix is of 4×4, as there are 4 possible inputs and 4 possible outputs.

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$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \longrightarrow (1)$$

As it has H-plane Tee section,

$$S_{23} = S_{13} \longrightarrow (2)$$

As it has E-plane Tee section,

$$S_{24} = -S_{14} \longrightarrow (3)$$

The E-arm port and H-arm port are so isolated that the other won't deliver an output, if an input is applied at one of them. Hence

$$S_{34} = S_{43} = 0 \Rightarrow (4)$$

From the symmetry property, we have

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21}, \; S_{13} = S_{31}, \; S_{14} = S_{41}$$
$$S_{23} = S_{32}, \; S_{24} = S_{42}, \; S_{34} = S_{43} \longrightarrow (5)$$

If ports 3 and 4 are perfectly matched to junction, then

$$S_{33} = S_{44} = 0 \longrightarrow (6)$$

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Substituting all the above equations in eq (1), to obtain the [S] matrix,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \longrightarrow (7)$$

From unitary property, $[S][S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \longrightarrow (8)$

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{4}|^2 = 1 \longrightarrow (9)$

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow (10)$

R_4C_4 : $|S_{14}|^2 + |S_{14}|^2 = 1 \longrightarrow (11)$

From eq (10)

$2|S_{13}|^2 = 1$

$\therefore S_{13} = \frac{1}{\sqrt{2}} \Rightarrow (12)$

From eq (11)

$2|S_{14}| = 1$

$\therefore S_{14} = \frac{1}{\sqrt{2}}$

$\longrightarrow (13)$

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From eq (8) and (9)

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2}$$

$$|S_{11}|^2 = |S_{22}|^2$$

$$S_{11} = S_{22} \longrightarrow (14)$$

using values of eq (12) and (13) in eq (8),

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$1 - \frac{1}{4}$

$$|S_{11}|^2 + |S_{12}|^2 = 1 - 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$\therefore S_{11} = 0, \quad S_{12} = 0 \longrightarrow (15)$$

From eq (9)

$$|S_{11}|^2 + |S_{22}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{22}|^2 = 0$$

$$\therefore S_{11} = 0, S_{22} = 0 \longrightarrow (16)$$

$\therefore$ port 1 and port 2 are perfectly matched to the junction.

The junction where all the four ports are perfectly matched is called a Magic-Tee junction

By substituting all the values in eq (7)

$$[S] = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ +\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

3. Precision Phase Shifter

6.4.15.1 Precision phase shifter

A precision phase shifter can be designed as a rotary type as shown in Fig. 6.27. This uses a section of circular waveguide containing a lossless dielectric plate of length $2l$ called halfwave (180°) section. This section can be rotated over 360° precisely between two sections of circular to rectangular waveguide transitions each containing lossless dielectric plates of length l called quarterwave (90°) sections oriented at an angle of 45° with respect to the broad wall of the rectangular waveguide ports at the input and output. The incident TE_{10} wave in the rectangular guide becomes a TE_{11} wave in the circular guide. The halfwave section produces a phase shift equal to twice its rotation angle θ with respect to the quarterwave section. The dielectric plates are tapered through a length of quarter wavelength at both ends for reducing reflection due to discontinuity.

The principle of operation of the rotary phase shifter can be explained as follows. The TE_{11} mode incident field E_i in the input quarterwave section can be decomposed into two transverse components, one E_1, polarised parallel and other, E_2 perpendicular to quarterwave plate. After propagation through the quarter wave plate these components are

$$E_1 = E_i \cos 45^\circ \, e^{-j\beta_1 l} = E_o \, e^{-j\beta_1 l} \tag{6.86}$$

$$E_2 = E_i \sin 45^\circ \, e^{-j\beta_2 l} = E_o \, e^{-j\beta_2 l} \tag{6.87}$$

where, $E_o = E_i/\sqrt{2}$. The length l is adjusted such that these two components will have equal magnitude but a differential phase change of $(\beta_1 - \beta_2)l = 90^\circ$. Therefore, after propagation through the quarterwave plate these field components become

$$E_1 = E_o \, e^{-j\beta_1 l} \tag{6.88}$$

$$E_2 = jE_o \, e^{-j\beta_1 l} = jE_1 = E_1 \, e^{j\pi/2} \tag{6.89}$$

Thus the quarterwave sections convert a linearly polarised TE_{11} wave to a circularly polarised wave and vice-versa.

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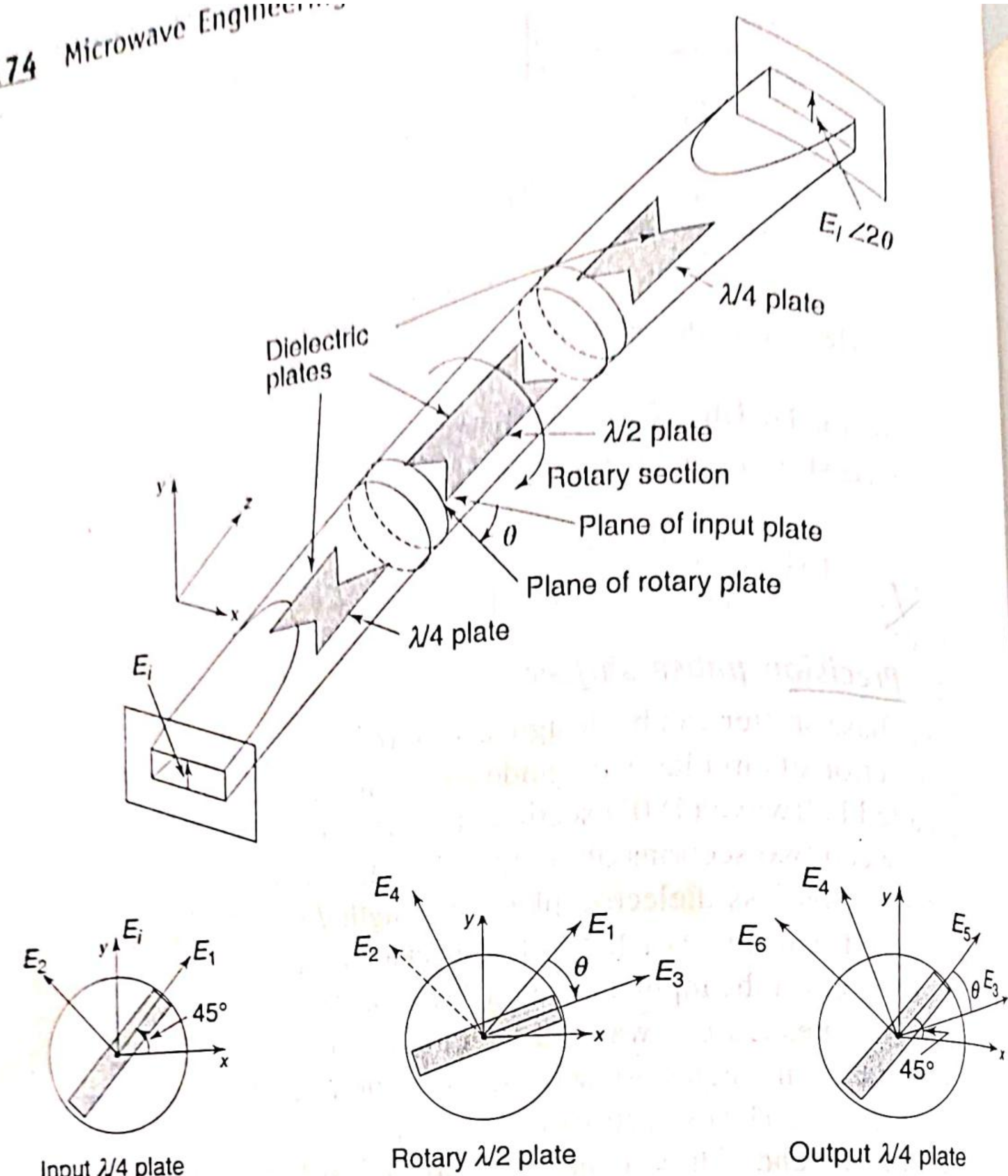


