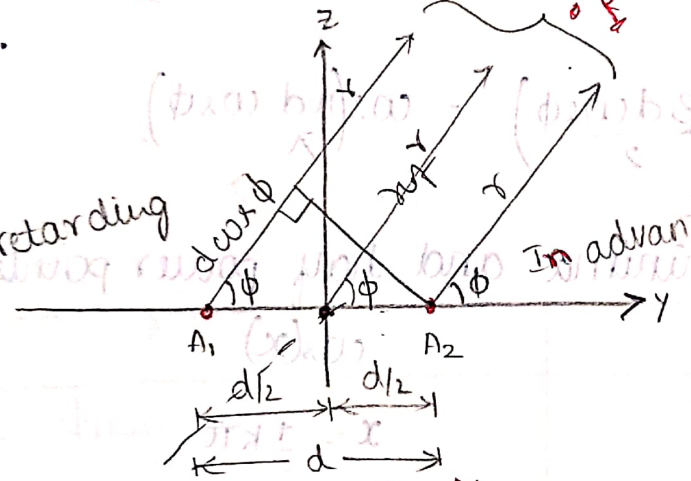
4. Parasitic array

Q1A

Array of point sources — Case-1

The two point sources with current equal in magnitude and phase. (Example for broadside array)



If we change ref. A_1 to origin

$\boxed{E_T = 2E_0 \cos\psi/2 \cdot e^{j\psi/2}}$ — phase change

Consider the two point source A_1 and A_2 separated by a distance d.

Path difference between two sources $= d\cos\phi$

In terms of wavelength, path difference $= \frac{d\cos\phi}{\lambda}$

Phase angle between the two ~~array phase~~ fields at point P is (Point Source)

phase angle $\psi = 2\pi$ (Path difference) — Venturing a phase

By option

$$= \frac{2\pi}{\lambda} \cdot d\cos\phi$$

$$\boxed{\psi = \beta \cdot d\cos\phi}$$

$E_1 = E_0 e^{-j\psi/2}$

E_1 is the far field at P due to point source A_1

$E_2 = E_0 e^{+j\psi/2}$

E_2 is the far field at P due to point source A_2

The phase of the wave at a distant point is measured wr. the centre. The total field is

$$E_T = E_1 + E_2$$

$$= E_0\left(e^{-j\psi/2} + e^{+j\psi/2}\right)$$

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$$= 2E_0\cos\left(\frac{\psi}{2}\right) ✓$$

$$E_T = 2E_0\cos\left(\frac{\beta d\cos\phi}{2}\right)$$

26/02/2013

Array factor

It is the ratio of magnitude of resultant field to the magnitude of maximum field.

Array factor $A.F = \frac{|E_T|}{|E_{max}|}$

$E_T = 2E_0\cos\left(\frac{\psi}{2}\right)$

where, $\boxed{E_T = 2E_0\cos\left(\frac{\beta d\cos\phi}{2}\right)}$ $\boxed{E_T = 2E_0\cos\left(\frac{\beta d\cos\phi}{2}\right)}$

$\boxed{E_{max} = 2E_0}$ $\beta = \frac{2\pi}{\lambda}\ \frac{d}{2}$

$$\Rightarrow A.F = \cos\left(\frac{\beta d\cos\phi}{2}\right) = \cos\left(\frac{\pi d}{\lambda}\cos\phi\right)$$ x

To find the maxima, (Null) minima and half power points in the array

	$\sin(x)$	$\cos(x)$
Maxima	$x = \pm(2k+1)\frac{\pi}{2}$	$x = \pm k\pi$
Minima (Null)	$x = \pm k\pi$	$x = \pm(2k+1)\frac{\pi}{2}$
HPBW	$x = \pm(2k+1)\frac{\pi}{4}$	$x = \pm(2k+1)\frac{\pi}{4}$

Q 1B

(i) For a maxima

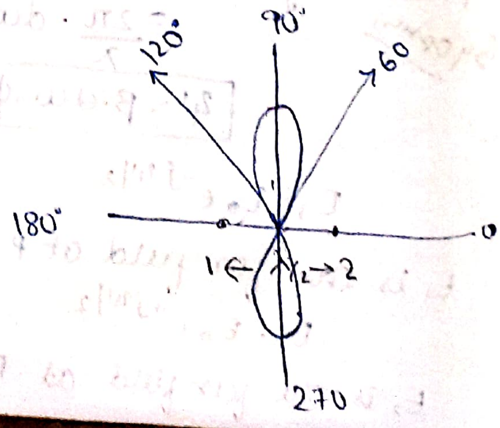
$$\frac{\pi}{\lambda}d\cos\phi = \pm k\pi$$

$$\cancel{E_T = 2E_0\cos\left(\frac{\beta d\cos\phi}{2}\right)}$$

$$\beta d = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}$$

$$\beta d = \pi$$

$$\frac{\pi}{\lambda}d\cos\phi = \pm k\pi$$



$$\frac{\cancel{\pi}}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{2} \cos\phi_{max} = \pm k\cancel{\pi}$$

$k = 0 \ldots$

$\phi_{max} = \cos^{-1}(\pm 1) \quad \cos\phi = 1$

$\cos\phi = 0$ for $k=0$

$\phi_{max} = \cos^{-1}(0)$

HPBW $= \pi/2$ or 270°

(ii) For ~~maxima~~

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1/\sqrt{2}$$

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1/\sqrt{2}$$

Put $d = \lambda/2$, $\beta = 2\pi/\lambda$

$$\frac{\pi}{2}\cos\phi = \pm(2k+1)\frac{\pi}{4}$$

put $k = 0$

$$\pi/2 \cos\phi = \pm\frac{\pi}{4}$$

~~$\phi_{min} = \cos^{-1} \pi/4$~~

~~$= 0^\circ / 180^\circ$~~

~~HPBW~~ $\phi_{HPBW} = 60^\circ$ or 120° → This is an example of broad side array.

(iii) For minima (null)

$$\frac{\pi}{2}\cos\phi_{min} = \pm(2k+1)\frac{\pi}{2}$$

put $k=0$

$$\frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$$

$$\phi_{min} = \cos^{-1}(\pm 1)$$

$$\phi_{min} = 0^\circ \text{ or } 180^\circ$$

assuming $d = \lambda/2$

let the spacing b/n two point source be $\lambda/2 = d$

$\cos\phi_{max} = 0$

$\phi_{max} = \cos^{-1}(0)$

$= 90^\circ$

All s.t.c.

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$V_m = V_s\sqrt{2}$

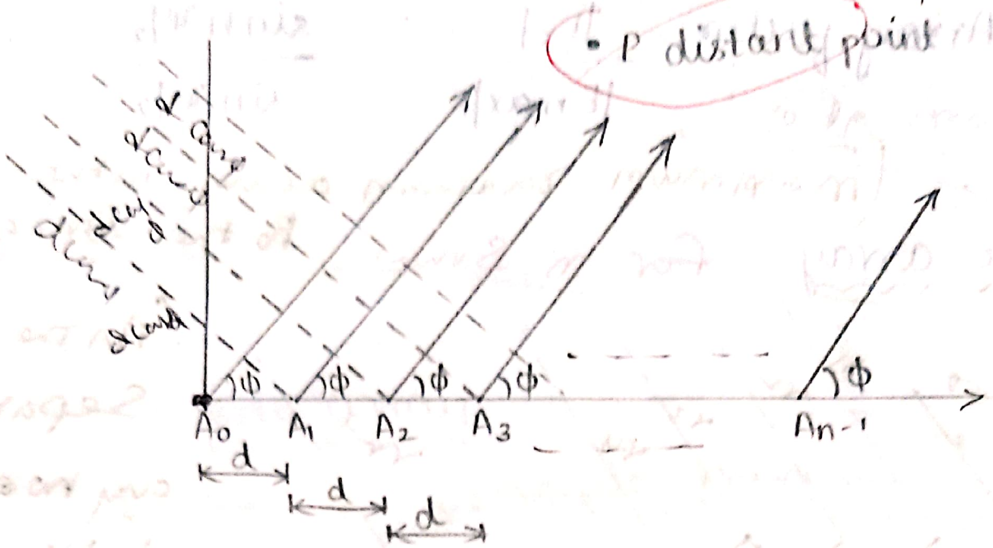
$\phi_{HPBW} = 60^\circ$ or 120°

or 180°. (If $\alpha = 0$ we get n element broad side array. If $\alpha = 180^\circ$, we get uniform end fire array)

Linear array of n isotropic point source derive expression for

n-element uniform linear array

Q2



w.r.t. A_0

$$E_T = E_0 + E_0 e^{j\psi} + E_0 e^{2j\psi} + \dots\dots + E_0 e^{j(n-1)\psi}$$

$$E_T = E_0\left[1 + e^{j\psi} + e^{2j\psi} + \dots\dots + e^{j(n-1)\psi}\right] \quad \text{------(1)}$$

Multiply by $e^{j\psi}$

$$E_T e^{j\psi} = E_0\left[e^{j\psi} + e^{2j\psi} + \dots\dots + e^{jn\psi}\right] \quad \text{------(2)}$$

Subtract (2) from (1)

$$E_T - E_T e^{j\psi} = E_0\left\{\left[1 + e^{j\psi} + e^{2j\psi} + \dots + e^{j(n-1)\psi}\right] - \left[e^{j\psi} + e^{2j\psi} + \dots + e^{j(n-1)\psi} + e^{jn\psi}\right]\right\}$$

$$E_T(1 - e^{j\psi}) = E_0(1 - e^{jn\psi})$$

$$E_T = \frac{E_0(1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

$$= E_0\left[\frac{e^{jn\psi/2}\left(e^{-jn\psi/2} - e^{jn\psi/2}\right)}{e^{j\psi/2}\left(e^{-j\psi/2} - e^{j\psi/2}\right)}\right]$$

wkt $e^{j\psi'} - e^{-j\psi'} = -2j\sin\psi'$

$$E_T = E_0\left[\frac{\left(-2j\sin n\psi'/2\right)e^{nj\psi'/2}}{\left(-2j\sin\psi'/2\right)e^{j\psi'/2}}\right]$$

$$E_T = E_0\,\frac{\sin\left(\frac{n\psi'}{2}\right)}{\sin\left(\frac{\psi'}{2}\right)}\,e^{j\psi'\left(\frac{n-1}{2}\right)}$$

$$\cancel{\therefore E_T = E_0\left[\frac{\sin n\frac{\psi'}{2}}{\sin\frac{\psi'}{2}}\right]}$$

