CMR INSTITUTE OF TECHNOLOGY

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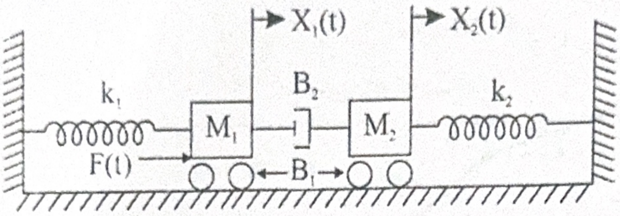
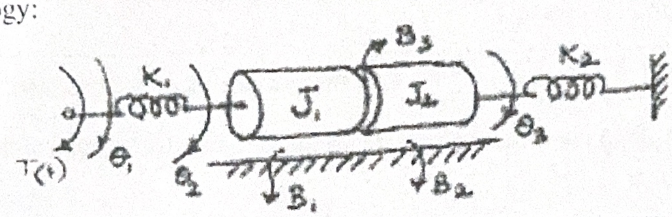
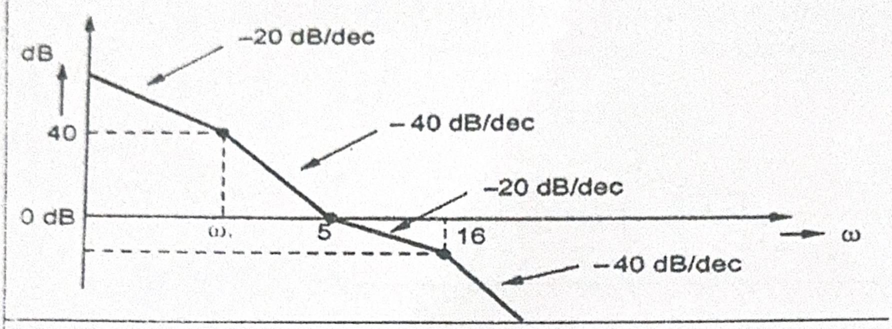
Internal Assessment Test – III

Sub:	Control Systems				Code:	BEC403
Date: 07/ 08/ 2024	Duration: 90 mins	Max Marks:	50	Sem: 4th	Branch:	ECE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1.	Draw the equivalent mechanical system and write the differential equations governing the behaviour of the mechanical system given below. Also Find the Force-Voltage and Force-Current analogous electrical network with their differential equations. 	[10]	CO1	L3
2.	Draw the mechanical network for the system shown in Figure below. Write the equations of performance and draw its analogous circuit based on Force-Current and Force-Voltage analogy: 	[10]	CO1	L3
3.	Analyze the range of K for which the system with closed loop transfer function $\frac{K}{S(S+2)(S^2+S+1)+K}$ is stable using R-H criteria. For what value of K the system oscillates and what is the corresponding frequency of oscillation.	[10]	CO4	L3
4	Sketch the root locus for a negative feedback control system with G(s)H(s) = $\frac{K}{S(S+4)(S^2+4S+20)}$.	[10]	CO4	L3

PTO...

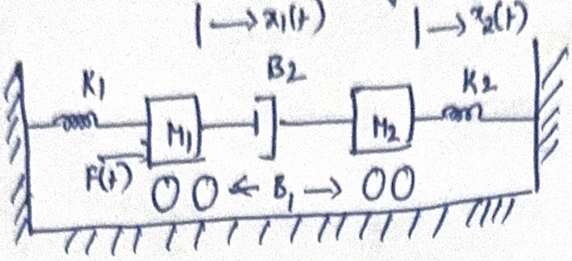
5	a) Analyze the Bode plot shown in Fig below to estimate the transfer function of a control system: 	[10]	CO5	L3
6	Sketch the Bode plot for open loop transfer function. $G(s)H(s) = \frac{K}{S(S+1)(0.1S+1)}$. Determine the value of K for gain margin (GM) of 10 dB.	[10]	CO5	L3
7.	For the state equation $X = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} X + \begin{pmatrix} 0 \\ 1 \end{pmatrix} U$ with the unit step input and the initial conditions are $X(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Find the following (a) State transition matrix (b) Solution of the state equation.	[10]	CO5	L3

