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Internal Assesment Test-III									
Sub:	Electromagnetic Theory							Code:	BEC401
Date:	06/08/2024	Duration:	90 mins	Max Marks:	50	Sem:	4th	Branch:	ECE(A,B,C,D)
Answer any **FIVE FULL** Questions									

		Marks	OBE CO	OBE RBT
1.(a)	Obtain the boundary conditions at the interface between two magnetic materials.	[07]	CO4	L2

** Magnetic Boundary conditions:-

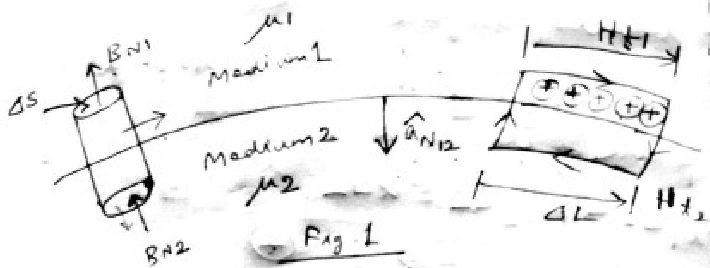


Fig. 1

Note

$(\vec{H}_1 - \vec{H}_2) \times \hat{a}$

$\hat{a}_y = k$

$(-\hat{a}_x)$

Gauss's law of magnetostatics, $\oint\!\!\oint \vec{B}\cdot\vec{ds} = 0$

$B_{N1}\Delta S - B_{N2}\Delta S = 0$

or $B_{N1}\Delta S = B_{N2}\Delta S$

or $\boxed{B_{N1} = B_{N2}}$ .. (1)

Normal comp. of B is continuous

Note

B_{N1}, H_{t1}

B_{N2}, H_{t2}

B_N, H_N

B_t, H_t

The fig. 1 shows a boundary b/w two isotropic homogeneous linear materials with permeabilities μ_1 and μ_2.

Now, $\vec{B} = \mu\vec{H}$

$\therefore B_{N1} = \mu_1 H_{N1}$ and $B_{N2} = \mu_2 H_{N2}$

$\therefore$ From (1), $\mu_1 H_{N1} = \mu_2 H_{N2}$

or $\boxed{H_{N2} = \frac{\mu_1}{\mu_2} H_{N1}}$ .. (2)

$\vec{M} = \chi_m \vec{H}$

$M_{N1} = \chi_{m1} H_{N1}$ and $M_{N2} = \chi_{m2} H_{N2}$

$$\frac{M_{N2}}{\chi_{m2}} = \frac{\mu_1}{\mu_2}\frac{M_{N1}}{\chi_{m1}}$$

or $\boxed{M_{N2} = \frac{\mu_1}{\mu_2}\cdot\frac{\chi_{m2}}{\chi_{m1}}\cdot M_{N1}}$ .. (3)

(b) Find magnetization in magnetic material where: [03] CO4 L3

i. $\mu = 1.8\times10^{-5}$ H/m, $\mathbf{H} = 120$ A/m

ii. $\mu_r = 22$ there are 8.3×10^{28} atoms/m^3 and each atom has a dipole moment of 4.5×10^{-27} A-m^2.

Soln.

(i) $\mu = 1.8\times10^{-5}$ H/m and $\vec{H} = 120$ A/m

$\underline{\vec{M} = \chi_m \vec{H}}$

$\mu = \mu_0\mu_r$ or $1.8\times10^{-5} = 4\pi\times10^{-7}\cdot\mu_r$

or $\underline{\underline{\mu_r}} = \dfrac{1.8\times10^{-5}}{4\pi\times10^{-7}} = 14.32$

$\mu_r = (1+\chi_m)$

or $\chi_m = \mu_r - 1 = 13.32$.