Phase angle of resultant field at point P

$$\theta = \left(\frac{n-1}{2}\right)\psi' = \left(\frac{n-1}{2}\right)(\beta d\cos\phi + \alpha)$$

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$\psi =$

If reference is shifted to center of array ∴ phase angle $\left(\frac{n-1}{2}\right)\psi = 0$ is automatically eliminated from and Eqn.

reduce to

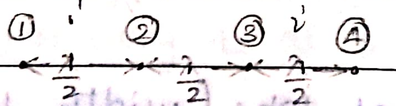
Pattern multiplication

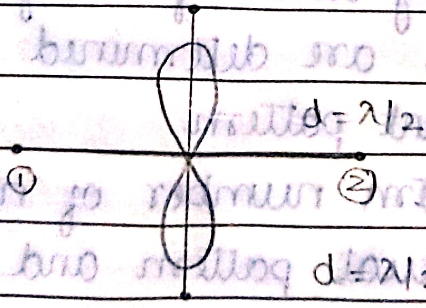
Q2 A simple method to sketch the complicated pattern of array just by inspection. This is a useful tool for designing an array. The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source pattern and the pattern of an array of isotropic point sources each located at the phase center of individual source and having related amplitude and phase whereas the total phase pattern is the addition of phase pattern of the individual sources and that of array of isotropic point sources.

$$E = \underbrace{\{E_i(\theta,\phi) \times E_a(\theta,\phi)\}}_{\text{product of field pattern}} \Big| \underbrace{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)}_{\text{Sum of phase pattern}}$$

Ex:

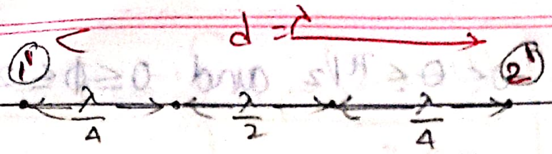
Radiation pattern of 4-isotropic element fed in phase spaced λ/2 apart.





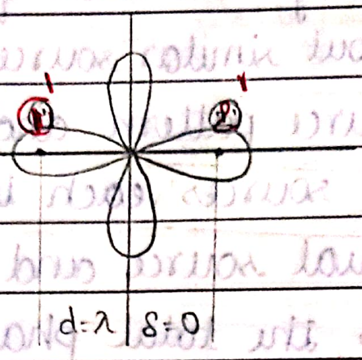
Field pattern of 2 isotropic point sources spaced λ/2 apart

Individual array



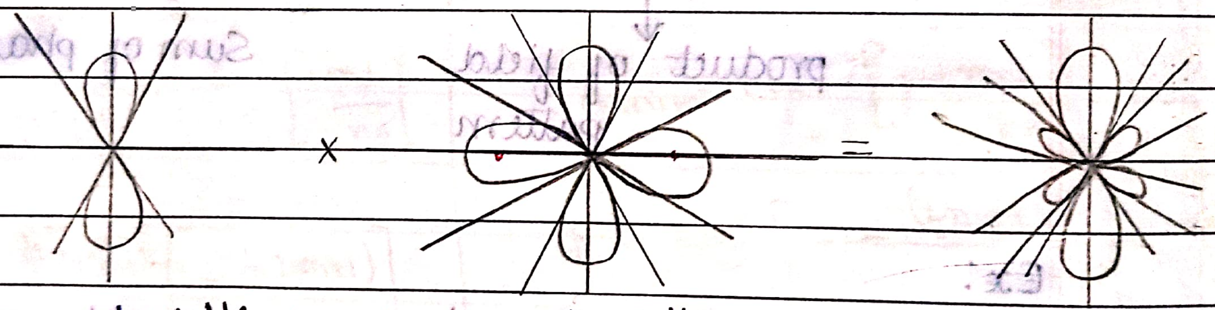
2 units array where one unit spaced λ

Pattern of 2 isotropic point sources with spacing λ



Group array.

Resultant pattern of 4-isotropic elements:



unit pattern Group pattern Resultant pattern of 4 isotropic elements

The width of the principal lobe (width between the nulls) and the corresponding width of array pattern are same. The secondary lobes are determined from the number of nulls in the resultant pattern.

In the resultant pattern number of nulls are the sum of the nulls of individual pattern and array pattern.

(3) Show that the directivity of the source with unidirectional radiation pattern given by $U = U_m \cos^n\theta$ can be represented as $D = 2(n+1)$

$$P_T = \int_0^{\pi}\int_0^{2\pi} U \, d\Omega$$

$$= \int_0^{\pi/2}\int_0^{2\pi} U_m \cos^n\theta \sin\theta \, d\phi \, d\theta \qquad [\because \text{unidirectional pattern}]$$

$$= U_m \int_0^{\pi/2} \cos^n(\theta) \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= 2\pi U_m \int_0^{\pi/2} (\sin\theta \cos\theta) \cos^{n-1}\theta \, d\theta$$

$$= 2\pi U_m \int_0^{\pi/2} \frac{\sin 2\theta}{2} \cos^{n-1}\theta \, d\theta$$

$$= U_m \pi \int_0^{\pi/2} \sin 2\theta \cos^{n-1}\theta \, d\theta$$

$$= 2U_m \pi \int_0^{\pi/2} \sin\theta \cos^n\theta \, d\theta$$

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Let $\cos\theta = x$

$-\sin\theta \, d\theta = dx$

when $\theta = 0$, $x = 1$

$\theta = \pi/2$, $x = 0$

$$P_T = 2\pi U_m \int_1^0 x^n (-dx)$$

$$= 2\pi U_m \left[-\frac{x^{n+1}}{n+1} \right]_1^0$$

$$= \frac{2\pi U_m}{n+1} (1^{n+1})$$

$$P_T = \frac{2\pi U_m}{n+1} \quad \text{— (1)}$$

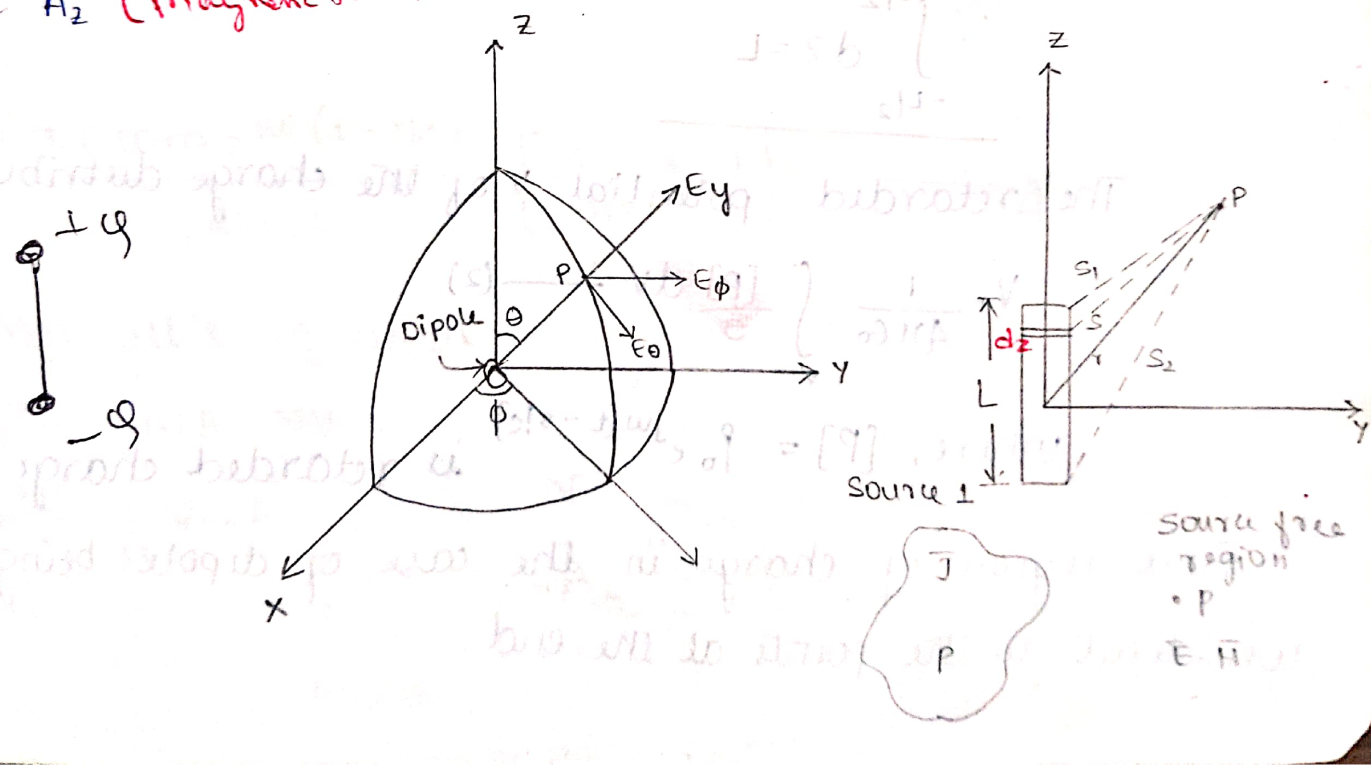
$$\frac{2\pi U_m}{n+1} = 4\pi U_0$$

$$\text{Directivity} = \frac{U_m}{U_0} = \frac{4\pi(n+1)}{2\pi}$$

$$\boxed{D = 2(n+1)}$$

4 (a) Derive the expression for the field component of a short dipole starting with expression of scalar electric potential and vector magnetic potential. Also determine far field component.