CI CCI HOD

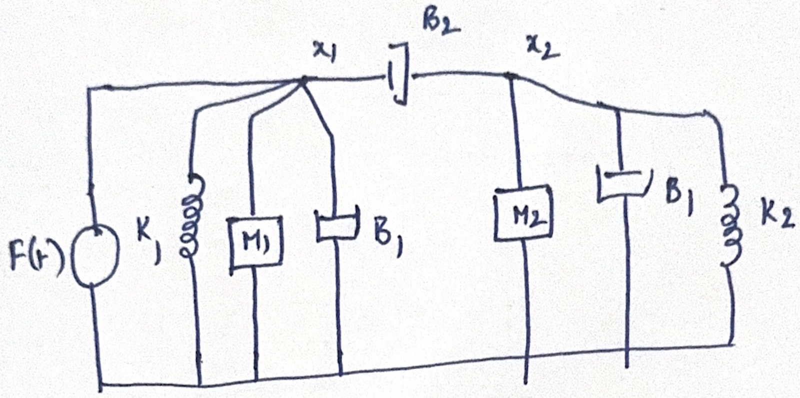
0.6989

Control Systems IAT-3 Solutions

1



Equivalent Mechanical System



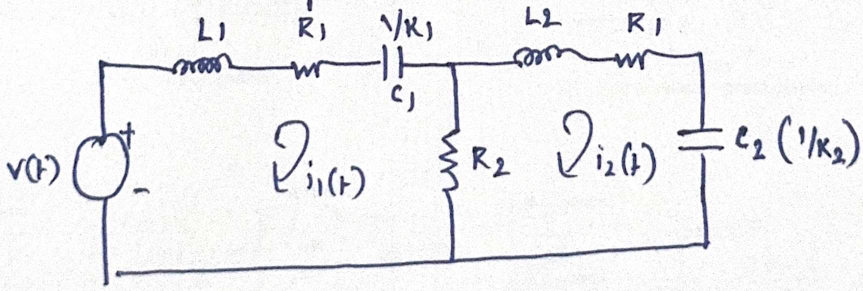
Differential Equations

$$F(t) = M_1 \cdot \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{d}{dt} x_1(t) + K_1 x_1(t) + B_2\left[\frac{d}{dt}(x_1(t)) - \frac{d}{dt} x_2(t)\right]$$

$$M_2 \cdot \frac{d^2 x_2(t)}{dt^2} + B_1 \frac{d}{dt} x_2(t) + K_2 x_2(t) + B_2\left[\frac{d}{dt}(x_2(t) - \frac{d}{dt}(x_1(t))\right] = 0$$