Fig. 6.27 *Precision rotary phase shifter*

After emergence from the halfwave section, the field components parallel and perpendicular to the halfwave plate can be represented as

$$E_3 = (E_1 \cos\theta - E_2 \sin\theta)\, e^{-j2\beta_1 l} = E_o\, e^{-j\theta}\, e^{-j3\beta_1 l} \quad (6.90)$$

$$E_4 = (E_2 \cos\theta + E_1 \sin\theta)\, e^{-j2\beta_2 l} = E_o\, e^{-j\theta}\, e^{-j3\beta_1 l}\, e^{-j\pi/2} \quad (6.91)$$

since,

$$2(\beta_1 - \beta_2)l = \pi \quad \text{or} \quad -2\beta_2 l = \pi - 2\beta_1 l \quad (6.92)$$

After emergence from the halfwave section the field components E_3 and E_4 may again be decomposed into two TE_{11} modes, polarised parallel and perpendicular to the output quarterwave plate. At the output end of this quarterwave plate the field components parallel and perpendicular to the quarterwave plate can be written as

(6.93)

Therefore, the parallel component E_5 and perpendicular component E_6 at the output end of the quarterwave plate are equal in magnitude and in phase to produce a resultant field which is a linearly polarised TE_{11} wave

$$
\begin{aligned}
E_{out} &= \sqrt{2}\ E_o\, e^{-j2\theta}\ e^{-j4\beta_1 l} \\
&= E_i\, e^{-j2\theta}\ e^{-j4\beta_1 l}
\end{aligned} \tag{6.95}
$$

having the same direction of polarisation as the incident field E_i with a phase change of $2\theta + 4\beta_1 l$. Since θ can be varied and $4\beta_1 l$ is fixed at a given frequency and structure, a phase shift of 2θ can be obtained by rotating the halfwave plate precisely through an angle of θ with respect to the quarterwave plates.

4. (i)

Example 6.3 A 20 mW signal is fed into one of collinear port 1 of a lossless *H*-plane *T*-junction. Calculate the power delivered through each port when other ports are terminated in matched load.

Solution

Since ports 2 and 3 are matched terminated, $a_2 = a_3 = 0$, $S_{11} = 1/2$. The total effective power input to port 1 is

$$
\begin{aligned}
P_1 &= |a_1|^2\,(1 - |S_{11}|^2) \\
&= 20\,(1 - 0.5^2) = 15 \text{ mW}
\end{aligned}
$$

The power transmitted to port 3 is

$$
\begin{aligned}
P_3 &= |a_1|^2\,|S_{31}|^2 \\
&= 20 \times (1/\sqrt{2})^2 = 10 \text{ mW}
\end{aligned}
$$

The power transmitted to port 2 is

$$
\begin{aligned}
P_2 &= |a_1|^2\,|S_{21}|^2 \\
&= 20 \times (1/2)^2 = 5 \text{ mW.}
\end{aligned}
$$

Therefore, $P_1 = P_3 + P_2$

4.(ii)

6.4.2 Coaxial Connectors and Adapters

Coaxial cables are terminated or connected to other cables and components by means of shielded standard connectors. The outer shield makes a 360 degree extremely low impedance joint to maintain shielding integrity. These connectors are of various types depending on the frequency range and the cable diameter. Commonly used microwave connectors are type N (male/female), BNC (male/female), TNC (male/female), APC (sexless), etc. Adapters, having different connectors at the two ends, are also made for interconnection between two different ports in a microwave system. The basic schematic diagrams of these connectors and adapters are shown in Fig. 6.6. The type N (Navy) connector is 50 and 75 ohms connector which was designed for military system applications during World War II. This is suitable for flexible or rigid cables in the frequency range of 1–18 GHz. The BNC (Bayonet Navy Connector) is suitable for 0.25 inch 50 ohm or 75 ohm flexible cables used up to 1 GHz. The TNC (Threaded Navy Connector) is like BNC, except that, the outer conductor has thread to make firm contact in the mating surface to minimise radiation leakage at higher frequencies. These connectors are used up to 12 GHz.

The SMA (Sub-Miniature A) connectors are used for thin flexible or semi-rigid cables. The higher frequency is limited to 24 GHz because of generation of higher order modes beyond this limit. All the above connectors can be of male or female configurations except the APC-7(Amphenol Precision Connector-7 mm) which provides coupling without male or female configurations. The APC-7 is a very accurate 50 ohm, low VSWR connector which can operate up to 18 GHz.

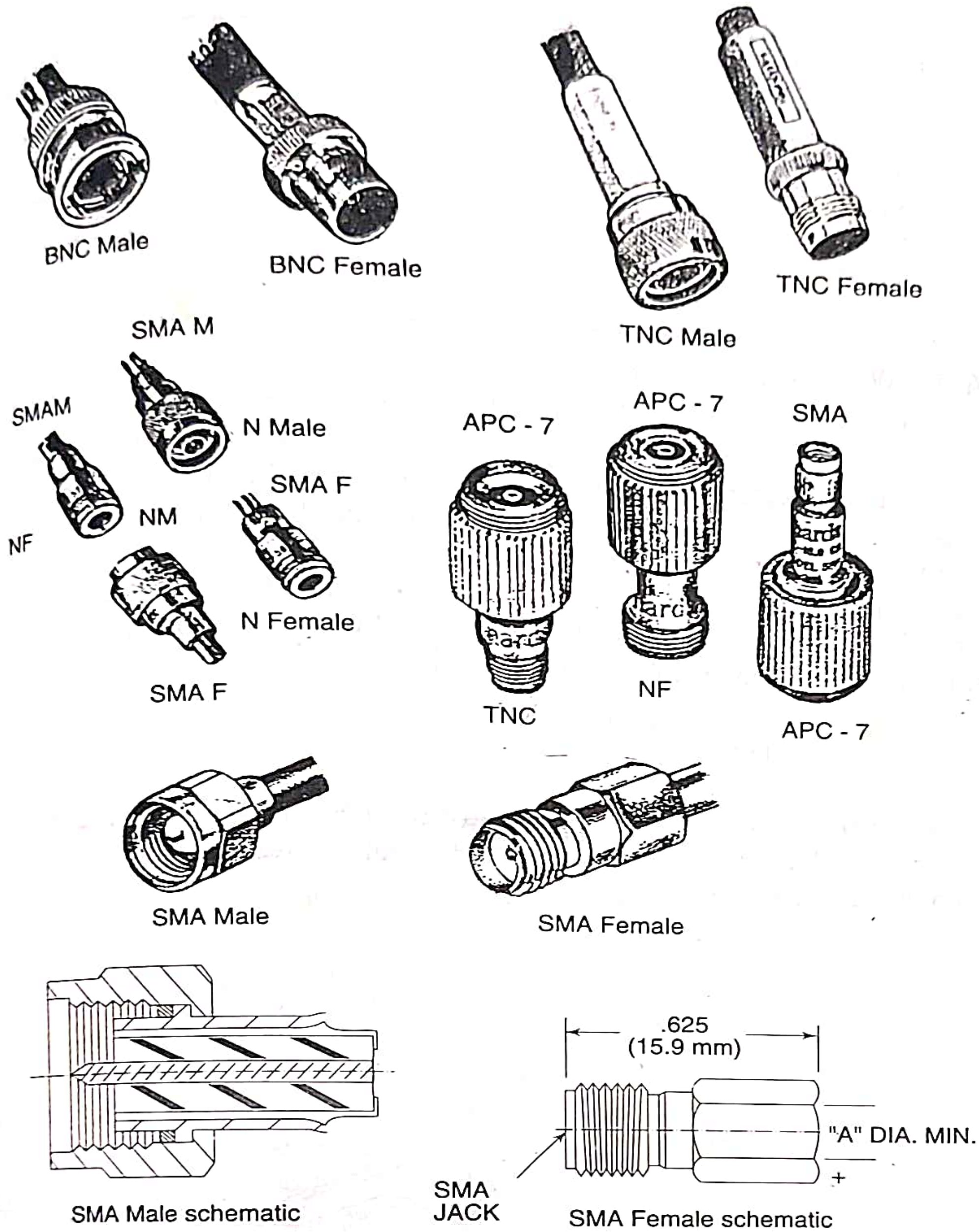


Fig. 6.6 *Coaxial connectors and adapters*

Another APC-3.5 connector is a high precision 50 ohm, low VSWR connector which can be either the male or female and can operate up to 34 GHz. It can mate with the oppositely sexed SM connector . Table 6.2 shows the type, dielectric in mating space and impedance of some of the above standard connectors.