A short linear conductor is often called a short dipole. The length L is very short ($L << \lambda$) compared to wavelength. The current I along the entire length is assumed to be uniform. Consider a dipole of length L placed along the z-axis with its centre at the origin. The electric and magnetic field due to dipole can be expressed in terms of vector and scalar potential. Since, we are interested in finding the far field we must use retarded potential. i.e., expressions involving $[t - r/c]$. For a short dipole shown in the figure, the retarded vector potential of the electric current has only one component namely $\vec{A_z}$ (Magnetic field)

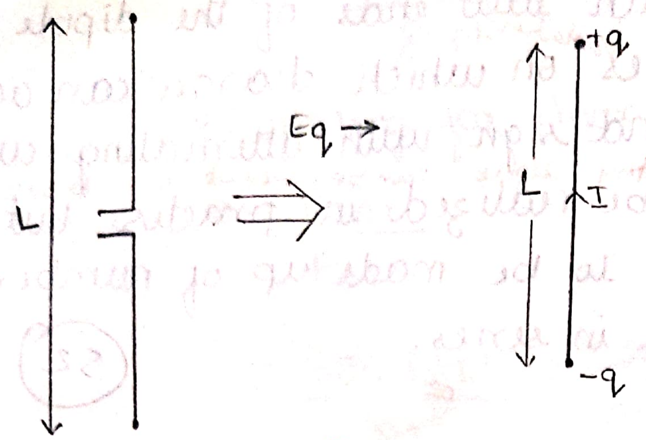


Short dipole

$L << \lambda$

$L << \frac{\lambda}{10} = 0.1\lambda$

(3)



magnetic po

The vector potential A_z

$t - r/c$, $A_z = \frac{\mu_0}{4\pi} \int_{-L/2}^{L/2} \frac{[I]}{S} dz$

$t - r/(r/t)$, where, $[I] \rightarrow$ retarded current delay

$[I] = I_0 e^{j\omega(t - r/c)}$ C, vel. r/r

Since $r >> L$; $\lambda >> L$ $\therefore S \simeq r$

$A_z = \frac{\mu_0}{4\pi} I_0 \frac{e^{j\omega[t - (r/c)]}}{r} \int_{-L/2}^{L/2} dz$

I $\boxed{A_z = \frac{\mu_0 I L e^{j\omega[t - r/c]}}{4\pi r}}$ ——— (1)

$\because \int_{-L/2}^{L/2} dz = L$

The retarded potential V of the charge distribution

$V = \frac{1}{4\pi\epsilon_0} \int_v \frac{[\rho]}{S} dv$ ——— (2)

where, $[\rho] = \rho_0 e^{j\omega[t - s/c]}$ is retarded charge density

Since region of charge in the case of dipole being considered to the points at the end.

Equation (2) reduces to

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{|q|}{S_1} - \frac{|q|}{S_2}\right] \quad — ③$$

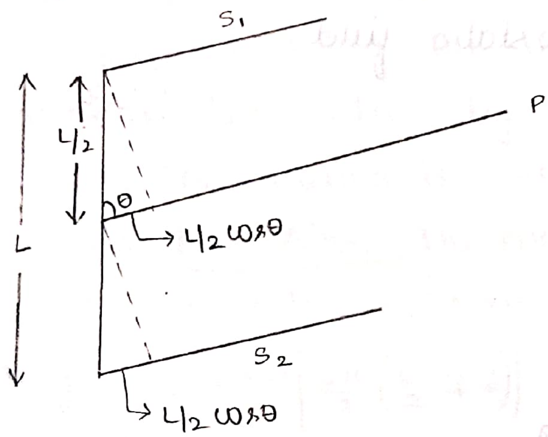
But we have

$$[q] = \int [I]\, dt \qquad 54$$

$$= I_0 \int e^{j\omega[t - s/c]}\, dt$$

$$= \frac{[I]}{j\omega}$$

$$V = \frac{I_0}{4\pi\epsilon_0(j\omega)}\left[\frac{e^{j\omega(t - S_1/c)}}{S_1} - \frac{e^{j\omega(t - S_2/c)}}{S_2}\right]$$



From the figure,

$$S_1 = r - \frac{L}{2}\cos\theta$$

$$S_2 = r + \frac{L}{2}\cos\theta$$

II

$$V = \frac{I_0 L\cos\theta\, e^{j\omega(t - r/c)}}{4\pi\epsilon_0 c}\left[\frac{c}{j\omega r^2} + \frac{1}{r}\right] \quad — ④$$

Maxwell's equation

$$\vec{E} = -j\omega\vec{A} - \nabla V$$

$$\vec{H} = \frac{1}{\mu}\nabla \times \vec{A}$$

V

A_L

$$E_r = \frac{I_0 L \cos\theta \, e^{jw(t-r/c)}}{2\pi\epsilon_0} \left[\frac{1}{c r^2} + \frac{1}{jw r^3}\right]$$

I_0 = r.m.s current of short dipole
L = length of antenna
~~I current~~
r - distance Tx Rx
c = velocity

$$E_\phi = 0$$

$$E_\theta = \frac{I_0 L \sin\theta}{4\pi\epsilon_0} e^{jw(t-r/c)} \left[\frac{jw}{c^2 r} + \frac{1}{c r^2} + \frac{1}{jw r^3}\right]$$

$r \uparrow$ $E \downarrow$

$$H_r = 0 \quad H_\phi = \frac{I_0 L \sin\theta}{4\pi} e^{jw(t-r/c)} \left[\frac{1}{r^2} + \frac{jw}{c r}\right]$$

$$H_\theta = 0$$

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Imp term

1. $\frac{1}{r} \propto w \rightarrow$ R.F [Radiation field]

distance $\uparrow$ field $\downarrow$ should be decreased linearly

2. Independent of w or $f \rightarrow$ induction field [I.F]

3. $\frac{1}{r^3} \propto \frac{1}{w} \rightarrow$ Electrostatic field

$$\frac{w}{\cancel{c} r} = \frac{1}{\cancel{c} r^2}$$

$$\frac{2\pi \cancel{f}}{\lambda \cancel{f}} = \frac{1}{r}$$

$$r = \frac{\lambda}{2\pi} \approx \frac{\lambda}{6}$$

(a) $r \lll \lambda/6$ N.F.R [Near field region]

(b) $r \ggg \lambda/6$ F.F.R [Far field region]

For far field region, neglect $1/r^2$ and $1/r^3$ components

$$E_\theta = \frac{jw I_0 L \sin\theta}{4\pi\epsilon_0 c^2 r} e^{jw(t-r/c)}$$

r.m.s value

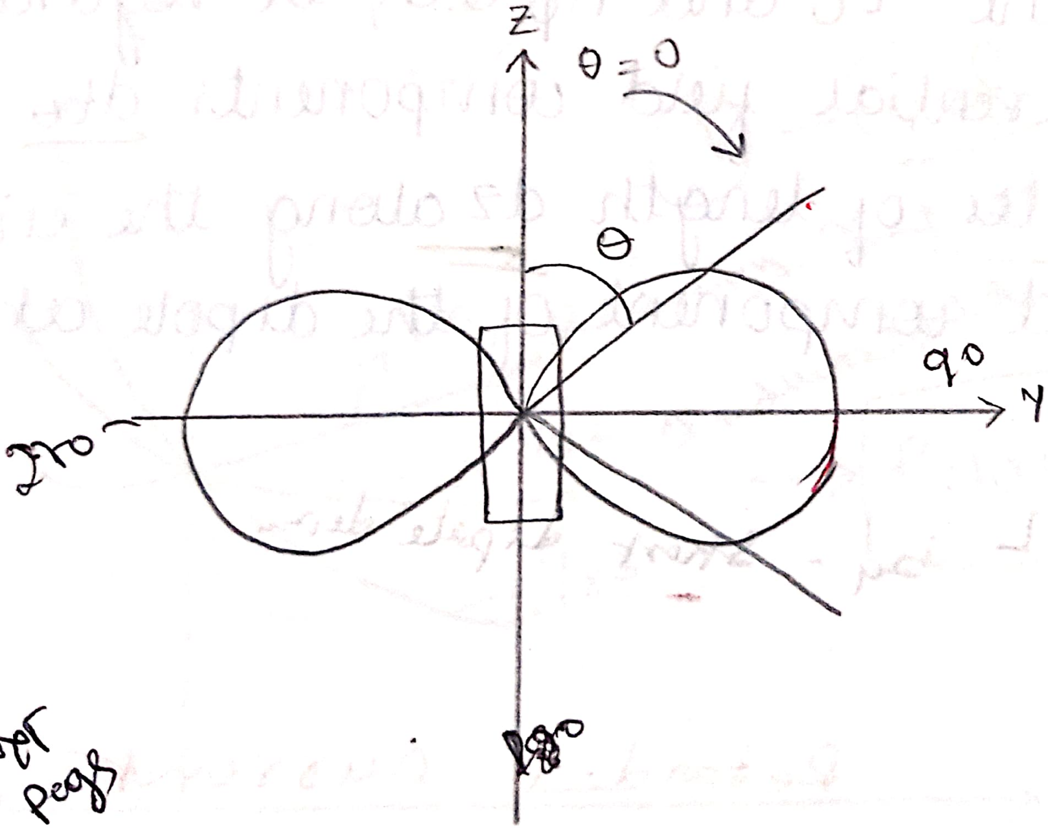
$$H_\phi = \frac{jw I_0 L \sin\theta}{4\pi c r} e^{jw(t-r/c)}$$

Curl Eqn

$$H_r = 0, \; H_\theta = 0$$

E_θ and $H_\phi \propto \sin\theta$

From the above equation, E_θ and H_ϕ are in the same time ~~space~~ phase in the far field with the field pattern of both proportional to sin θ. The pattern is independent of