$\therefore \vec{M} = \chi_m\vec{H} = (13.32\times120)$ A/m $= 1598.8$ A/

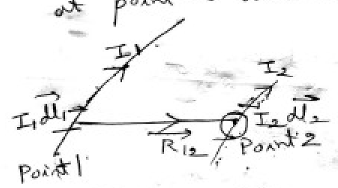
(ii) $M = (8.3\times10^{28})\times(4.5\times10^{-27}) = 373.5$ A/m.

$\boxed{M = nm}$

2.(a) Derive an expression of differential force on differential current element. [06] CO4 L2

Force b/w differential current elements

Using Biot-Savart law, the mag. field at point 2 due to current element at point 1,



$$\vec{dH_2} = \frac{I_1\vec{dl_1}\times\hat{a}_{R_{12}}}{4\pi R_{12}^2} \quad \text{--- (1)}$$

Then at point 2, $\vec{dB_2} = \mu_0 \vec{dH_2}$

$$= \mu_0\frac{I_1\vec{dl_1}\times\hat{a}_{R_{12}}}{4\pi R_{12}^2} \quad \text{--- (2)}$$

we choose second current element $I_2\vec{dl_2}$.

$\therefore$ The differential force on element 2 is

$$d(\vec{dF_2}) = I_2\vec{dl_2}\times\vec{dB_2}$$

$$= I_2\vec{dl_2}\times\left(\mu_0\frac{I_1\vec{dl_1}\times\hat{a}_{R_{12}}}{4\pi R_{12}^2}\right)$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}\vec{dl_2}\times(\vec{dl_1}\times\hat{a}_{R_{12}})$$

$\therefore$ The total force b/w two filamentary circuit is,

$$\boxed{\vec{F_2} = \frac{\mu_0 I_1 I_2}{4\pi}\left[\oint\vec{dl_2}\times\oint\frac{\vec{dl_1}\times\hat{a}_{R_{12}}}{R_{12}^2}\right]}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}\oint\left[\oint\frac{\hat{a}_{R_{12}}\times\vec{dl_1}}{R_{12}^2}\right]\times\oint\vec{dl_2}$$

(b) Write the equations to compare electric circuit and magnetic circuit parameters. [04] CO4 L2

Electric Circuit	Magnetic Circuit
$\mathbf{E} = -\nabla V$	$\mathbf{H} = -\nabla V_m$
$V_{AB} = \int_A^B \mathbf{E} \cdot d\mathbf{L}$	$V_{m\,AB} = \int_A^B \mathbf{H} \cdot d\mathbf{L}$
$\mathbf{J} = \sigma \mathbf{E}$	$\mathbf{B} = \mu \mathbf{H}$
$I = \int_S \mathbf{J} \cdot d\mathbf{S}$	$\Phi = \int_S \mathbf{B} \cdot d\mathbf{S}$
$V = IR$	$V_m = \Phi \Re$
$R = \frac{d}{\sigma S}$	$\Re = \frac{d}{\mu S}$
$\oint \mathbf{E} \cdot d\mathbf{L} = 0$	$\oint \mathbf{H} \cdot d\mathbf{L} = I_{\text{total}}$ $\oint \mathbf{H} \cdot d\mathbf{L} = NI$

3. Using Maxwell's equation derive an expression for a uniform plane wave in free space. [10] CO5 L2

TEM wave propagation in free space

For free space, medium is sourceless,

$\rho_v = 0, \quad J = 0$

∴ Maxwell's equations are,

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \ldots (1)$$

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \ldots (2)$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \ldots (3)$$

$$\vec{\nabla} \cdot \vec{H} = 0 \quad \ldots (4)$$

Note: $\vec{\nabla} \cdot \vec{D} = 0$ [∵ ρ_v ... ∴ $\epsilon \vec{\nabla} \cdot \vec{E} = 0$

From (1), if $\vec{E}$ is changing with time, then at ~~the~~ same point $\vec{H}$ has curl.

Uniform plane wave where $\vec{E}$ and $\vec{H}$ fields lies in the transverse plane.