5.

11-1-1 Characteristic Impedance of Microstrip Lines

Microstrip lines are used extensively to interconnect high-speed logic circuits in digital computers because they can be fabricated by automated techniques and they provide the required uniform signal paths. Figure 11-1-1 shows cross sections of a microstrip line and a wire-over-ground line for purposes of comparison.

In Fig. 11-1-1(a) you can see that the characteristic impedance of a microstrip

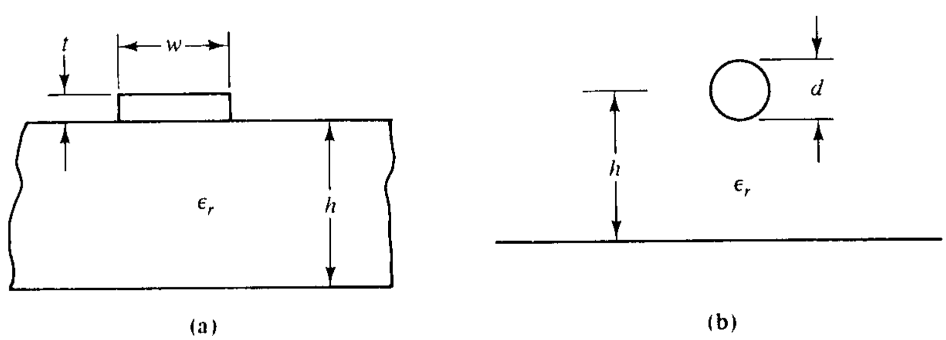


Figure 11-1-1 Cross sections of (a) a microstrip line and (b) a wire-over-ground line.

line is a function of the strip-line width, the strip-line thickness, the distance between the line and the ground plane, and the homogeneous dielectric constant of the board material. Several different methods for determining the characteristic impedance of a microstrip line have been developed. The field-equation method was employed by several authors for calculating an accurate value of the characteristic impedance [3 to 5]. However, it requires the use of a large digital computer and is extremely complicated. Another method is to derive the characteristic-impedance equation of a microstrip line from a well-known equation and make some changes [2]. This method is called a *comparative*, or an *indirect*, method. The well-known equation of the characteristic impedance of a wire-over-ground transmission line, as shown in Fig. 11-1-1(b), is given by

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \frac{4h}{d} \qquad \text{for } h \gg d \qquad (11\text{-}1\text{-}1)$$

where ϵ_r = dielectric constant of the ambient medium
h = the height from the center of the wire to the ground plane
d = diameter of the wire

If the effective or equivalent values of the relative dielectric constant ϵ_r of the ambient medium and the diameter d of the wire can be determined for the microstrip line, the characteristic impedance of the microstrip line can be calculated.

Effective dielectric constant ϵ_{re}. For a homogeneous dielectric medium, the propagation-delay time per unit length is

$$T_d = \sqrt{\mu\epsilon} \tag{11-1-2}$$

where μ is the permeability of the medium and ϵ is the permittivity of the medium. In free space, the propagation-delay time is

$$T_{df} = \sqrt{\mu_0\epsilon_0} = 3.333 \quad \text{ns/m} \quad \text{or} \quad 1.016 \quad \text{ns/ft} \tag{11-1-3}$$

where

$$\mu_0 = 4\pi \times 10^{-7}\ \text{H/m, or } 3.83 \times 10^{-7}\ \text{H/ft}$$

$$\epsilon_0 = 8.854 \times 10^{-12}\ \text{F/m, or } 2.69 \times 10^{-12}\ \text{F/ft}$$

In transmission lines used for interconnections, the relative permeability is 1. Consequently, the propagation-delay time for a line in a nonmagnetic medium is

$$T_d = 1.106\sqrt{\epsilon_r} \qquad \text{ns/ft} \tag{11-1-4}$$

The effective relative dielectric constant for a microstrip line can be related to the relative dielectric constant of the board material. DiGiacomo and his coworkers discovered an empirical equation for the effective relative dielectric constant of a microstrip line by measuring the propagation-delay time and the relative dielectric constant of several board materials, such as fiberglass-epoxy and nylon phenolic [6].

Transformation of a rectangular conductor into an equivalent circular conductor. The cross-section of a microstrip line is rectangular, so the rectangular conductor must be transformed into an equivalent circular conductor. Springfield discovered an empirical equation for the transformation [7]. His equation is

$$d = 0.67w\left(0.8 + \frac{t}{w}\right) \tag{11-1-6}$$

where d = diameter of the wire over ground
w = width of the microstrip line
t = thickness of the microstrip line

The limitation of the ratio of thickness to width is between 0.1 and 0.8, as indicated in Fig. 11-1-3.

Characteristic impedance equation. Substituting Eq. (11-1-5) for the dielectric constant and Eq. (11-1-6) for the equivalent diameter in Eq. (11-1-1) yields

$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right] \qquad \text{for } (h < 0.8w) \tag{11-1-7}$$

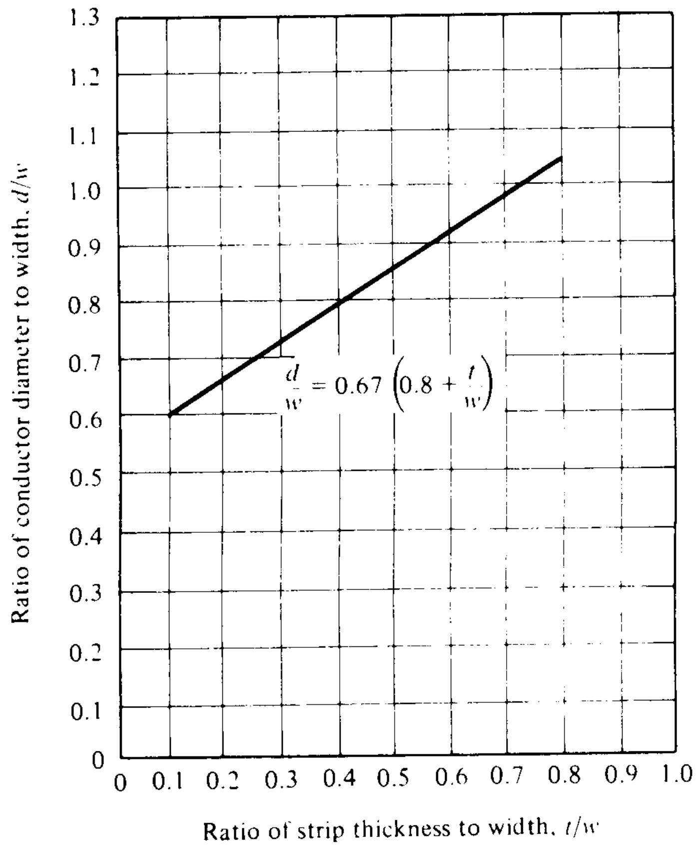


Figure 11-1-13 Relationship between a round conductor and a rectangular conductor far from its ground plane. (*After H. R. Kaupp [2]; reprinted by permission of IEEE, Inc.*)

where ϵ_r = relative dielectric constant of the board material
h = height from the microstrip line to the ground
w = width of the microstrip line
t = thickness of the microstrip line

Equation (11-1-7) is the equation of characteristic impedance for a narrow microstrip line. The velocity of propagation is

$$v = \frac{c}{\sqrt{\epsilon_{re}}} = \frac{3 \times 10^8}{\sqrt{\epsilon_{re}}} \quad \text{m/s} \tag{11-1-8}$$

The characteristic impedance for a wide microstrip line was derived by Assadourian and others [8] and is expressed by

$$Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{377}{\sqrt{\epsilon_r}}\frac{h}{w} \quad \text{for } (w \gg h) \tag{11-1-9}$$

6. (i) Radiation Pattern

BASIC ANTENNA PARAMETERS

(1) Radiation pattern

* Plot of the relative field strength of the radio waves emitted by the antenna at different angles.

