CBCS SCHEME

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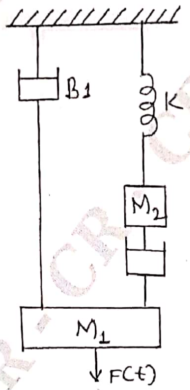
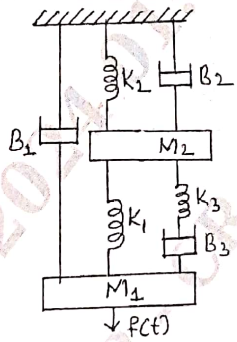
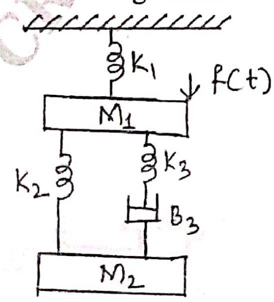
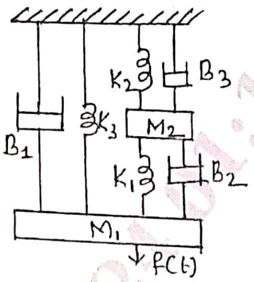
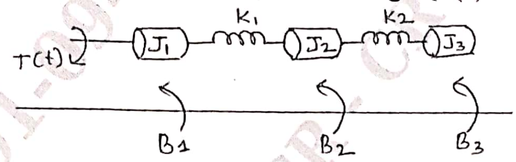
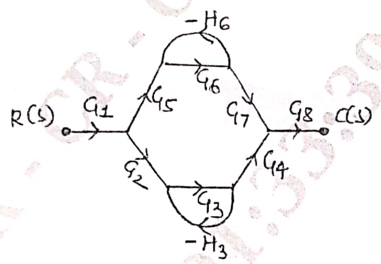
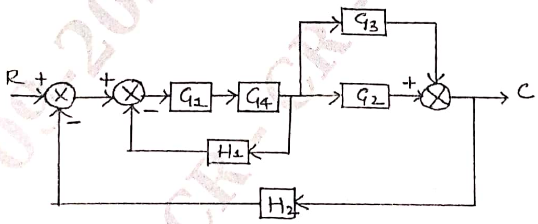
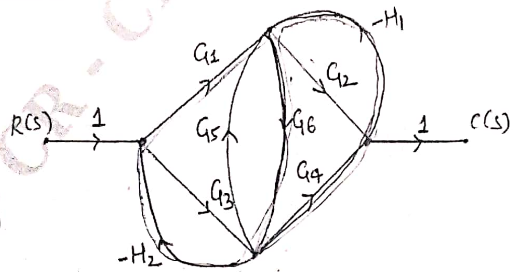
BEC403

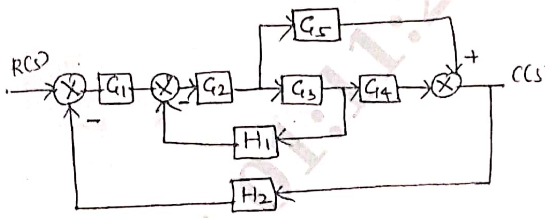
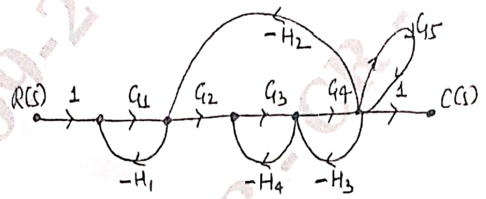
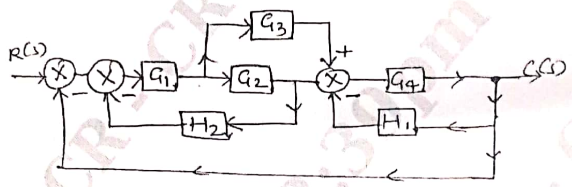
Fourth Semester B.E./B.Tech. Degree Examination, June/July 2024
Control Systems