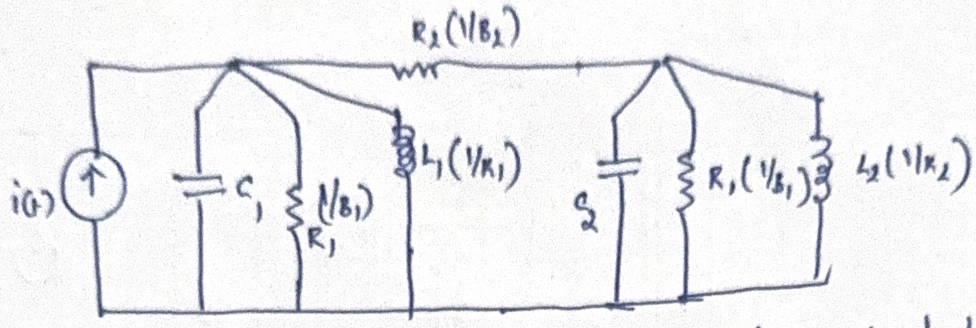
Force-Voltage Analogy

Force – voltage, Mass – L, B – R, K – $\frac{1}{C}$



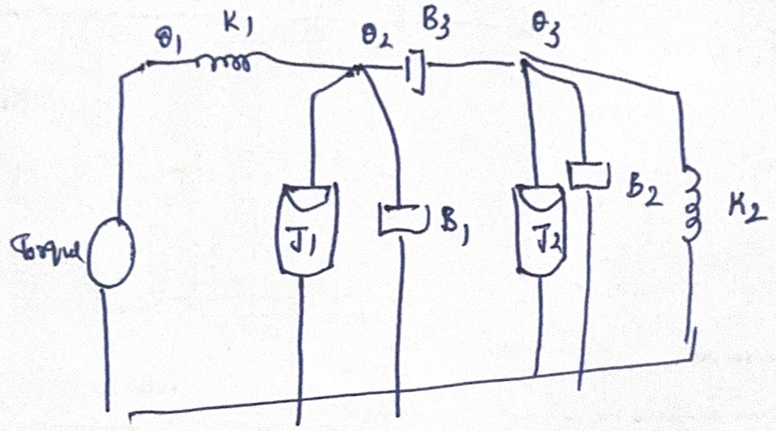
Force – Current Analogy

Force – Current, Mass – C, B – 1/R, K – $\frac{1}{L}$



Write the Corresponding differential equations in both the analogies.

(2)

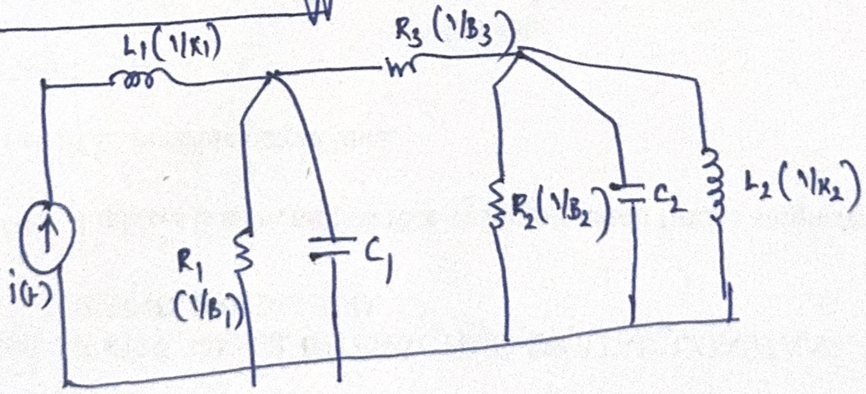


$$T(t) = K_1(\theta_1 - \theta_2)$$

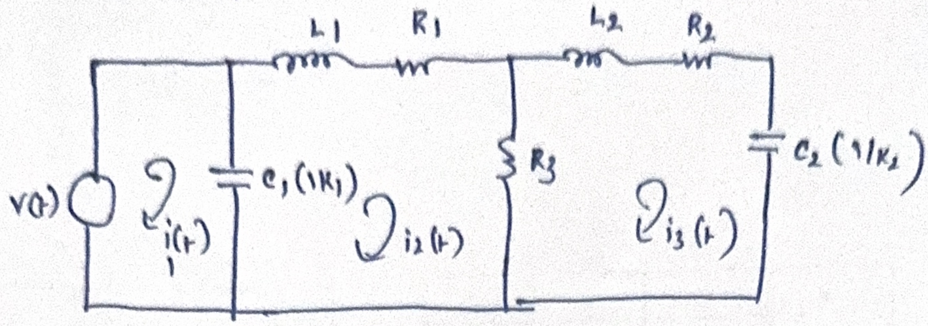
$$J_1 \cdot \frac{d^2\theta_2(t)}{dt^2} + B_1 \frac{d\theta_2(t)}{dt} + B_3\left[\frac{d}{dt}(\theta_2) - \frac{d}{dt}\theta_3\right] = 0$$

$$J_2 \cdot \frac{d^2\theta_3(t)}{dt^2} + B_2 \cdot \frac{d}{dt}\theta_3(t) + K_2 \cdot \theta_3(t) + B_3\left\{\frac{d}{dt}\left[\theta_3(t)\right] - \frac{d}{dt}\theta_2(t)\right] = 0$$