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Parallel

No - X axis

See next page

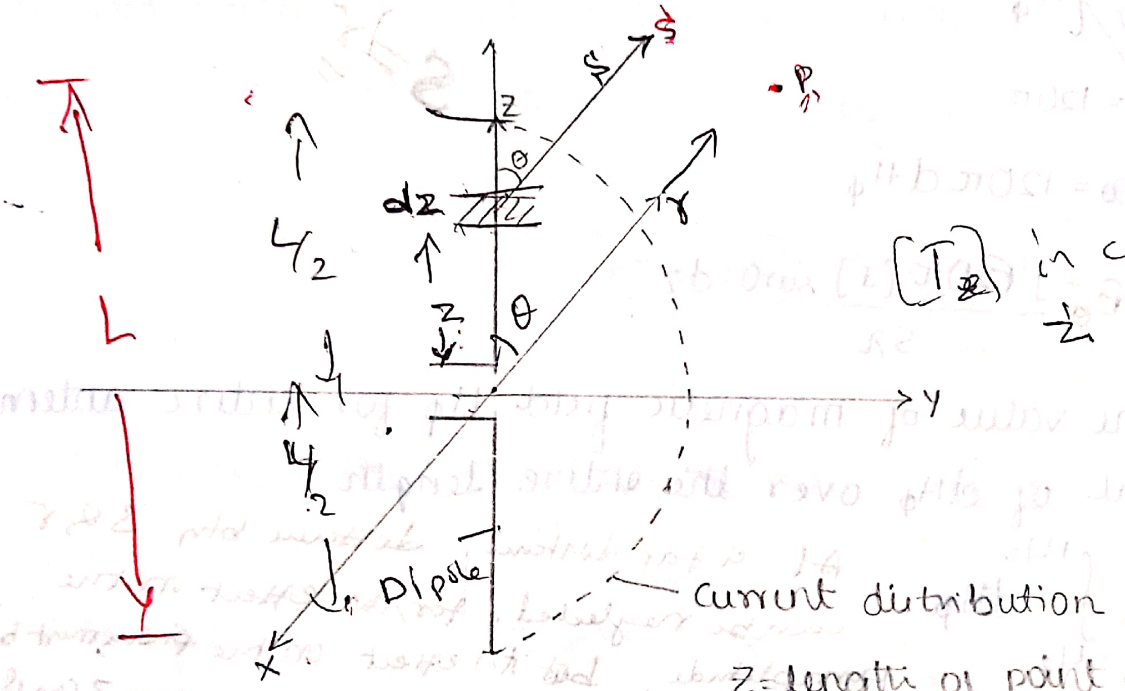
Short dipole

4(b)

Field due to thin linear antenna

$d \ll \frac{\lambda}{100}$

(4) For a thin linear antenna, a sinusoidal current distribution is assumed along the length of the antenna. The antenna is fed at the centre by a balanced two wire transmission line. The antenna is thin i.e., when the conductor diameter is less than $\lambda/100$. The magnitude of the current at any point on the antenna could be expressed as mathematically $I = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} \mp z\right)\right]$



$[I_z]$ is current at any point z of the antenna

z = length of point along the dipole

$$[I]_z = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} \mp z\right)\right] \cdot e^{j\omega[t - s/c]}$$

In this expression negative sign is used when z is positive and positive sign is used when z is negative. The value of the current at any point z on the antenna refer to a point at a distance S is as given above.

The far field component E_θ and H_ϕ can be regarded as the total effect of differential field components dE_θ and dH_ϕ created by different dipoles of length dz along the entire length of the antenna. The field component of the dipole at a distance can be written as :

$$H_\phi = \frac{j\omega\,[I]\sin\theta\cdot L}{4\pi c\,S}$$ by - short dipole derin.

$[I] = I_0\, e^{j\omega(t - S/c)}$ — Retarded current

time difference S/c is the interval required for the disturbance to travel distance S in free space at the velocity of light $c = 3\times10^8$ m/s

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$$H_\phi = \frac{j\,[I]\sin\theta\cdot L}{S\cdot 4\pi\, c/\omega}$$

$$= \frac{j\,[I]\sin\theta\cdot L}{S\cdot 4\pi\cdot \frac{c}{2\pi f}}$$

$$H(\phi) = \frac{j\,[I]\sin\theta\cdot L}{2\cdot S\cdot\lambda}$$

$$\therefore dH_\phi = \frac{j[I]}{2\cdot S\lambda}\sin\theta\, dz$$

$\lambda = \frac{2\pi}{\beta}$

$$E_\theta = \eta H_\phi$$

$$\eta = 120\pi$$

$S \approx r$

$$dE_\theta = 120\pi\, dH_\phi$$

$$dE_\theta = \frac{j\,60\pi\,[I]\sin\theta\cdot dz}{S\lambda}$$

The value of magnetic field H_ϕ for entire antenna is the integral of dH_ϕ over the entire length

$$H_\phi = \int_{-L/2}^{+L/2} dH_\phi$$

At a far distance difference b/w S & r can be neglected. for its effect on the amplitude, but its effect on the phase must be included; and in the phase part $S = r - z\cos\theta$

putting value of $[I]$ (Integrating)

$$H(\phi) = \frac{j[I]}{2\pi r}\left[\frac{\cos\left(\frac{\beta L\cos\theta}{2}\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right]$$

$$E_\theta = \eta H_\phi = 120\pi H_\phi$$

$$E_\theta = \frac{j60[I]}{r}\left[\frac{\cos\left(\frac{\beta L\cos\theta}{2}\right) - \cos\left(\frac{\beta L}{2}\right)}{\sin\theta}\right]$$

$\frac{2\pi}{\lambda}\times\frac{\lambda/2}{2}$ +1

$\cos(\pi/2)$

$= 90°$

Q5

Radiation resistance of short dipoles

A short dipole — a short linear conductor is often called a short dipole. Length l is very short compared to the wavelength. Current I along the entire length is assumed to be uniform.

$$S_r = \vec{E} \times \vec{H}$$

$$S_r = \frac{E^2}{\eta} \ W/m^2$$

Radial compont. of poynting vector. depends on θ, & ϕ

Total power radiated by short dipole $\quad P$ or $W = \int S_r \cdot ds \quad W$

Substituting the value of S_r and ds

$$P \text{ or } W = \int S_r \cdot r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int \frac{E^2}{\eta} r^2 \sin\theta \, d\theta \, d\phi$$

$d\Omega = \sin\theta \, d\theta \, d\phi$

For a short dipole, field at a distance r is.

$$W = \frac{E^2}{\eta} = \frac{30\pi I_0^2 L^2 \sin^2\theta}{\lambda^2 r^2}$$

Substituting

$$W = \int \frac{30\pi I_0^2 L^2 \sin^2\theta}{\lambda^2 \cancel{r^2}} \cdot \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$= \frac{30\pi I_0^2 L^2}{\lambda^2} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta \, d\phi$$

A 9

$$= \frac{30\pi I_0^2 L^2 (2\pi)}{\lambda^2} \left[\cancel{\frac{-\sin^2\theta \cos\theta}{3}}^{\,0} - \frac{2}{3}\cos\theta \right]_0^{\pi}$$

$$= \frac{\cancel{30}^{10}\pi I_0^2 L^2 (2\pi)}{\lambda^2} (4/\cancel{3})$$

$$W = \frac{80\pi^2 I_0^2 L^2}{\lambda^2} \quad \text{—(1)}$$

$$W = I_0^2 R_r \quad \text{—(2)}$$

I_0 rms current of dipole ✓ Short dipole

R_r - Radiation Resistance of dipole

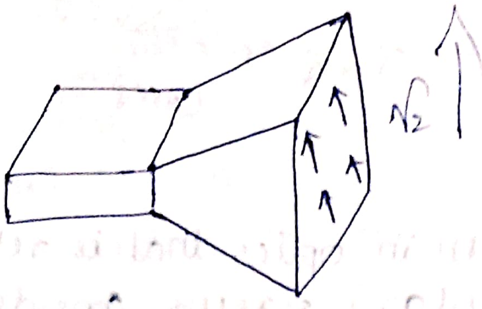
Equation (1) & (2) implies

$$\frac{80\pi^2 \cancel{I_0^2} L^2}{\lambda^2} = \cancel{I_0^2} R_r$$

$$\boxed{R_r = \frac{80\pi^2 L^2}{\lambda^2}} \longrightarrow \text{radiation resistance}$$

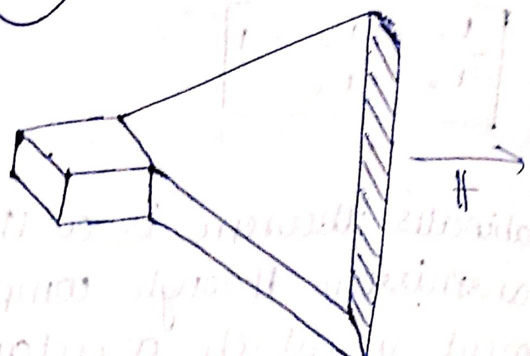
Types - Horn antennas

6(a)



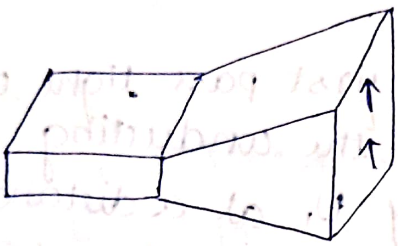
E- plane sectoral plane

Flaring in E-plane



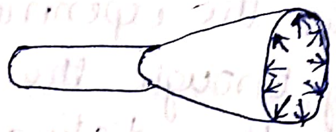
H. plane sectoral plane

Flaring in H- plane



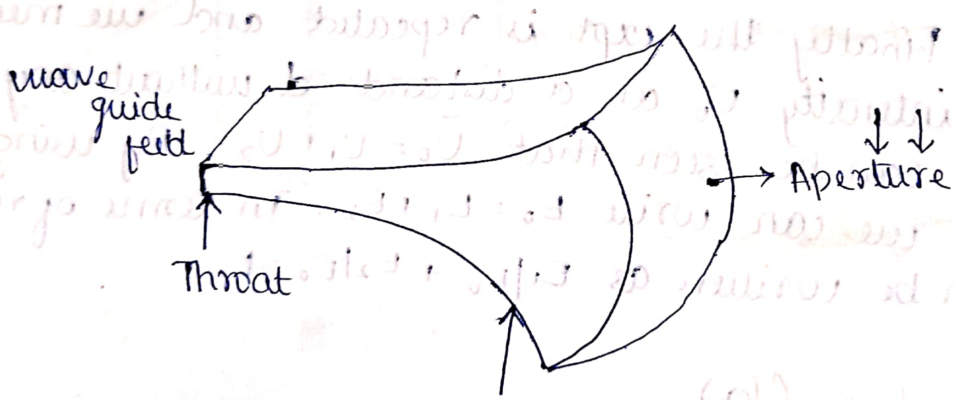
Pyramidal horn

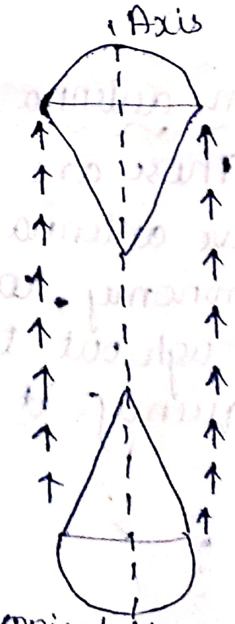
Flaring in both plane (E & H)



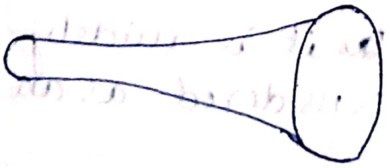
Conical Horn

These are most widely used M wave antenna. These are considered as the aperture antenna.