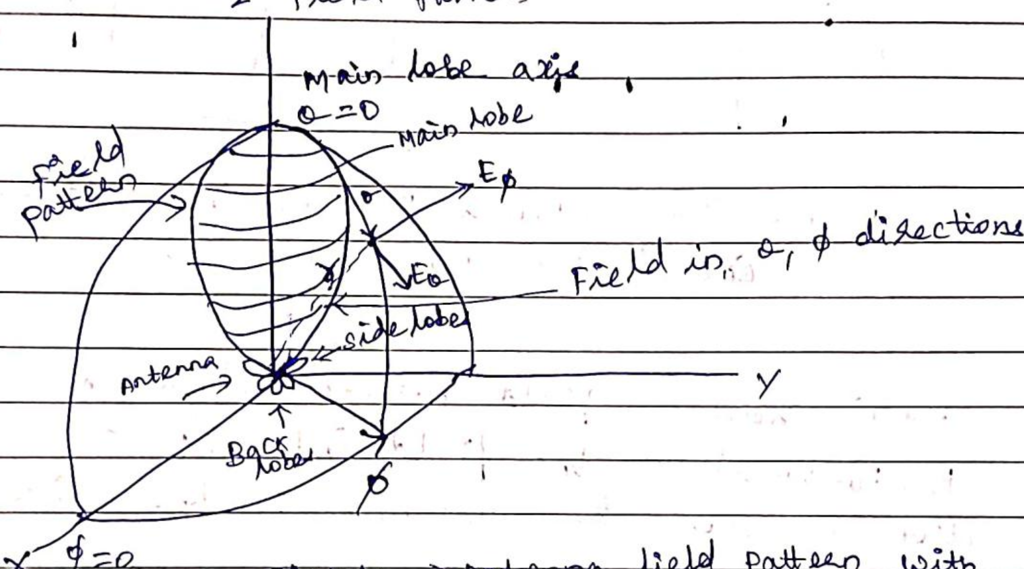
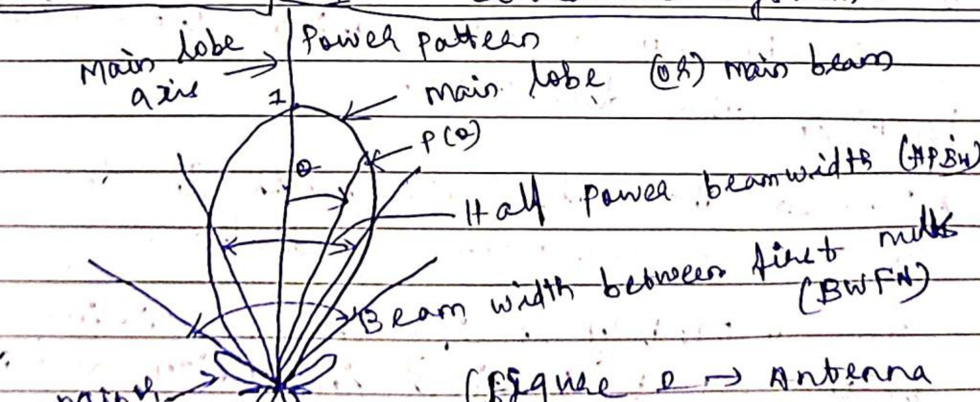


Figure 1 → Antenna field pattern with coordinate system.



(Figure 2 → Antenna power pattern in polar coordinates (linear scale)

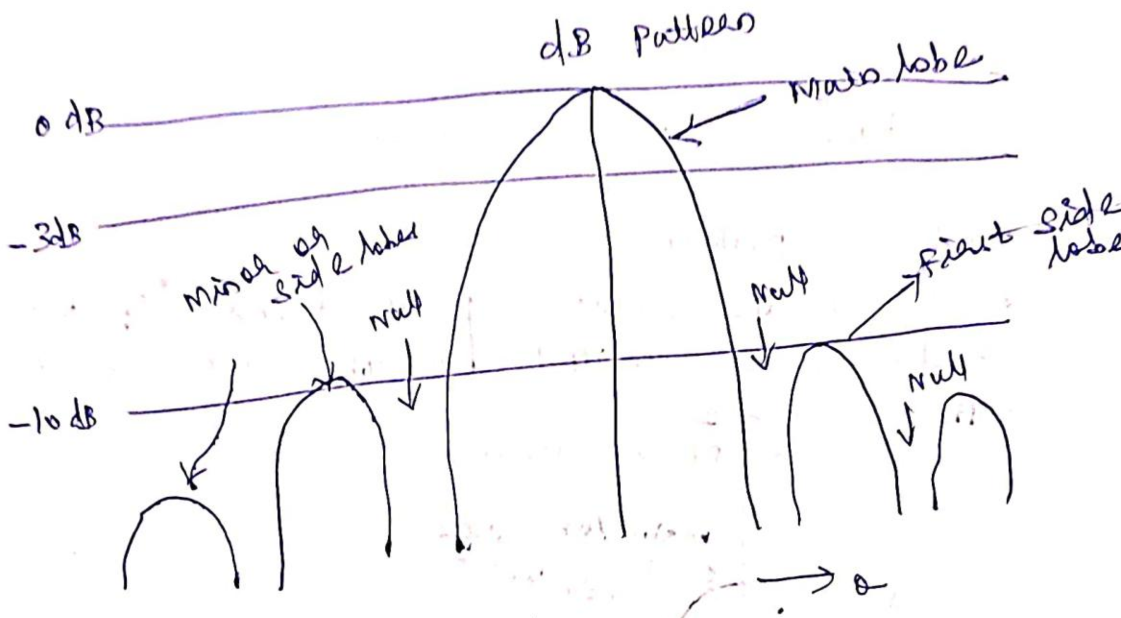


Figure 3 → Antenna patterns in rectangular coordinates and decibel scale

Fig (1) shows a field pattern where r is proportional to the field intensity at a certain distance from the antenna in the direction θ, ϕ.

* The pattern has its main-lobe maximum in the Z direction (θ = 0) with minor lobes (side and back) in other directions.

* Between the lobes are nulls in the directions of zero or minimum radiation.

* To completely specify the radiation pattern with respect to field intensity and polarization requires three patterns:

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(i) The θ component of the electric field as a function of the angles θ and ϕ or $E_\theta(\theta, \phi)$ V/m

(ii) The ϕ component of the electric field as a function of the angles θ and ϕ or $E_\phi(\theta, \phi)$ V/m

(iii) The phases of these fields as a function of the angles θ and ϕ or $\delta_\theta(\theta, \phi)$ and $\delta_\phi(\theta, \phi)$ (radians or degrees)

Normalized field pattern → dividing a field component by its maximum value. It is a dimensionless number with a maximum value of unity.

Normalized field pattern for the θ component of the electric field is given by,

$$E_\theta(\theta, \phi)_n = \frac{E_\theta(\theta, \phi)}{E_\theta(\theta, \phi)_{max}} \longrightarrow (1)$$

These patterns are measured at far field conditions.

2

* Patterns may also be expressed in terms of the power per unit area (or) poynting vector $S(\theta,\phi)$ at a certain distance from the antenna.

* Normalized power pattern is given by,

$$P_n(\theta,\phi) = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}} \quad \text{(dimensionless)} \longrightarrow (2)$$

where $S(\theta,\phi)$ = poynting vector

$$= \frac{\left[E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)\right]}{Z_0} \quad W/m^2$$

$S(\theta,\phi)_{max}$ = maximum value of $S(\theta,\phi)$ W/m^2

Z_0 = intrinsic impedance of free space

$= 377\ \Omega$.

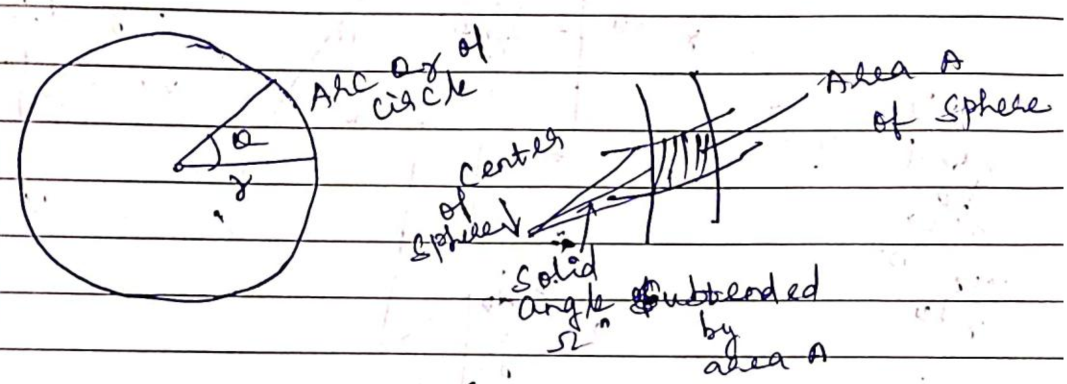
* Any of these field or power patterns → 3-dimensional spherical coordinates (Figure 1)

Principal plane patterns → Two cuts at right angles (XZ and YZ plane) Figure. 2

Fig (3) → In decibel scale
$dB = 10 \log_{10} P_n(\theta,\phi)$

(ii) Beam Area

(ii) Beam Area or Beam Solid Angle



The arc of a circle as seen from the center of the circle subtends an angle.
From fig (1), the arc length θr subtends the angle θ.