– Such a wave is called TEM wave.

We assume, $\vec{E} = E_x \hat{a}_x$ and $\vec{H} = H_y \hat{a}_y$

Then, $\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \hat{a}_y \frac{\partial E_x}{\partial z} - \hat{a}_z \frac{\partial E_x}{\partial y}$

$\therefore$ From eqn. 2

$$\vec{\nabla} \times \vec{E} = \frac{\partial E_x}{\partial z}\, \hat{a}_y = -\mu_0 \frac{\partial H}{\partial t}$$

or $$\frac{\partial E_x}{\partial z}\, \hat{a}_y = -\mu_0 \frac{\partial H_y}{\partial t}\, \hat{a}_y \quad \text{-- (5)}$$

Similarly from (1),

$$\vec{\nabla} \times \vec{H} = -\frac{\partial H_y}{\partial z}\, \hat{a}_x = \epsilon_0 \frac{\partial E_x}{\partial t}\, \hat{a}_x \quad \text{-- (6)}$$

Equations (5) and (6),

$$\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H_y}{\partial t} \quad \text{-- (7)}$$

$$\frac{\partial H_y}{\partial z} = -\epsilon_0 \frac{\partial E_x}{\partial t} \quad \text{-- (8)}$$

Differentiating (7) w.r.t z,

$$\frac{\partial^2 E_x}{\partial z^2} = -\mu_0 \left(\frac{\partial^2 H_y}{\partial z\, \partial t}\right) \quad \text{-- (9)}$$

Differentiating (8) w.r.t t,

$$\left(\frac{\partial^2 H_y}{\partial z\, \partial t}\right) = -\epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \quad \text{-- (10)}$$

Substituting (10) into (9)

$$\boxed{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}} \quad \text{-- (11)}$$

— x-polarized TEM electric field in free space

Differentiating (7) w.r.t t,

$$\left(\frac{\partial^2 E_x}{\partial t\, \partial z}\right) = -\mu_0 \frac{\partial^2 H_y}{\partial t^2} \quad \text{-- (12)}$$

Differentiating (8) w.r.t z,

$$\frac{\partial^2 H_y}{\partial z^2} = -\epsilon_0 \left(\frac{\partial^2 E_x}{\partial t\, \partial z}\right) \quad \text{-- (13)}$$

$$\therefore \frac{\partial^2 H_y}{\partial z^2} = \epsilon_0 \mu_0 \left(\frac{\partial^2 H_y}{\partial t^2}\right)$$

$$\boxed{\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2}} \quad \text{-- (14)}$$

Wave equation for the magnetic field.

4. Discuss the uniform plane wave propagation in a good conducting medium. Explain skin effect and skin depth. [10] CO5 L2

Propagation in good conductor (Skin effect)

For a conductor, $\frac{\epsilon''}{\epsilon'} >> 1$

or $\frac{\sigma}{\omega\epsilon'} >> 1$ $\left[\because \epsilon'' = \frac{\sigma}{\omega}\right]$

$\therefore$ The general expression for propagation constant,

$$jk = j\omega\sqrt{\mu\epsilon'\left(1 - j\frac{\epsilon''}{\epsilon'}\right)}$$

$$= j\omega\sqrt{\mu\epsilon'\left(1 - \frac{j\sigma}{\omega\epsilon'}\right)}$$

$$jk = j\omega\sqrt{\mu\epsilon}$$

$$= j\omega\sqrt{\mu(\epsilon' - j\epsilon'')}$$

$$= j\omega\sqrt{\mu\epsilon'\left(1 - j\frac{\epsilon''}{\epsilon'}\right)}$$

$$= j\omega\sqrt{\mu\epsilon'\left(\frac{-j\sigma}{\omega\epsilon'}\right)}$$

$$= j\sqrt{\frac{\omega^{\cancel{2}}\mu\cancel{\epsilon'}(-j)\sigma}{\cancel{\omega}\cancel{\epsilon'}}} = j\sqrt{(-j)\omega\mu\sigma}$$