Torque – Current Analogy



Torque – voltage



(5)

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+2)(s^2+s+1)+K}$$

The characteristic Equation is

$$s(s+2)(s^2+s+1) + K = 0$$

$$s^4 + 3s^3 + 3s^2 + 2s + K = 0$$

s^4	1	3	K
s^3	3	2	
s^2	7/3	K	
s^1	$\frac{\frac{14}{3} - 3K}{7/3}$	0	
s^0	K		

→ Row of zeros

For the system to be stable

$K > 0$

$$\frac{\frac{14}{3} - 3K}{7/3} > 0$$

$$\frac{14}{3} > 3K$$

$$\frac{14}{9} > K$$

$$K < \frac{14}{9}$$

$K < 1.55$

The range of K is $0 < K < 1.55$

for system to be stable.

For $K = 1.55$ s^1 row becomes row of zeros.

Marginal value of $K = 1.55$

Auxiliary Equation is $\frac{7}{3}s^2 + K_{mar} = 0$

$$\frac{7}{3}s^2 + 1.55 = 0$$

$$s^2 = -1.55 \times \frac{3}{7}$$

$s = \pm j\,0.8163$.

Freq of oscillations $\boxed{\omega = 0.8163 \text{ rad/sec}}$

(4) $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+20)}$

open loop poles are

$P = 4$ $s=0,\ s=-4\ \ s=-2+j4\ \ s=-2-j4$

open loop zeros are $z=0$

$P>z$ Number of root locus branches $N=P=4$.

All root locus branches terminate at ∞.

Centroid $\sigma = -2$

Angles of asymptotes $\phi_1 = 45^0\ \ \phi_2 = 135^0\ \ \phi_3 = 225^0\ \ \phi_4 = 315^0$

Break away point

$s=-2 \quad s = -2 \pm j2.45$

as $K\big|_{s=-2} = 64$

$K\big|_{s=-2\pm j2.45} = 100$

all 3 are valid break away points.