The total angle in the circle is 2π radians and the total arc length is $2\pi r$.

* An area A of the surface of a sphere as seen from the center of the sphere subtends a solid angle Ω.

* The total solid angle subtended by the sphere is 4π steradians or square radians, abbreviated sr.

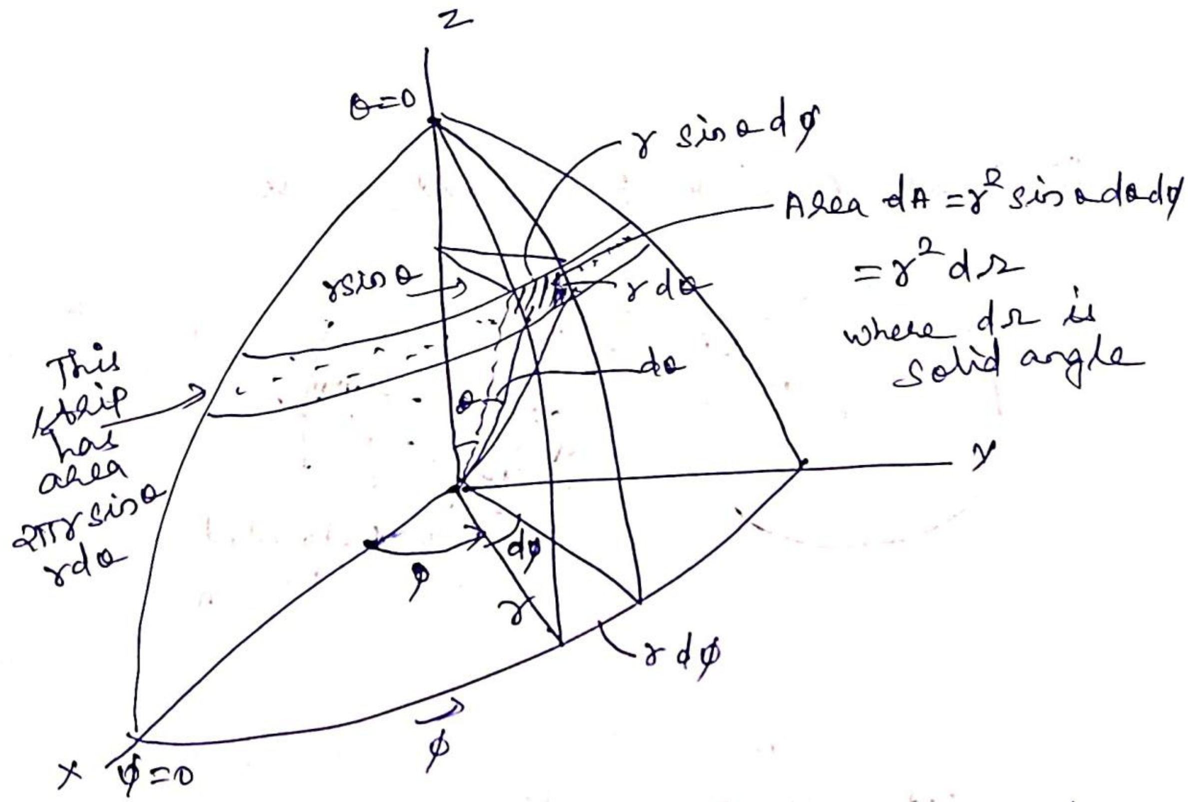


Figure → Spherical coordinates in relation to the area dA of solid angle $d\Omega = \sin\theta \, d\theta \, d\phi$

* The incremental area dA of the surface of a sphere is given by,

$$dA = (r \sin\theta \, d\phi)(r \, d\theta)$$

$$= r^2 \sin\theta \, d\theta \, d\phi$$

$$= r^2 \, d\Omega \quad \longrightarrow (1)$$

where $d\Omega$ = solid angle subtended by ~~a sphere~~ the area dA.

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The area of the strip of width $r\,d\theta$ extending around the sphere at a constant angle θ is given by $(2\pi r \sin\theta)(r\,d\theta)$.
Integrating this for θ values from 0 to π yields the area of the sphere.

$$\therefore \text{Area of sphere} = 2\pi r^2 \int_0^{\pi} \sin\theta \, d\theta$$

$$= 2\pi r^2 \left[-\cos\theta\right]_0^{\pi} = 2\pi r^2 \left[-\cos\pi + \cos 0\right]$$

$$= 2\pi r^2 \left[-(-1)+1\right]$$

$$= 4\pi r^2 \longrightarrow (2)$$

Where 4π is solid angle subtended by a sphere in steradians.

Thus 1 steradian $= 1\ sr = \frac{\text{Solid angle of sphere}}{4\pi}$

$$= 1\ rad^2 = \left(\frac{180}{\pi}\right)^2 (deg)^2$$

$$= 3282.8064 \text{ square degrees} \longrightarrow (3)$$

$\therefore 4\pi$ steradians $= 3282.8064 \times 4\pi$

$= 41252.96$

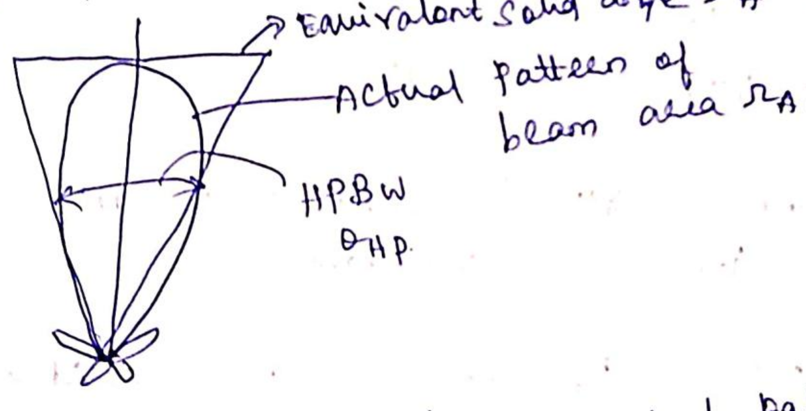
≈ 41253 square degrees $\longrightarrow (4)$

= solid angle in a sphere

Beam area (or) Beam solid angle Ω_A for an antenna is given by the integral of the normalized power pattern over a sphere. (4π Sr)

$$\Omega_A = \int_0^{2\pi} \int_0^{\pi} P_n(\theta, \phi)\, d\Omega \text{ steradian} \longrightarrow (5)$$

where $d\Omega = \sin\theta\, d\theta\, d\phi$.



The beam area Ω_A of an actual pattern is equivalent to the same solid angle subtended by the spherical cap of the cone-shaped (triangular cross section) pattern.

This solid angle can often be described approximately in terms of the angles subtended by the half-power points of the main lobe in the two principal planes as given by,

$$\Omega_A = \theta_{HP} \phi_{HP} \rightarrow (6)$$

where θ_{HP} and ϕ_{HP} are the half-power Beamwidths (HPBW) in the two principal planes, minor lobes being neglected.

(iii) **Directivity**

(V) DIRECTIVITY

The directivity D of an antenna is given by the ratio of the maximum radiation intensity (power per unit solid angle) $U(\theta, \phi)$ to the average radiation intensity U_{av} (averaged over a sphere).