Biconical Horn



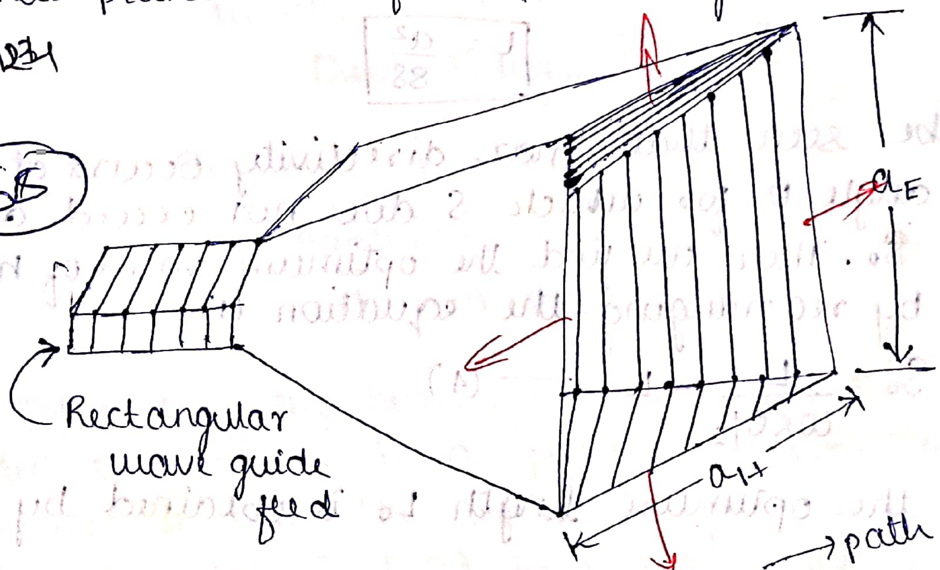
Exponentially tapered conical

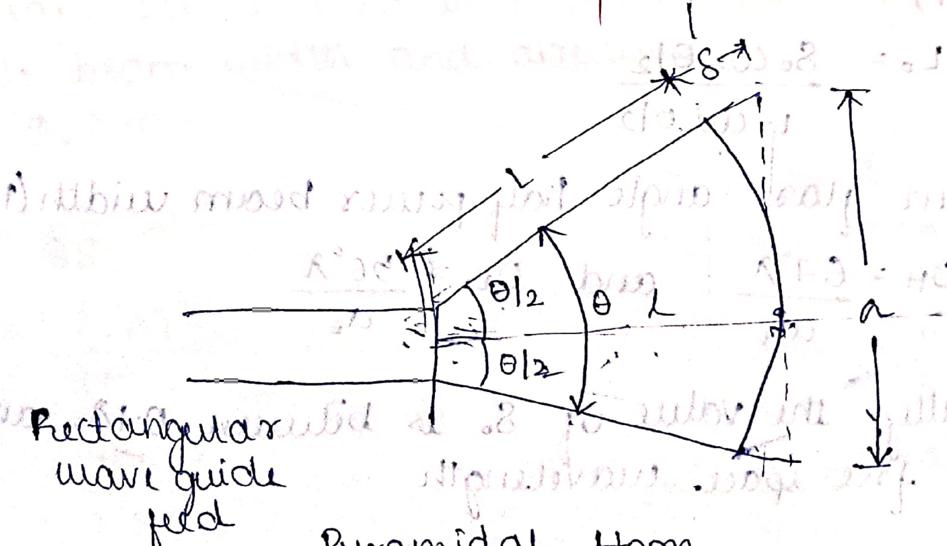
It is possible for a waveguide to radiate energy if one end is suitably excited and the other end is opened out. Only a small portion of energy is radiated. As there is a discontinuity and matching is not proper and most of the energy is reflected back. The impedance mismatch is overcome by flaring the open end of the waveguide. The flared structure is called horned antenna. In addition to matching the impedance ~~and~~ it reduces the SWR to an acceptable level, the flare gives good directivity, narrow bandwidth. Since there are no resonant parts the horn antenna works over a wide frequency ranges.

Fermat's principle or design of Horn antenna

The field over the plane surface of the antenna (mouth of the horn) can be made everywhere in phase by shaping that surface so that all path from the source to that plane are of equal length electrically.

24/04/24





Pyramidal Horn

$$\cos\frac{\theta}{2} = \frac{L}{L+\delta} \quad \text{—— (1)}$$

$$\sin\theta/2 = \frac{a}{2(L+\delta)} \quad \text{--- (2)}$$

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$$\tan\theta/2 = \frac{a/2}{L} = \frac{a}{2L} \quad \text{--- (3)}$$

where, $\theta \rightarrow$ flare angle open end

Flare angle in E plane

$$\theta_E = 2\tan^{-1}\left(\frac{a}{2L}\right)$$

$\theta/2 = \tan^{-1}\frac{a/2}{L}$

$\theta = 2\tan^{-1}\left(\frac{a}{2L}\right)$

Flare angle in H-plane

$$\theta_H = 2\cos^{-1}\left(\frac{L}{L+\delta}\right)$$

From the fig,

$$L^2 + \left(\frac{a}{2}\right)^2 = (L+\delta)^2$$

$$L^2 + \frac{a^2}{4} = L^2 + 2\delta L + \delta^2$$

δ^2 ↳ neglect

$$\frac{a^2}{4} = 2\delta L$$

$$\boxed{L = \frac{a^2}{8\delta}}$$

It can be seen that max. directivity occurs at the largest flare angle θ for which δ does not exceed a certain value δ_0. Then we find the optimum value of horn dimension δ_0 by rearranging the equation (1)....

$$\delta_0 = \frac{L}{\cos\theta/2} - L \quad \text{--- (4)}$$

Similarly the optimum length L_0 is obtained by equation (4)

$$L_0 = \frac{\delta_0 \cos\theta/2}{1-\cos\theta/2}$$

Optimum flare angle half power beam width (HPBW),

$$\theta_H = \frac{67^\circ\lambda}{a_H} \quad \text{and} \quad \theta_E = \frac{56^\circ\lambda}{a_E}$$

Practically the value of δ_0 is between $0.1\lambda_0$ and $0.4\lambda_0$ where λ_0 is free space wavelength

Rectangular Horn antenna

w.k.t.,

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \eta_{ap} A_p}{\lambda^2}$$

$$\eta_{ap} = \frac{A_e}{A_p} = \text{apperture efficiency}$$

A_e → effective apperture

A_p – physical apperture

$\eta = \frac{A_e}{A_p}$

For a rectangular horn antenna

$A_p = a_E \cdot a_H$

$\eta A_p = A_e$

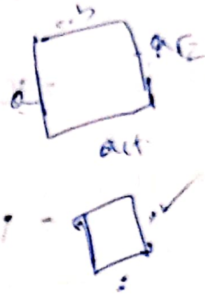
a_E = E plane apperture

a_H = H plane apperture

Assume $a_H = a_E = \lambda$

$\eta_{ap} = 0.6$

$$D = \frac{4\pi(0.6) A_p}{\lambda^2} = \frac{7.5 A_p}{\lambda^2}$$



In dB,

$$D_{dB} = 10 \log_{10}\left[\frac{7.5 A_p}{\lambda^2}\right]$$

Similarly for conical horn

$A_p = \pi r^2$

r = radius of apperture

08 Determine L of the horn, H-plane apperture and flare angles θ_E and θ_H in a pyramidal horn for which E-plane apperture is 10λ. The horn is fed with a rectangular waveguide. Let $\delta = 0.2\lambda$ in E plane and 0.375λ in H-plane. Calculate beam width and directivity.

$a_E = 10\lambda$

$$L = \frac{a_E^2}{8\delta} = \frac{(10\lambda)^2}{8(0.2\lambda)} = \frac{100\lambda^2}{1.6\lambda}$$

$L = 62.5\lambda$

Flare angle in E plane

$$\theta_E = 2\tan^{-1}\left(\frac{a}{2L}\right) = 2\tan^{-1}\left(\frac{10\lambda}{2(62.5\lambda)}\right)$$

$\theta_E = 0.159^{rad} = 9.147°$

Flare angle in H-plane

$$\theta_H = 2\cos^{-1}\left(\frac{L}{L+\delta}\right)$$

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$$= 2\cos^{-1}\left(\frac{62.5\lambda}{62.5\lambda + 0.375\lambda}\right)$$

$$\theta_H = 0.218\,\text{rad} = 12.512^\circ$$

HPBW $\theta_E = \dfrac{56^\circ\lambda}{a_E} = \dfrac{56^\circ\lambda}{10\lambda} = 5.6^\circ$

$$\theta_H = \frac{67\lambda}{13.7\lambda = a_H} = 4.8905^\circ$$

$$D \text{ in db} = 10\log_{10}\left(\frac{7.5A_p}{\lambda^2}\right)$$

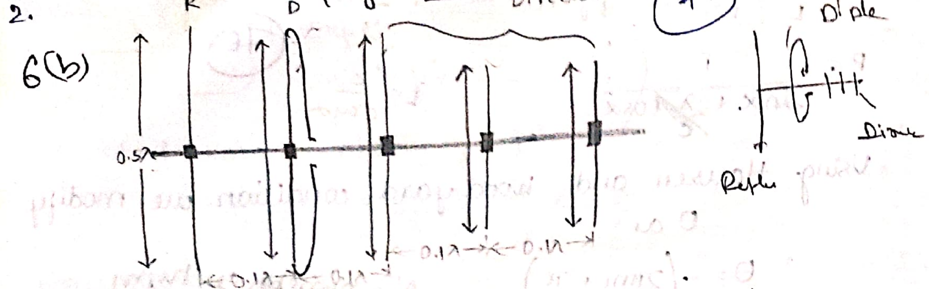
$$= 10\log_{10}\left(\frac{7.5 \times a_E \times a_H}{\lambda^2}\right)$$