$$-j = 1\angle -90^\circ$$

$$\sqrt{1\angle -90^\circ} = 1\angle(-45^\circ) = \frac{1}{\sqrt{2}}(1 - j)$$

$$jk = j\sqrt{\omega\mu\sigma}\,\frac{(1-j)}{\sqrt{2}} = \sqrt{\frac{\omega\mu\sigma}{2}}\,j(1-j)$$

$$= j\sqrt{\frac{\omega\mu\sigma}{2}} + \sqrt{\frac{\omega\mu\sigma}{2}} = \alpha + j\beta$$

comparing, $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{\cancel{2}\pi f\mu\sigma}{\cancel{2}}} = \sqrt{\pi f\mu\sigma} = \beta$

If only E_x component travelling along z-direction,

$$E_x = E_{x0}\, e^{-\alpha z} \cos(\omega t - \beta z), \quad \left[\begin{array}{l} z>0 \text{ good conductor} \\ z<0 \text{ perfect dielectric} \end{array}\right.$$

$$\text{or } E_x = E_{x0}\underbrace{\left[e^{-(\sqrt{\pi f\mu\sigma})z}\right]}_{\text{Indicates a decrease in conduction current density and electric field intensity with penetration along z-direction}} \cos\left(\omega t - (\sqrt{\pi f\mu\sigma}).z\right)$$

If $z = \frac{1}{\sqrt{\pi f\mu\sigma}}$ ✓

Skin depth, $\boxed{\delta = \frac{1}{\sqrt{\pi f\mu\sigma}} = \frac{1}{\alpha} = \frac{1}{\beta}}$

The distance at which the ~~source~~ electric field intensity reduces to $e^{-1} = 0.368$ of its original value is called skin depth.

Intrinsic impedance of a good conductor

$$\eta = \sqrt{\frac{\mu}{\epsilon' - j\epsilon''}} = \sqrt{\frac{\mu}{\epsilon'}} \cdot \frac{1}{\sqrt{\left(1 - j\frac{\epsilon''}{\epsilon'}\right)}}$$

$$= \sqrt{\frac{\mu}{\epsilon'}} \cdot \sqrt{\frac{1}{\left(1 - j\frac{\sigma}{\omega\epsilon'}\right)}} = \sqrt{\frac{j\omega\mu}{j\omega\epsilon'\left(1 + \frac{\sigma}{j\omega\epsilon'}\right)}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\epsilon')}}$$

$$\therefore \text{ when } \sigma >> \omega\epsilon', \quad \eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

5. State and prove Poynting's theorem. Define Poynting's vector. [10] CO5 L2

Poynting's Theorem :-

For a conductive medium,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking dot product of $\vec{E}$ on both sides,

$$\vec{E}.(\vec{\nabla} \times \vec{H}) = \vec{E}.\vec{J} + \vec{E}.\frac{\partial \vec{D}}{\partial t}$$

$$\text{or } \vec{E}.(\vec{\nabla} \times \vec{H}) = \vec{E}.\vec{J} + \epsilon\left(\vec{E}.\frac{\partial \vec{E}}{\partial t}\right) \quad \text{-- (1)}$$

According to vector identity,

$$\vec{\nabla}.(\vec{E} \times \vec{H}) = -\vec{E}.(\vec{\nabla} \times \vec{H}) + \vec{H}.(\vec{\nabla} \times \vec{E})$$

$$\text{or } \vec{E}.(\vec{\nabla} \times \vec{H}) = \vec{H}.(\vec{\nabla} \times \vec{E}) - \vec{\nabla}.(\vec{E} \times \vec{H}) \quad \text{-- (2)}$$