Also at a certain distance from the antenna, the directivity may be expressed as the ratio of the maximum to average poynting vector.

$$D = \frac{U(\theta, \phi)_{max}}{U(\theta)_{average}} = \frac{S(\theta, \phi)_{max}}{S_{average}} \text{ (dimensionless)} \rightarrow (1)$$

Both radiation intensity and poynting vector values should be measured in the far field of the antenna.

$\therefore$ Directivity $D = \dfrac{1}{\frac{1}{4\pi}\iint \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}\, d\Omega}$

$$= \frac{1}{\frac{1}{4\pi}\iint P_n(\theta,\phi)\, d\Omega} \rightarrow (3)$$

or $D = \dfrac{4\pi}{\Omega_A}$

The smaller the beam solid angle, the greater the directivity.

(vi) <u>Examples of Directivity</u>

If an antenna could be isotropic, that radiate the same in all directions,

$P_n(\theta,\phi) = 1$ for all θ and ϕ $\rightarrow (1)$

then $\Omega_A = 4\pi \rightarrow (2)$

$\rightarrow (3)$

$D = 1$

Smallest directivity antenna can have is 1.

$\boxed{\Omega_A \leq 4\pi \text{ and } D \geq 1}$

Neglecting the effect of minor lobes,

$$D = \frac{4\pi}{\Omega_A}$$

$$D \approx \frac{4\pi}{\theta_{HP}\,\phi_{HP}} \approx \frac{41,000}{\phi^{\circ}_{HP}\,\phi^{\circ}_{HP}} \rightarrow (4)$$

4π steradians $= 41,253$ square degrees
$\approx 41,000$ "

where θ_{HP} = half power beamwidth in θ plane, radians

ϕ_{HP} = half power beamwidth in ϕ plane, radians

θ°_{HP} = half power beamwidth in θ plane, degrees

ϕ°_{HP} = " " " ϕ " , "

(v) Beam Efficiency

(iv) BEAM EFFICIENCY :

The (total) beam area Ω_A (beam solid angle) consists of the main beam area Ω_M plus the minor lobe area Ω_m. Thus,

$$\Omega_A = \Omega_M + \Omega_m \longrightarrow (1)$$

The ratio of the main beam area to the total beam area is called the beam efficiency ε_M. Thus,

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A} = \text{Beam efficiency} \longrightarrow (2)$$

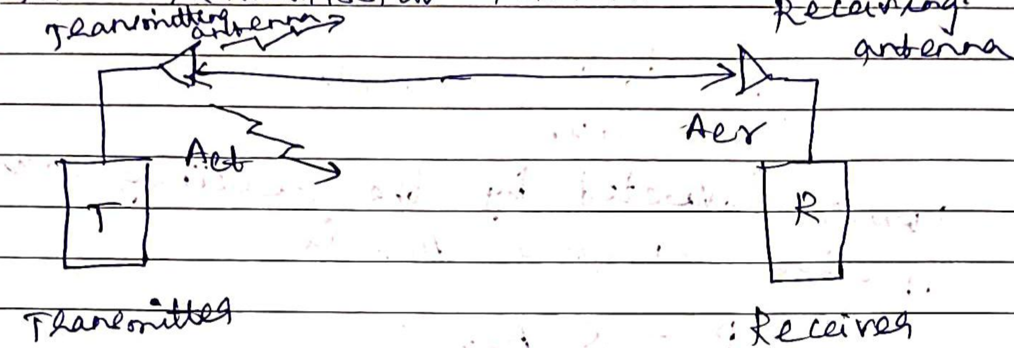
The ratio of the minor lobe area to the total beam area is called the stray factor. Thus,

$$\varepsilon_m = \frac{\Omega_m}{\Omega_A} = \text{stray factor} \longrightarrow (3)$$

Also $\varepsilon_M + \varepsilon_m = 1 \longrightarrow (4)$

7. **(i) Friis Transmission Formula**

FRIIS TRANSMISSION FORMULA



This formula gives the power received over a radio communication circuit.

Let the transmitter T feed a power P_t, to a transmitting antenna of effective power A_{et}. At a distance r, a receiving antenna of effective Aperture A_{er} intercepts some of the power radiated by the transmitting antenna and delivers it to the Receiver R.

Assumption → Transmitting antenna is isotropic, the power per unit area at the receiving antenna is,

$$S_r = \frac{P_t}{4\pi r^2} \longrightarrow (1)$$

If the antenna has gain G_t, the power per unit area at the receiving antenna will be increased in proportion as given by,

$$S_r = \frac{P_t G_t}{4\pi r^2} \text{ watts} \rightarrow (2)$$

Power collected by the receiving antenna of effective aperture A_{er} is,

$$P_r = S_r A_{er}$$

$$= \frac{P_t G_t A_{er}}{4\pi r^2} \text{ watts} \rightarrow (3)$$

The gain of the transmitting antenna can be expressed as,

$$G_t = \frac{4\pi A_{et}}{\lambda^2} \rightarrow (4)$$

Substituting this in eq (3),

$$P_r = \frac{P_t \, 4\pi A_{et} A_{er}}{4\pi r^2 \lambda^2}$$

$$\boxed{P_r = \frac{P_t A_{er} A_{et}}{r^2 \lambda^2}} \text{ (watts)} \rightarrow \text{Friis transmission formula}$$

where

P_r = received Power in watts

P_t = power into transmitting antenna, wa

A_{et} = effective aperture of transmitting antenna in m^2

A_{er} = effective aperture of receiving antenna in m^2

r = distance between antennas, m

λ = wavelength in m

It is assumed that each antenna is in the far field of the other.

7. (ii) $d = 0.5$ km

$= 0.5 \times 10^3$ m

$G_t = 25$ dB

$10 \log_{10} G_t = 25$ dB

$\log_{10} G_t = \frac{25}{10}$

$G_t = 10^{\frac{25}{10}} = 316.23$

$G_r = 20$ dB

$10 \log_{10} G_r = 20$ dB

$\log_{10} G_r = \frac{20}{10}$

$G_r = 10^{\frac{20}{10}} = 100$

$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{1 \times 10^9} = \frac{3}{10} = 0.3$ m

$$P_r = \frac{P_t G_t G_r \lambda^2}{16 \pi^2 d^2}$$

$$P_t = \frac{P_r \, 16 \pi^2 d^2}{G_t G_r \lambda^2}$$

$$= \frac{(1.8 \times 10^{-3})(16)(\pi)^2 (0.5 \times 10^3)^2}{(316.23)(100)(0.3)^2}$$

$$= \frac{426366.9101}{2846.07}$$

$= 149.809$

$\boxed{P_t \approx 150 \text{ watts}}$

11-1-2 Losses in Microstrip Lines

Microstrip transmission lines consisting of a conductive ribbon attached to a dielectric sheet with conductive backing (see Fig. 11-1-4) are widely used in both microwave and computer technology. Because such lines are easily fabricated by printed-circuit manufacturing techniques, they have economic and technical merit.

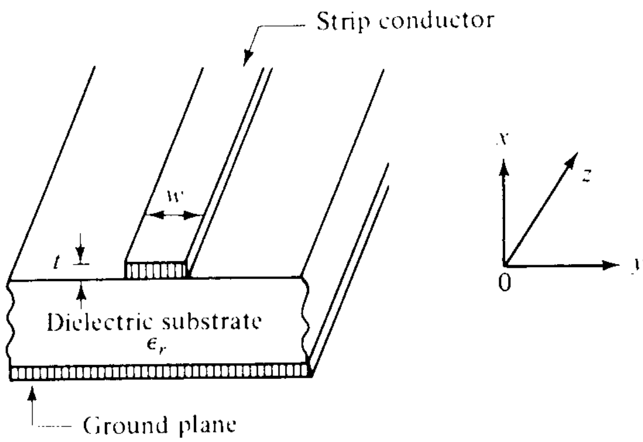


Figure 11-1-4 Schematic diagram of a microstrip line.

The characteristic impedance and wave-propagation velocity of a microstrip line was analyzed in Section 11-1-1. The other characteristic of the microstrip line is its attenuation. The attenuation constant of the dominant microstrip mode depends on geometric factors, electrical properties of the substrate and conductors, and on the frequency. For a nonmagnetic dielectric substrate, two types of losses occur in the dominant microstrip mode: (1) dielectric loss in the substrate and (2) ohmic skin loss in the strip conductor and the ground plane. The sum of these two losses may be expressed as losses per unit length in terms of an attenuation factor α. From ordinary transmission-line theory, the power carried by a wave traveling in the positive z direction is given by

$$P = \tfrac{1}{2} VI^* = \tfrac{1}{2}(V_+ e^{-\alpha z} I_+ e^{-\alpha z}) = \tfrac{1}{2}\frac{|V_+|^2}{Z_0} e^{-2\alpha z} = P_0 e^{-2\alpha z} \qquad (11\text{-}1\text{-}10)$$

where $P_0 = |V_+|^2/(2Z_0)$ is the power at $z = 0$.