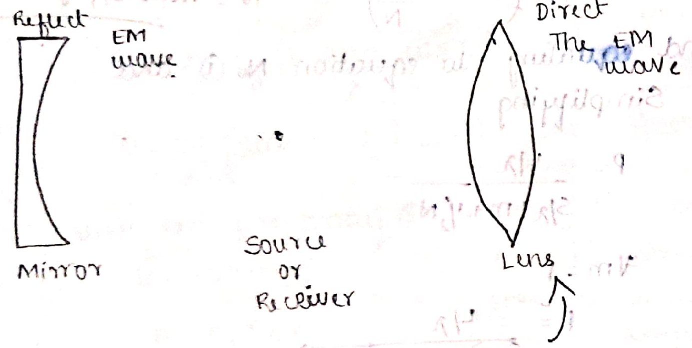
$$= 10\log_{10}\left(\frac{7.5 \times 10\lambda \times 13.7\lambda}{\lambda^2}\right)$$

D in dB = 30.117 dB

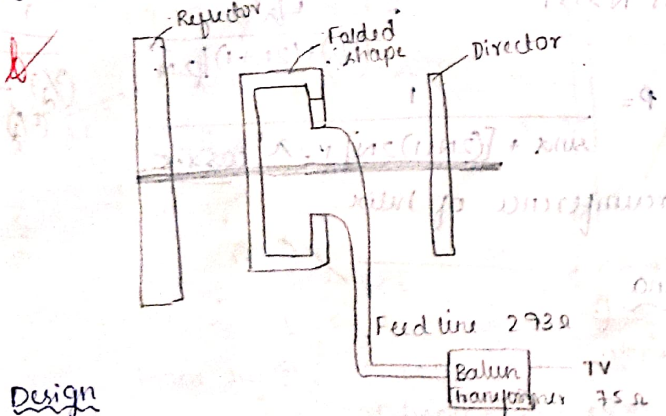
2. Yagi-uda-antenna

6(b)





Optical equivalent of Yagi-Uda antenna



Design

Length of active element

1. $L_a = 0.46\lambda$

2. Length of reflector

$L_r = 0.475\lambda$

3. Director length

$L_{d1} = 0.44\lambda$

$L_{d2} = 0.43\lambda$

$L_{d3} = 0.42\lambda$

⋮

$l_a = 0.46\lambda$

$L_r = 0.475\lambda$

$L_{d1} = 0.44\lambda$

$S_L = 0.25\lambda$

$S_d = 0.31\lambda$

$D = 0.01\lambda$

1.5λ

4. Spacing between reflector and folded dipole

5. Spacing between directors

$S_d = 0.31\lambda$

6. Diameter of element = 0.01λ

7. Array length = 1.5λ

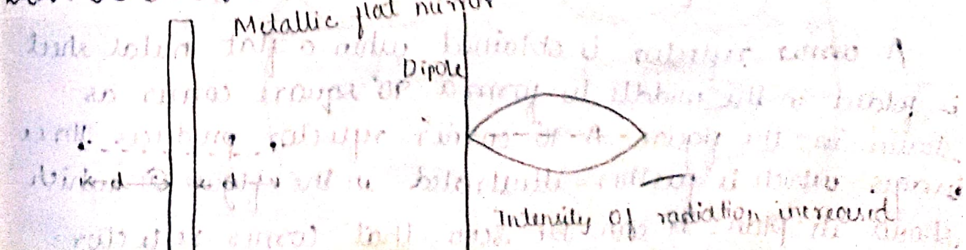
Yagi-Uda antenna is a five element system operating in the range of 40-300MHz. We find that directors and reflector elements are parasitic elements. A parasitic element is a conductor kept near the dipole such that the current gets induced in it by the field in the dipole. It has been found that when the parasitic element kept near a dipole ($\lambda/2$) of length longer than $\lambda/2$ as shown in the figure, it will act as a reflector. The longer element is found to be inductive in nature. Similarly shorter length conductor as shown in the figure will act as a capacitor element and hence as a director. In practical situation, we need to use only one reflecting element as more number does not have any effect on the radiation pattern. However we can use more than one director element to produce a very strong directive action on the radiation. It has also been found that more than 16 elements will not have much effect on the directivity of the antenna. The typical gain of the yagi-uda antenna is 15dB.

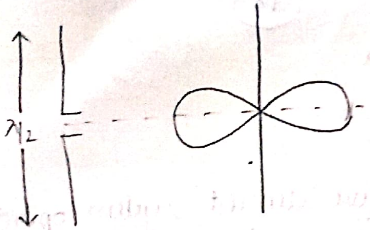
Application: used as receiving antenna in television system

Reflector antennas 6(c)

Reflectors are widely used to modify the radiation pattern of radiating element.

Flat shield reflector





Corner reflector

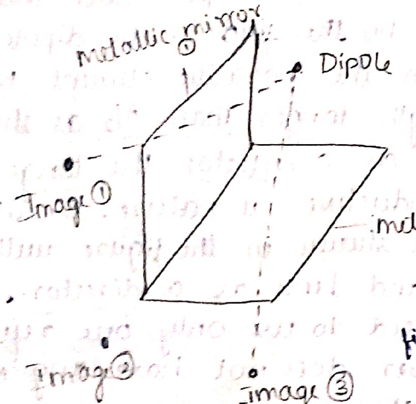


fig ①

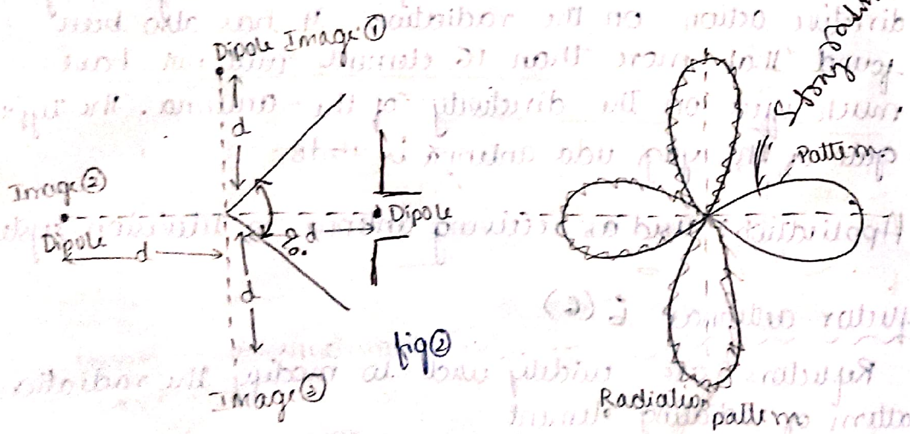


fig ②

$$E_\phi = 2KI_1\left[\cos\left(\frac{2\pi d}{\lambda}\cos\phi\right) - \cos\left(\frac{2\pi d}{\lambda}\sin\phi\right)\right]$$

A corner reflector is obtained when a flat metal sheet is folded in the middle to form a 90° square corner as shown in the figure. A 90° corner reflector produces three images which is further illustrated in the figure ② which produces more intense beam than a flat sheet reflector. This is because in the corner reflector there are 3 images that vertically add to produce the radiation pattern instead of one image in the flat sheet reflector. The field intensity is computed using the expression E_ϕ.

Number of images in a corner reflector is $n = \frac{360°}{\theta}$

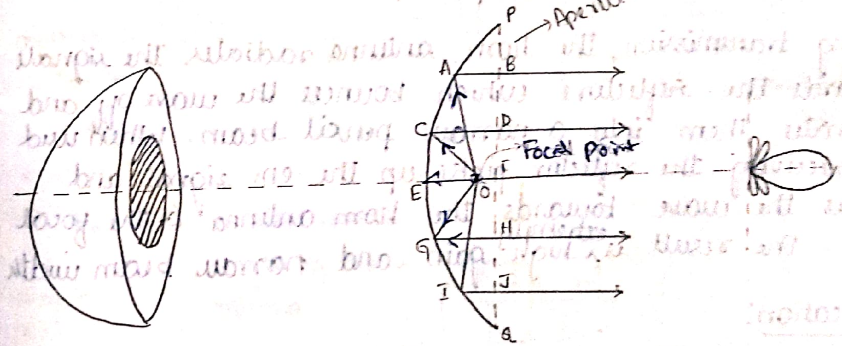
If $\theta = 90° = \frac{360°}{90°} = 4$

If $\theta = 60° = \frac{360°}{60°} = 6$

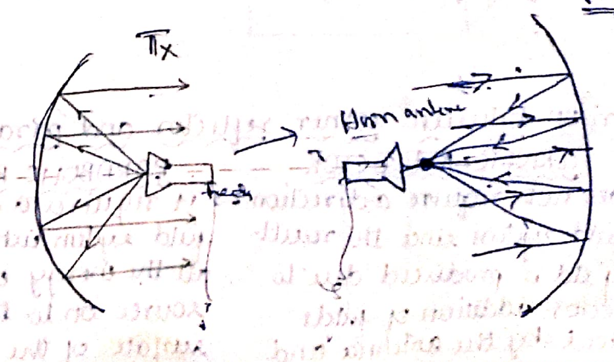
The mirror of any size and shape can convert an isotropic radiator into a directive source producing stronger radiation in a particular direction.