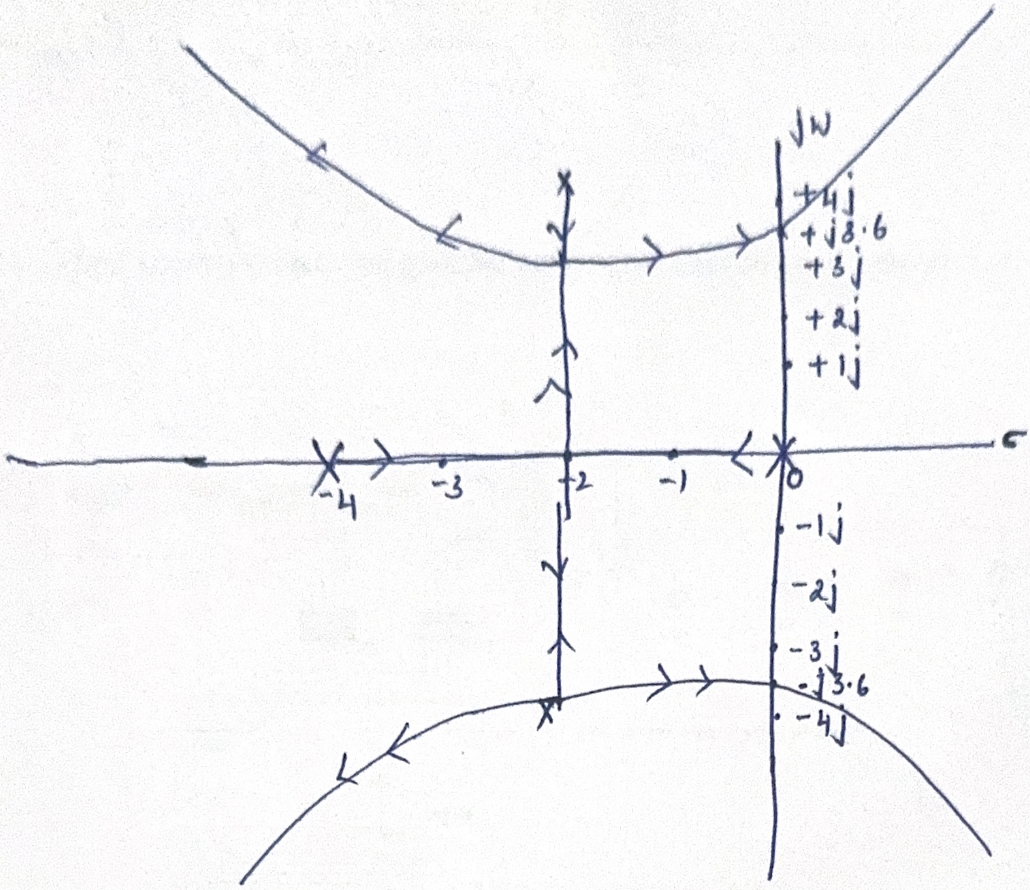
Intersection of root locus with imaginary axis

$\omega = \pm\sqrt{10}$ rad/sec.

Angle of departure

ϕ_d at $-2+j4 = -90^0$

ϕ_d at $-2-j4 = 90^0$



(5)

Starting slope — $-20db \longrightarrow \frac{k}{s}$

Change of slope at w $-20db \longrightarrow \frac{1}{1+sq.} \longrightarrow \frac{1}{1+s\cdot\left(\frac{1}{w}\right)}$

$q = \frac{1}{w}$

change of slope at $w=5 \to +20db \longrightarrow \left(1+s\cdot\frac{1}{5}\right)$

$= 1+0.2s$

change of slope at $w=16 \to -20 \to \frac{1}{1+s\cdot\left(\frac{1}{16}\right)}$

The Transfer function is

$$G(s)H(s) = \frac{k\cdot(1+0.2s)}{s\left(1+\frac{s}{w}\right)\left(1+\frac{s}{16}\right)}$$

(6) $G(s)H(s) = \frac{K}{s(s+1)(0.1s+1)}$

It is in standard time Constant form.

$G(j\omega)H(j\omega) = \frac{K}{j\omega(1+j\omega)(0.1j\omega+1)}$ Assume $K=1$.

Term	Corner freq	slope(db/dec)	change of slope
1	—	0	
$\frac{1}{\omega}$	—	-20	-20
$\frac{1}{1+j\omega}$	$\omega_{c_1} = 1$	-20	-40
$\frac{1}{1+0.1j\omega}$	$\omega_{c_2} = 10$	-20	-60

$\omega_l = 0.5$ $\omega_h = 50$

Mag in db for the factor $\left(\frac{1}{\omega}\right)$

$= 20\log\left(\frac{1}{\omega}\right)$

at $\omega = \omega_l = 0.5$ Mag = 6.02 db

at $\omega = \omega_{c_1} = 1$ Mag = 0 db

at $\omega_{c_2} = 10$: $-40\log(10) + 0$

$= -40$ db

at $\omega_h = 50$: $-60\log\left(\frac{50}{10}\right) - 40$

$= -81.93$ db.