Using (2) into eqn. (1),

$$\vec{H}.(\vec{\nabla} \times \vec{E}) - \vec{\nabla}.(\vec{E} \times \vec{H}) = \vec{E}.\vec{J} + \epsilon\left(\vec{E}.\frac{\partial \vec{E}}{\partial t}\right)$$

$$\text{or } \vec{H}.\left(-\frac{\partial \vec{B}}{\partial t}\right) - \vec{\nabla}.(\vec{E} \times \vec{H}) = \vec{E}.\vec{J} + \epsilon\left(\vec{E}.\frac{\partial \vec{E}}{\partial t}\right)$$

$$\left[\because (\vec{\nabla} \times \vec{E}) = -\left(\frac{\partial \vec{B}}{\partial t}\right)\right]$$

Faraday's law,

$$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}$$

or $$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \mu\,\vec{H}\cdot\frac{\partial\vec{H}}{\partial t},$$

$$\mu\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)$$

and $$\epsilon\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)$$

Note

$$\frac{\partial}{\partial t}(\vec{H}\cdot\vec{H}) = \vec{H}\cdot\frac{\partial\vec{H}}{\partial t} + \frac{\partial\vec{H}}{\partial t}\cdot\vec{H} = 2\left(\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}\right)$$

or $$\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(\vec{H}\cdot\vec{H})$$

$$\therefore -\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \mu\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}.$$

i.e. $$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right) \quad \text{--- (3)}$$

Integrating over a volume,

$$-\iiint \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dv = \iiint(\vec{E}\cdot\vec{J})\,dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)dv$$

or $$-\oint\!\!\!\oint(\vec{E}\times\vec{H})\cdot\vec{ds} = \iiint(\vec{E}\cdot\vec{J})\,dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)dv. \quad \text{-- (4)}$$

The total power flowing out of the volume is,

$$\oint\!\!\oint (\vec{E} \times \vec{H}) \cdot \vec{ds} \text{ Watt}$$

The cross product, $(\vec{E} \times \vec{H}) \equiv \vec{S}$ W/m^2

Poynting's vector

In case of uniform plane wave,

$$E_x \hat{a}_x \times H_y \hat{a}_y = S_z \hat{a}_z$$

$$E_x = E_{x0} \cos(\omega t - \beta z)$$

$$H_y = \frac{E_{x0}}{\eta} \cos(\omega t - \beta z)$$

$$S_z = \frac{E_{x0}^2}{\eta} \cos^2(\omega t - \beta z)$$

The time-average power density, $\langle S_z \rangle$

For lossy dielectric, $\langle S_z \rangle = \frac{1}{2} \frac{E_{x0}^2}{\eta} e^{-2\alpha z} \cos\theta_n$

where, $\eta = |\eta| \angle \theta_n$.

6.(a) What is the inconsistency of Ampere's law with the continuity of the current equation? Derive a modified form of Ampere's law for time-varying fields. [07] CO5 L2

Ampere's circuital law for time-varying magnetic field :-

According to Ampere's law,

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad \text{(1)}$$

$$\therefore \quad \vec{\nabla}.(\vec{\nabla} \times \vec{H}) \equiv \vec{\nabla}.\vec{J} = 0$$

$$[\because \vec{\nabla}.(\vec{\nabla} \times \vec{H}) = 0 \text{ identically}]$$

But According to continuity of current eqn,

$$\vec{\nabla}.\vec{J} = -\frac{\partial \rho_v}{\partial t} \quad \text{(2)}$$

For time varying fields we add an unknown term G to eqn. (1)