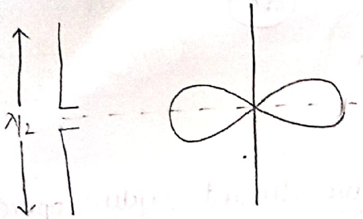
02/04/24

Parabolic Reflectors:



$OAB = OCD = OEF = OGH = OIJ$





Corner reflector

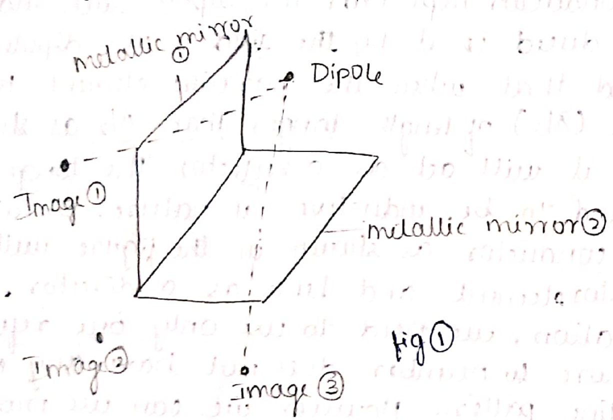


fig ①

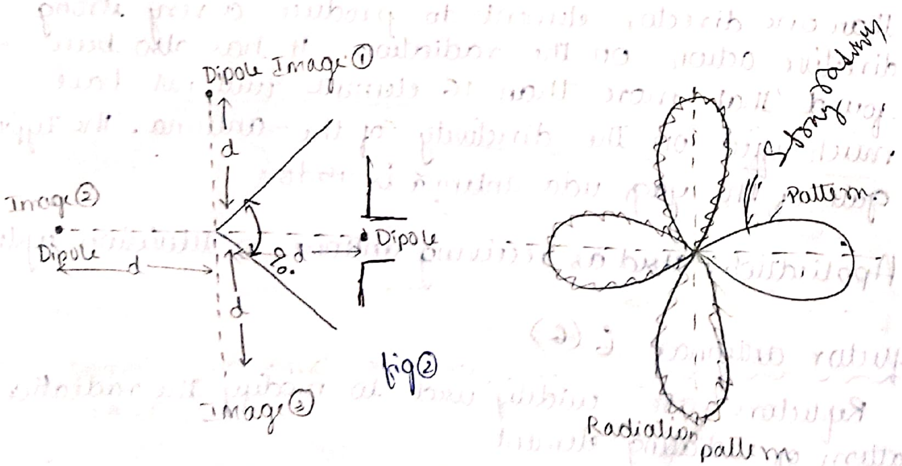


fig ②

$$E_\phi = 2kI_1\left[\cos\left(\frac{2\pi d}{\lambda}\cos\phi\right) - \cos\left(\frac{2\pi d}{\lambda}\sin\phi\right)\right]$$

A corner reflector is obtained when a flat metal sheet is folded in the middle to form a 90° square corner as shown in the figure. A 90° corner reflector produces three images which is further illustrated in the figure ② which shows its plan. It can be seen that corner reflector produces more intense beam than a flat sheet reflector. This is because in the corner reflector there are 3 images that vertically add to produce the radiation pattern instead of one image in the flat sheet reflector. The field intensity is computed using the expression E_ϕ.

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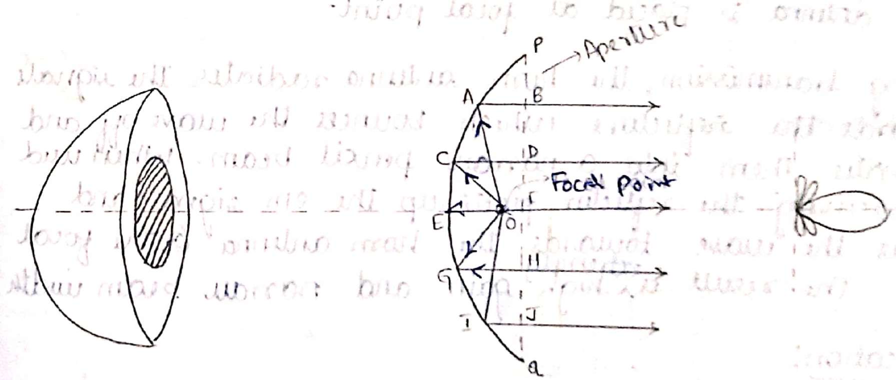
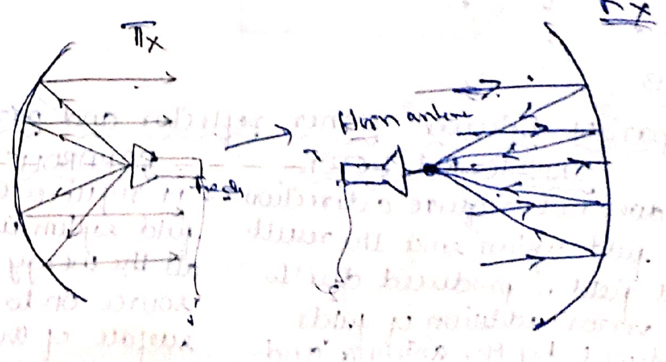
02/04/24

Parabolic Reflectors:



OAB = OCD = OEF = OGH = OIJ



The figure shows construction of parabolic reflector. It can be seen that a parabolic reflector can be constructed by smoothening, the corner reflector to form into a curved surface. In this transformation the curved surface is to follow the mathematical theory on which a parabola is formed. For a parabola distance OAB = OCD = OEF = OGH = OIJ. The point O is called focus and PQ is the aperture of the parabola. Thus it can be seen that all the rays emitting from the point O traverse the same distance to reach the aperture of the curved surface after reflection from that surface. Thus all the reflected rays, such as AB, CD, EF, GH and IJ can be seen to be parallel to each other.

Working:

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Transmission

The figure 4 shows how a parabolic reflector is used in conjuction with a horn antenna for transmission. The horn antenna is placed at focal point.

- During transmission, the horn antenna radiates the signals towards the reflectors which bounces the wave off and collimates them into a narrow pencil beam. When used for receiving the reflector picks up the em signal and bounces the wave towards the horn antenna @ the focal point. The result is extremely high gain and narrow beam width.

Application:

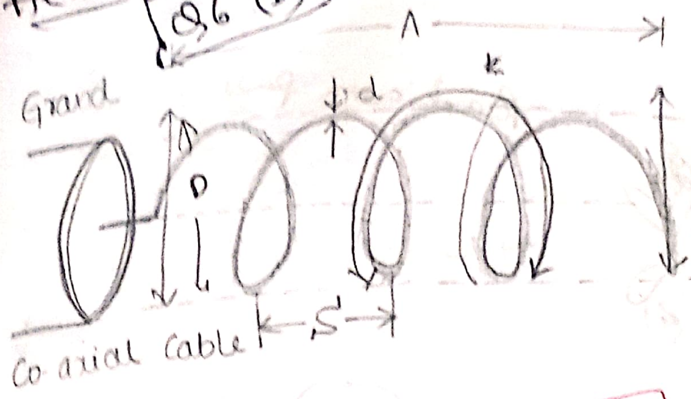
→ Antenna for receiving broadcast directly from satellite, radar direction finding, microwave commn link, pt to pt sources etc.,

03/05/13

Comparison between corner reflector and parabolic reflector

CORNER REFLECTOR	PARABOLIC REFLECTOR
→ It does not require a directional field system since the resultant field is produced due to the vector addition of fields produced by the antenna and	→ It requires a directional field system which will direct all the energy emitted from the source on to the curved surface of the parabola

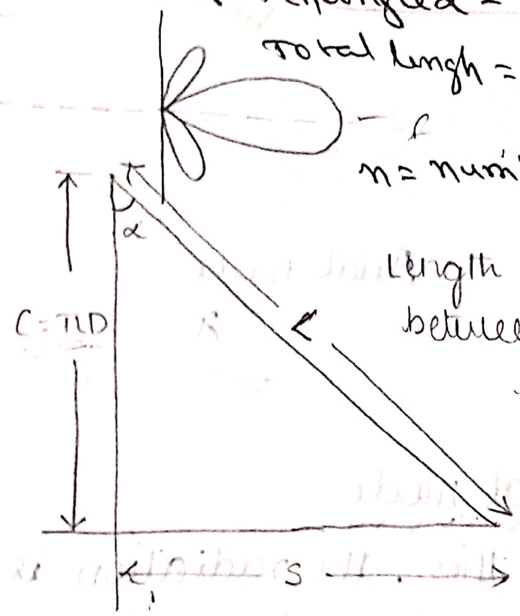
Helical antenna [Q6 (b)]



Circumference of helix $= C = \pi D$

Pitch angle $\alpha = S/\pi D$

Total length $= L = n \times S'$



n = number of turns

Length of wire between each turn

$\tan\alpha = \frac{S}{\pi D}$ — α pitch angle

$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right)$ $\left(\frac{S}{C}\right)$

Helical antenna can be used to produce and receive waves of circular polarization. It is a wideband antenna, super gain antenna, acts like an endfire array. It offers uniform input impedance over its operating frequency range.

construction:

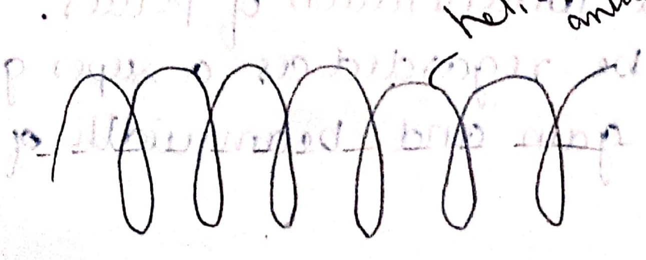
Suppose if you want to construct helical antenna first we decide number of turns required and diameter of each turn. Then we choose a copper wire of sufficient thickness d, length L and mechanical strength. This is then wound on a cylinder of appropriate diameter to form the desired helix. In winding the helix, we employ rotating and pulling forces simultaneously on the wire. [Q7]

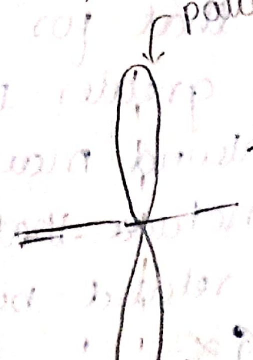
1) wide band antenna
2) Super gain antenna
3) acts an end fire antenna
4) uniform input impedance over operating freq range

Different modes of helical antenna

There are 2 modes.