The attenuation constant α can be expressed as

$$\alpha = -\frac{dP/dz}{2P(z)} = \alpha_d + \alpha_c \qquad (11\text{-}1\text{-}11)$$

where α_d is the dielectric attenuation constant and α_c is the ohmic attenuation constant.

The gradient of power in the z direction in Eq. (11-1-11) can be further expressed in terms of the power loss per unit length dissipated by the resistance and the power loss per unit length in the dielectric. That is,

$$
\begin{aligned}
-\frac{dP(z)}{dz} &= -\frac{d}{dz}(\tfrac{1}{2}VI^*) \\
&= \tfrac{1}{2}\left(-\frac{dV}{dz}\right)I^* + \tfrac{1}{2}\left(-\frac{dI^*}{dz}\right)V \\
&= \tfrac{1}{2}(RI)I^* + \tfrac{1}{2}\sigma V^* V \\
&= \tfrac{1}{2}|I|^2R + \tfrac{1}{2}|V|^2\sigma = P_c + P_d
\end{aligned}
\qquad (11\text{-}1\text{-}12)
$$

where σ is the conductivity of the dielectric substrate board.

Substitution of Eq. (11-1-12) into Eq. (11-1-11) results in

$$\alpha_d \simeq \frac{P_d}{2P(z)} \qquad \text{Np/cm} \qquad (11\text{-}1\text{-}13)$$

and

$$\alpha_c \simeq \frac{P_c}{2P(z)} \qquad \text{Np/cm} \qquad (11\text{-}1\text{-}14)$$

Dielectric losses. As stated in Section 2-5-3, when the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. In that case the dielectric attenuation constant, as expressed in Eq. (2-5-20), is given by

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \qquad \text{Np/cm} \qquad (11\text{-}1\text{-}15)$$

where σ is the conductivity of the dielectric substrate board in $\mho$/cm. This dielectric constant can be expressed in terms of dielectric loss tangent as shown in Eq. (2-5-17):

$$\tan\theta = \frac{\sigma}{\omega\epsilon} \qquad (11\text{-}1\text{-}16)$$

Then the dielectric attenuation constant is expressed by

$$\alpha_d = \frac{\omega}{2}\sqrt{\mu\epsilon}\,\tan\theta \qquad \text{Np/cm} \qquad (11\text{-}1\text{-}17)$$

Since the microstrip line is a nonmagnetic mixed dielectric system, the upper dielectric above the microstrip ribbon is air, in which no loss occurs. Welch and Pratt [9] derived an expression for the attenuation constant of a dielectric substrate. Later on, Pucel and his coworkers [10] modified Welch's equation [9]. The result is

$$
\begin{aligned}
\alpha_d &= 4.34\,\frac{q\sigma}{\sqrt{\epsilon_{re}}}\sqrt{\frac{\mu_0}{\epsilon_0}} \\
&= 1.634 \times 10^3\,\frac{q\sigma}{\sqrt{\epsilon_{re}}} \qquad \text{dB/cm}
\end{aligned}
\qquad (11\text{-}1\text{-}18)
$$

In Eq. (11-1-18) the conversion factor of 1 Np = 8.686 dB is used, ϵ_{re} is the effective dielectric constant of the substrate, as expressed in Eq. (11-1-5), and q denotes the dielectric filling factor, defined by Wheeler [3] as

$$q = \frac{\epsilon_{re} - 1}{\epsilon_r - 1} \tag{11-1-19}$$

We usually express the attenuation constant per wavelength as

$$\alpha_d = 27.3 \left(\frac{q\epsilon_r}{\epsilon_{re}}\right) \frac{\tan\theta}{\lambda_g} \qquad \text{dB}/\lambda_g \tag{11-1-20}$$

where $\lambda_g = \dfrac{\lambda_0}{\sqrt{\epsilon_{re}}}$ and λ_0 is the wavelength in free space, or

$\lambda_g = \dfrac{c}{f\sqrt{\epsilon_{re}}}$ and c is the velocity of light in vacuum.

If the loss tangent, $\tan\theta$, is independent of frequency, the dielectric attenuation per wavelength is also independent of frequency. Moreover, if the substrate conductivity is independent of frequency, as for a semiconductor, the dielectric attenuation per unit is also independent of frequency. Since q is a function of ϵ_r and w/h, the filling factors for the loss tangent $q\epsilon_n/\epsilon_{re}$ and for the conductivity $q/\sqrt{\epsilon_{re}}$ are also functions of these quantities. Figure 11-1-5 shows the loss-tangent filling factor against w/h for a range of dielectric constants suitable for microwave inte-

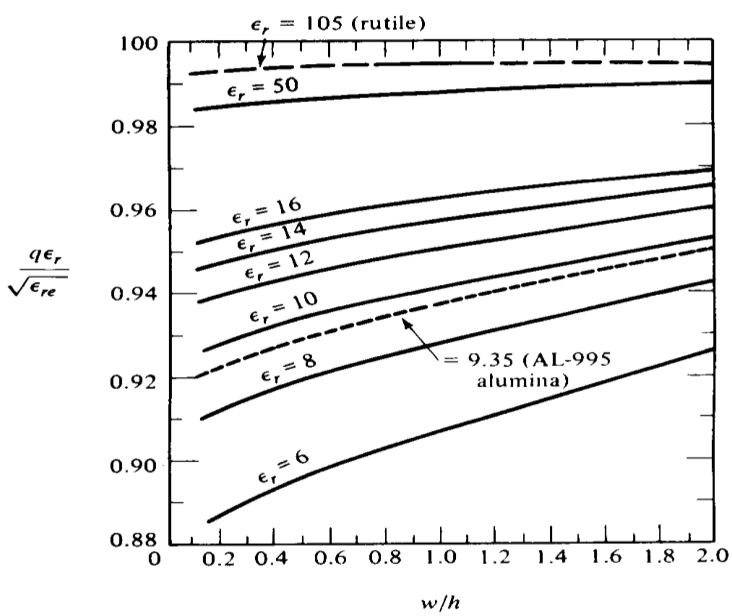


Figure 11-1-5 Filling factor for loss tangent of microstrip substrate as a function of w/h. (*After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.*)

grated circuits. For most practical purposes, this factor is considered to be 1. Figure 11-1-6 illustrates the product $\alpha_d\rho$ against w/h for two semiconducting substrates, silicon and gallium arsenide, that are used for integrated microwave circuits. For design purposes, the conductivity filling factor, which exhibits only a mild dependence on w/h, can be ignored.

Ohmic losses. In a microstrip line over a low-loss dielectric substrate, the predominant sources of losses at microwave frequencies are the nonperfect conductors. The current density in the conductors of a microstrip line is concentrated in a sheet that is approximately a skin depth thick inside the conductor surface and exposed to the electric field. Both the strip conductor thickness and the ground plane thickness are assumed to be at least three or four skin depths thick. The current density in the strip conductor and the ground conductor is not uniform in the transverse plane. The microstrip conductor contributes the major part of the ohmic loss. A diagram of the current density J for a microstrip line is shown in Fig. 11-1-7.