To find w

Consider the st. line Equation $y = mx + c$.

$$\text{Mag in dB} = \text{slope}(\log w) + c$$

at $w = 5$

$$0 = -40 \log 5 + c$$

$$c = 40 \log 5$$

$$c = 27.9$$

at $w = w$

$$40 = -40 \log w + 27.9$$

$$-40 \log w = 12.1$$

$$\log w = -0.3025$$

$$w = 0.4983$$

$$\underline{w = 0.5}$$

so the TF is

$$G(s)H(s) = \frac{K(1+0.2s)}{s(1+2s)(1+0.0625s)}$$

To find K.

Equate $20 \log K =$ Mag at $w = 1$.

$$\text{Mag in db} = -40(\log w) + c$$

$$= -40 \log(1) + 27.9$$

$$= 27.9$$

Mag at $w=1$ = 27.9 db.

$$20 \log K = 27.9$$

$$\log K = \frac{27.9}{20} = 1.395$$

$$\boxed{K = 24.8}$$

Resultant phase angle $\angle G(j\omega)H(j\omega) = -90 - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega)$

ω	$\angle G(j\omega)H(j\omega)$
0.1	-96.28
0.2	-102.45
0.5	-119.43
1	-140.7
2	-164.74
5	-195.25
10	-219.28
50	-257.54
100	-263.716

if we draw the bode plot

for $K=1$ GM = 20 db.

for given gain margin 10 db the magnitude plot has to be shifted upwards by 10dB.

$$20 \log K = 10$$

$$\boxed{K = 3.162}$$

(7) $A = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

as it is a non homogeneous equation
∴ solution is

$$x(t) = L^{-1}[sI-A]^{-1} x(0) + L^{-1}[sI-A]^{-1} B \cdot U(s).$$

$L^{-1}\{(sI-A)^{-1}\} = \phi(t)$ = state transition matrix

$$\phi(t) = \begin{bmatrix} \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t} & \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t} \\ -\frac{4}{3}e^{-t} + \frac{4}{3}e^{-4t} & -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t} \end{bmatrix}$$

$$L^{-1}[sI-A]^{-1}B \cdot U(s).$$

$$U(s) = \frac{1}{s}.$$

The solution is

$$L^{-1}[sI-A]^{-1}x(0) + L^{-1}[sI-A]^{-1}B \cdot U(s)$$

$$\begin{bmatrix} \frac{4}{3}e^{-t}+\frac{1}{3}e^{-4t} & \frac{1}{3}e^{-t}+\frac{1}{3}e^{-4t} \\ -\frac{4}{3}e^{-t}+\frac{4}{3}e^{-4t} & -\frac{1}{3}e^{-t}+\frac{4}{3}e^{-4t} \end{bmatrix}_{2\times2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times1} + L^{-1}\begin{bmatrix} \frac{s+5}{(s+1)(s+4)} & \frac{1}{(s+1)(s+4)} \\ \frac{-4}{(s+1)(s+4)} & \frac{s}{(s+1)(s+4)} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\cdot\frac{1}{s}$$

$$= \begin{bmatrix} \frac{5}{3}e^{-t} \\ -\frac{8}{3}e^{-t}+\frac{8}{3}e^{-4t} \end{bmatrix} + L^{-1}\begin{bmatrix} \frac{1}{s(s+1)(s+4)} \\ \frac{1}{(s+1)(s+4)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{3}e^{-t} \\ -\frac{8}{3}e^{-t}+\frac{8}{3}e^{-4t} \end{bmatrix} + L^{-1}\begin{bmatrix} \frac{\frac{1}{4}}{s} - \frac{\frac{1}{3}}{s+1} + \frac{\frac{1}{12}}{s+4} \\ \frac{\frac{1}{3}}{s+1} - \frac{\frac{1}{3}}{s+4} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{5}{3}e^{-t} \\ -\frac{8}{3}e^{-t}+\frac{8}{3}e^{-4t} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} - \frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t} \\ \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} \frac{1}{4} - \frac{4}{3}e^{-t} + \frac{1}{12}e^{-4t} \\ -\frac{7}{3}e^{-t} + \frac{7}{3}e^{-4t} \end{bmatrix}$$