1. Normal mode

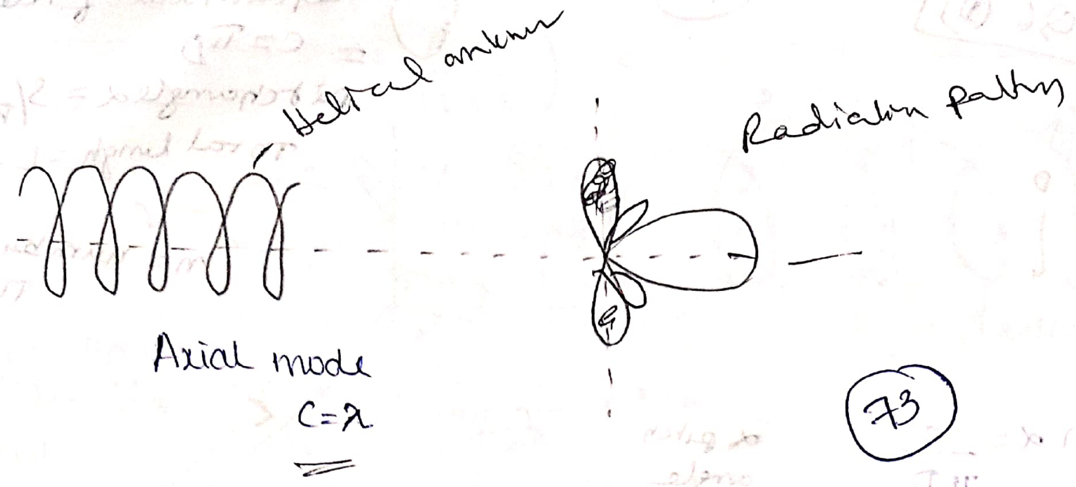




Apply

1) Space Comm
2) Satellite comm

2. Axial mode



73

Normal mode

In this, the radiation is maximum along the broad side to the helix axis under the condition that the circumference of the helix is smaller wrt wavelength λ.

Axial mode.

In this, the max. radiation is along the helix axis or an endfire axis. Under the condition that circumference of the helix is of the order of one wavelength. or more

Design consideration of monofilar (single turn) axial mode helical antenna

The following are the parameters considered for monofilar axial mode helical antenna.

1. Gain required to be produced by the antenna.
2. Impedance offered by the antenna.
3. Axial ratio of the helix.
4. Width of the beam to be produced

We know that for an array, the larger the gain of the array the greater the concentration of power. Since the helical antenna may be regarded as a super gain array, we conclude that gain and beamwidth of helical antenna are related by

$$G \propto \frac{1}{(HPBW)^2}$$

Beam width of helical antenna depends on

1. Number of turns in the helix
2. Gain of the antenna & DNF
3. Impedance of the antenna
4. Axial ratio of the antenna

The above parameters are found to be the function of

1. Size and shape of the ground plane
2. Diameter of helix conductor
3. Structure that support the helix
4. Feed arrangement

John. D. Kraus developed the empirical formula for beam width and directivity

$$HPBW = \frac{52}{\frac{c}{\lambda}\sqrt{n\left(\frac{S}{d}\right)}}$$

n = number of turns

S - pitch

$$FNBW = \frac{115}{\frac{c}{\lambda}\sqrt{n\left(\frac{S}{d}\right)}}$$

d = diag conductor

C = ~~Circumference of D~~ velocity of light

$$\text{directivity } D = 12n\left(\frac{c}{\lambda}\right)^2\left(\frac{S}{\lambda}\right)$$

29/04/2024

Obtain the expression for axial mode pattern of helical antenna

Wkt, for N element isotropic radiation

$$E = \frac{\sin(N\theta/2)}{\sin\theta/2}$$

$$\theta = \phi + \frac{2\pi d}{\lambda}\cos\alpha$$

ϕ = progressive phase angle, α = angle of field with

N-isotropic sources

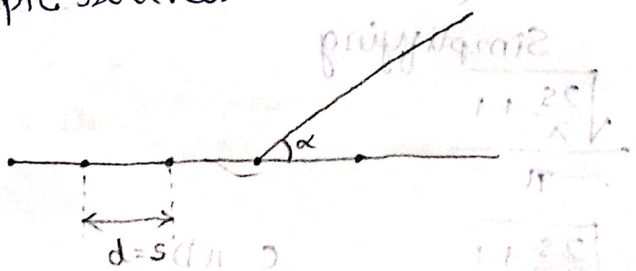


fig (1)

Axial axis

fig (2) Helical antenna equalled to N element array

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Internal Assesment Test - 3

Sub:	Microwave and Antennas							**Code:**	21EC62
Date:	27/07/2024	**Duration:**	90 mins	**Max Marks:**	50	**Sem:**	VI	**Branch:**	ECE

Answer any Five Questions

	Questions	Marks	CO	RBT
1.	Derive the expression for total field, in case of two isotropic point sources with the same amplitude and equal phase. Plot the field pattern for two isotropic point sources spaced $\lambda/2$ apart.	[10]	CO4	L2
2.	Derive the expression for total field, in case of n-linear isotropic point sources. What is pattern multiplication and discuss briefly with examples.	[10]	CO4	L3
3.	State and prove the power theorem & mention its applications. Prove that D=2(n+1) for a unidirectional pattern given by $U = U_m cos^n \theta$	[10]	CO4	L2,L3
4.	Derive field expression for Short Electric dipole/ Thin linear antenna.	[10]	CO4	L2
5.	Derive the expression for radiation resistance of short dipole and small loop antenna.	[10]	CO4,5	L3
6.	Write short notes on a) Horn Antennas, b) Yagi Uda array, c) Parabolic Reflector d) Helical antenna.	[10]	CO5	L2
7	Explain with diagram Helix geometry & Helix modes, What are Practical design considerations for mono-filar axial mode Helical Antenna.	[10]	CO5	L2
8	Determine the Length L, H- plane aperture and flare angle θ_E and θ_H of a pyramidal horn for which the E -plane aperture $a_E = 10\lambda$. The horn is fed by a rectangular waveguide with TE_{10} mode. Let $\delta = 0.2\ \lambda$ in the E –plane and $0.375\ \lambda$ in the H- plane. Also find Beam widths and directivity.	[10]	CO4	L3

CI

CCI

M. Pappa
HOD

Radiation Resistance of a loop antenna (for any size):

Derive an expr. for the radiation resistance of a loop ante.?

By Poynting thm, $P = \frac{1}{2} E H$ $\quad E \rightarrow H\eta_0$

$$= \frac{1}{2}(H\eta_0)H = \frac{1}{2}\frac{H^2}{\eta_0} \Rightarrow 120\pi$$

$$P = \frac{1}{2} H^2 \eta_0 \Rightarrow \boxed{P = 60H^2} \qquad \eta_0 = 120\pi.$$

From general expression for loop antennas ...

Generalised loop antenna field strength for any size:

②

Refer Kraus Pg - 249

$$E_\phi = \frac{60\pi \beta a I}{r} J_1\left(\frac{2\pi a \sin\theta}{\lambda}\right)$$

$$H_\theta = \frac{\beta a I}{2r} J_1\left(\frac{2\pi a \sin\theta}{\lambda}\right)$$

$$= \frac{\beta a I}{2r} J_1\left(\frac{2\pi a \sin\theta}{\lambda}\right)$$

elemental power

$$dP = \left[\frac{\beta a I}{2r} J_1\left(\frac{2\pi \sin\theta}{a}\right)\right]^2 \times 60\pi$$

$$= 15\pi\left[\frac{\beta a I}{r} J_1(\beta a \sin\theta)\right]^2$$

Consider $\int$ over spherical co-ordinates.

$$P = \int_0^{2\pi}\int_0^{2\pi} 15\pi\left[\frac{\beta a I}{r} J_1(\beta a \sin\theta)\right]^2 \sin\theta \, d\theta \, d\phi$$

$$= 30\pi^2(\beta a I)^2 \int_0^{\pi} J_1^2(\beta a \sin\theta)\sin\theta \, d\theta \rightarrow (1)$$

Case 1: For Small loop

$J_1(x) = \frac{x}{2}$ in ①

$$P = 30\pi^2(\beta a I)^2 \int_0^{\pi} \frac{(\beta a \sin\theta)^2}{4} \sin\theta \, d\theta$$

$$\boxed{P = 10\beta^4 A^2 I^2}$$

$A = \pi a^2$ (Loop Area)

$$\frac{1}{2} I^2 R_r = 10\beta^4 A^2 I^2$$

③

$$P = \frac{1}{2} I^2 R_r \Rightarrow \therefore R_r = 20\beta^4 A^2$$

$$R_r = 20 \times \left(\frac{2\pi}{\lambda}\right)^4 A^2 = 31171\left(\frac{A}{\lambda^2}\right) \text{ ohm}$$

$$R_r = 31171\left(\frac{nA}{\lambda^2}\right)^2 \text{ ohm}$$

n = number of Turns

Case 2: Large loop.

C_λ is perimeter of the loop

$C_\lambda > 5$ or $\beta a > 5$

a → radius of the loop

λ → wavelength.

Direct eqn : $R_r = 3721\left(\frac{a}{\lambda}\right)$ ohm

Directivity of Circular loop:

General : $D = \frac{2\beta a\left[J_1^2(\beta a \sin\theta)\right]_{max}}{\left[\int_0^{2\beta a} J_2(y)\,dy\right]}$

Case 1: Small loop $\left(\beta a << \frac{1}{3}\right)$

$$D = \frac{2\beta a\left[\left(\frac{\beta a}{2}\right)\sin\theta\right]^2}{\left(\frac{\beta a}{3}\right)^3} \rightarrow \boxed{D = 3/2}$$

Case 2: Large loop $(\beta a >> 5)$

$$J_1(\beta a \sin\theta) = 0.582$$

$$\boxed{D = 0.677\beta a}$$