Because of mathematical complexity, exact expressions for the current density of a microstrip line with nonzero thickness have never been derived [10]. Several researchers [8] have assumed, for simplicity, that the current distribution is uniform and equal to I/w in both conductors and confined to the region $|x| < w/2$. With this assumption, the conducting attenuation constant of a wide microstrip line is given by

$$\alpha_c \simeq \frac{8.686 R_s}{Z_0 w} \quad \text{dB/cm} \quad \text{for } \frac{w}{h} > 1 \tag{11-1-21}$$

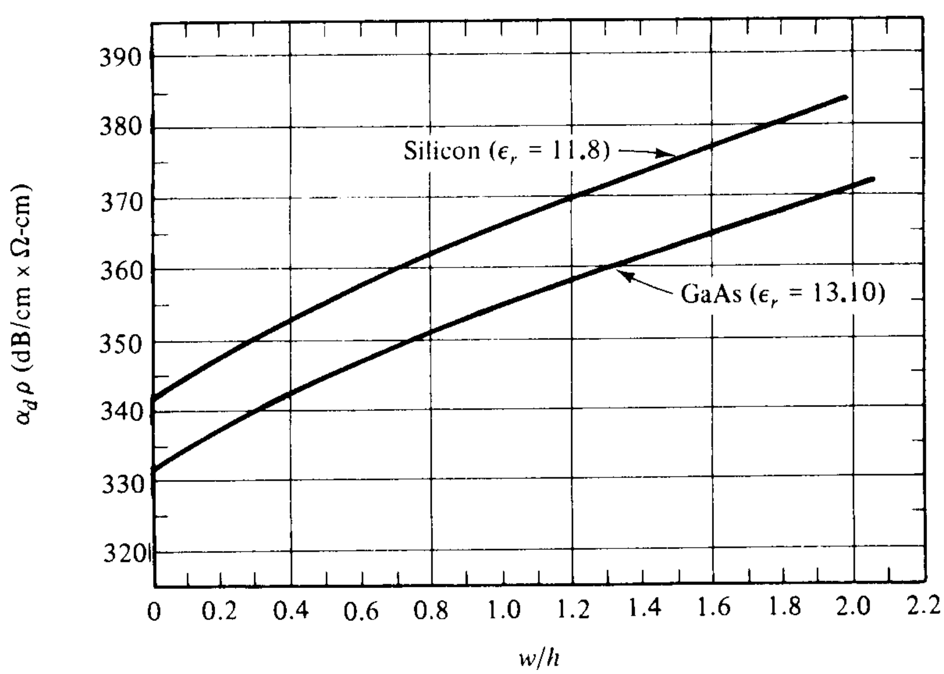


Figure 11-1-6 Dielectric attenuation factor of microstrip as a function of w/h for silicon and gallium arsenide substrates. (*After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.*)

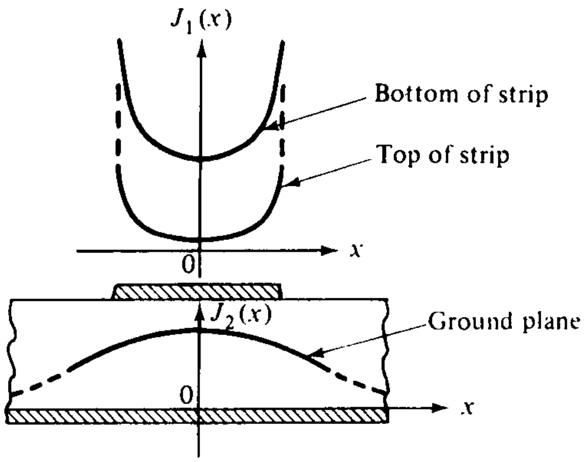


Figure 11-1-7 Current distribution on microstrip conductors. (*After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.*)

where $R_s = \sqrt{\dfrac{\pi f \mu}{\sigma}}$ is the surface skin resistance in Ω/square,

$R_s = \dfrac{1}{\delta\sigma}$ is Ω/square

$\delta = \sqrt{\dfrac{1}{\pi f \mu \sigma}}$ is the skin depth in cm

For a narrow microstrip line with $w/h < 1$, however, Eq. (11-1-21) is not applicable. The reason is that the current distribution in the conductor is not uniform, as assumed. Pucel and his coworkers [10, 11] derived the following three formulas from the results of Wheeler's work [3]:

Radiation losses. In addition to the conductor and dielectric losses, microstrip line also has radiation losses. The radiation loss depends on the substrate's thickness and dielectric constant, as well as its geometry. Lewin [12] has calculated the radiation loss for several discontinuities using the following approximations:

1. TEM transmission
2. Uniform dielectric in the neighborhood of the strip, equal in magnitude to an effective value
3. Neglect of radiation from the transverse electric (TE) field component parallel to the strip
4. Substrate thickness much less than the free-space wavelength

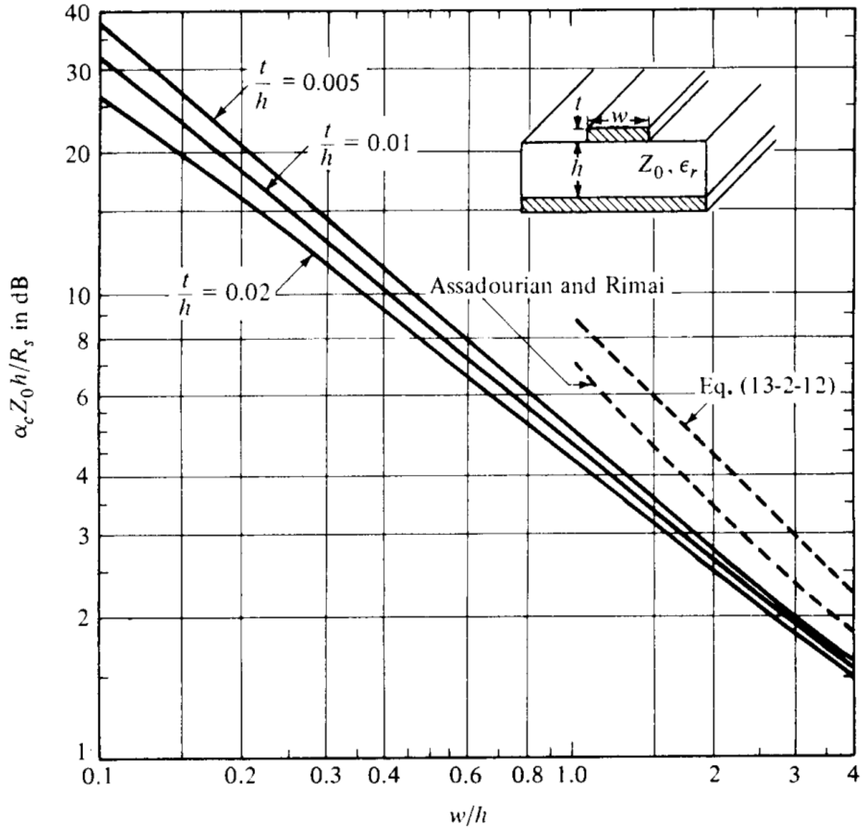


Figure 11-1-8 Theoretical conductor attenuation factor of microstrip as a function of w/h. (*After R. A. Pucel et al. [10]; reprinted by permission of IEEE, Inc.*)

Lewin's results show that the ratio of radiated power to total dissipated power for an open-circuited microstrip line is

$$\frac{P_{\text{rad}}}{P_t} = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 \frac{F(\epsilon_{re})}{Z_0} \tag{11-1-28}$$

where $F(\epsilon_{re})$ is a radiation factor given by

$$F(\epsilon_{re}) = \frac{\epsilon_{re} + 1}{\epsilon_{re}} - \frac{\epsilon_{re} - 1}{2\epsilon_{re}\sqrt{\epsilon_{re}}} \ln \frac{\sqrt{\epsilon_{re}} + 1}{\sqrt{\epsilon_{re}} - 1} \tag{11-1-29}$$

in which ϵ_{re} is the effective dielectric constant and $\lambda_0 = c/f$ is the free-space wavelength.

The radiation factor decreases with increasing substrate dielectric constant. So, alternatively, Eq. (11-1-28) can be expressed as

$$\frac{P_{\text{rad}}}{P_t} = \frac{R_r}{Z_0} \tag{11-1-30}$$

where R_r is the radiation resistance of an open-circuited microstrip and is given by

$$R_r = 240\pi^2 \left(\frac{h}{\lambda_0}\right)^2 F(\epsilon_{re}) \tag{11-1-31}$$

The ratio of the radiation resistance R_r to the real part of the characteristic impedance Z_0 of the microstrip line is equal to a small fraction of the power radiated from a single open-circuit discontinuity. In view of Eq. (11-1-28), the radiation loss decreases when the characteristic impedance increases. For lower dielectric-constant substrates, radiation is significant at higher impedance levels. For higher dielectric-constant substrates, radiation becomes significant until very low impedance levels are reached.