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{G}$$

$$\text{or} \quad \vec{\nabla}.(\vec{\nabla} \times \vec{H}) = \vec{\nabla}.\vec{J} + \vec{\nabla}.\vec{G}$$

$$0 = \vec{\nabla}.\vec{J} + \vec{\nabla}.\vec{G}$$

$$\text{or} \quad \vec{\nabla}.\vec{J} = -\vec{\nabla}.\vec{G} = \frac{-\partial \rho_v}{\partial t}$$

$$\text{or} \quad \therefore \vec{\nabla}.\vec{G} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\vec{\nabla}.\vec{D})$$

$$[\because \vec{\nabla}.\vec{D} = \rho_v]$$

$$\text{or} \quad \vec{\nabla}.\vec{G} = \vec{\nabla}.\frac{\partial \vec{D}}{\partial t}$$

$$\text{or} \quad \boxed{\vec{G} = \frac{\partial \vec{D}}{\partial t}}$$

$$\therefore \boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

$$\frac{\partial \vec{D}}{\partial t} = \vec{J}_D = \text{Displacement current density}$$

Point form of Ampere's law,

$$\oint \vec{H}.\vec{dl} = I + \iint \frac{\partial \vec{D}}{\partial t}.\vec{ds}$$

→ Integral form of Ampere's law

(b) Write Maxwell's equations in point and integral form. [03] CO5 L2

Point form of Maxwell equations

$\vec{\nabla}\cdot\vec{D} = \rho_v \longrightarrow$ Gauss's law of electrostatics

$\vec{\nabla}\cdot\vec{B} = 0 \longrightarrow$ Gauss's law of magnetostatics

$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \longrightarrow$ Faraday's law

$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \longrightarrow$ Ampere's law

$\vec{D} \longrightarrow$ Electric flux density

Integral form of Maxwell's equations

Gauss's law for electrostatics

$$\oiint \vec{D}\cdot\vec{ds} = \iiint \rho_v\, dv$$

Gauss's divergence theorem

$$\oiint \vec{D}\cdot\vec{ds} = \iiint (\vec{\nabla}\cdot\vec{D})\,dv$$

$$\vec{\nabla}\cdot\vec{D} = \rho_v$$

Gauss's Law for magnetostatics

$$\oiint \vec{B}\cdot\vec{ds} = 0$$

Faraday's Law

$$\oint \vec{E}\cdot\vec{dl} = -\iint \frac{\partial\vec{B}}{\partial t}\cdot\vec{ds}$$

Ampere's Law

$$\oint \vec{H}\cdot\vec{dl} = I + \iint \frac{\partial\vec{D}}{\partial t}\cdot\vec{ds}$$

7. A 9.375 GHz uniform plane wave is propagating in polyethylene εr = 2.26. If the amplitude of **E** is 500 V/m and the material is assumed to be lossless, find (a) Phase constant, (b) wavelength, (c) velocity of propagation, (d) intrinsic impedance, and (e) magnetic field intensity. [10] CO5 L3

solution For lossless medium, $\epsilon'' = 0$ $\quad \boxed{\epsilon = \epsilon_0 \epsilon_r}$

(i) $\beta = \omega\sqrt{\mu\epsilon'} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r'}$ $\quad \boxed{jk = \alpha + j\beta, \ \epsilon = \epsilon' - j\epsilon''}$

$$= \frac{2\pi f}{c}\sqrt{\mu_r\epsilon_r} = \frac{2\pi \times 9.375\times10^{9}}{3\times10^{8}}\sqrt{2.26}$$

$\left[\because c = \frac{1}{\sqrt{\mu_0\epsilon_0}}\right]$

$$= 295 \text{ rad/m}$$

(ii) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{295} = 2.13 \text{ cm}$

(iii) $v_p = \frac{\omega}{\beta} = 1.99\times10^{8} \text{ m/s}$

(iv) $\eta = \frac{\eta_0\sqrt{\mu_r'}}{\sqrt{\epsilon_r'}} = \frac{377\sqrt{1}}{\sqrt{2.26}} = 250.7\ \Omega$

note: $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \eta_0\sqrt{\frac{\mu_r}{\epsilon_r}}$

(v) $H_y = \frac{E_x}{\eta} = \frac{500}{250.7} = 1.99 \text{ A/m}$