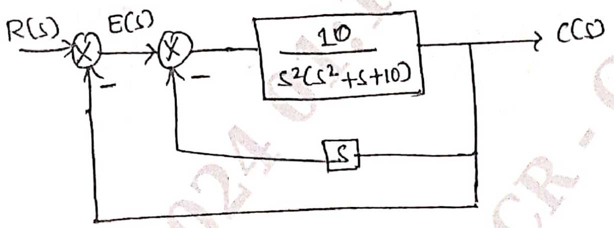
Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define Control system. Write down any four differences between Open Loop Control System and Closed Loop Control System.	4	L2	CO1
	b.	For the mechanical system shown in Fig. Q1(b), obtain the equivalent electrical system using Force – Voltage method.  Fig. Q1(b)	8	L2	CO1
	c.	For the mechanical system, shown in Fig. Q1(c), obtain the equivalent electrical system using Force – Current method.  Fig. Q1(c)	8	L2	CO1
		OR			
Q.2	a.	For the mechanical system shown in Fig. Q2(a), obtain the equivalent electrical system using Force – Voltage method.  Fig. Q2(a)	7	L2	CO1

	b.	For the mechanical system shown in Fig. Q2(b), obtain the equivalent electrical system using Force – Voltage method.  Fig. Q2(b)	7	L2	CO1
	c.	Draw the electrical network based on torque – current analogy and write performance equation for the mechanical system of Fig. Q2(c).  Fig. Q2(c)	6	L2	CO1
		Module – 2			
Q.3	a.	Find $\frac{C(s)}{R(s)}$ by Mason's gain formula for Fig. Q3(a).  Fig. Q3(a)	6	L3	CO3
	b.	Determine the transfer function $\frac{C(s)}{R(s)}$ of the system shown in Fig. Q3(b).  Fig. Q3(b)	6	L3	CO3
	c.	For the single flow graph of Fig. Q3(c), find the transfer function using Mason's gain formula.  Fig. Q3(c)	8	L3	CO3

		OR			
Q.4	a.	Reduce the block diagram to its canonical form and obtain C(s)/R(s) of the system of Fig. Q4(a).  Fig. Q4(a)	6	L3	CO3
	b.	Obtain the transfer function of the single flow graph shown in Fig. Q4(b), using Mason's gain formula.  Fig. Q4(b)	6	L3	CO3
	c.	Reduce the block diagram of Fig. Q4(c) to its simple form and obtain C(s)/R(s).  Fig. Q4(c)	8	L3	CO3
		Module – 3			
Q.5	a.	With the help of graphical representation and mathematical expression, explain the following test signals : i) Step signal ii) Ramp signal iii) Impulse signal iv) Parabolic signal.	8	L3	CO2
	b.	Find Kp , Kv , Ka and steady state error for a system with Open loop transfer function $G(s)\,H(s) = \frac{10(s+2)(s+3)}{s(s+1)(s+4)(s+5)}$, where $r(t) = 3 + t + t^2$.	6	L3	CO2
	c.	The Open loop transfer function of a servo system with unity feedback is given as $G(s) = \frac{10}{s(0.1s+1)}$. Find out static error constants and obtain steady state error when an input $r(t) = A_0 + A_1 t + \frac{A_2}{2} t^2$ is applied.	6	L3	CO2
		OR			
Q.6	a.	For a unity feedback control system with $G(s) = \frac{64}{s(s+9.6)}$, write the output response to a unit step input. Determine 1) The response at t = 0.1 set 2) Maximum value of response and the time at which it occurs. 3) Settling time.	10	L2	CO3

	b.	For the system shown in Fig. Q6(b), 1) Identify the type of C(s) / E(s) 2) Find values of Kp , Kv , Ka. 3) If r(t) = 10u(t), find steady state value of the output.  Fig. Q6(b)	10	L2	CO3
		Module – 4			
Q.7	a.	Find the number of roots with positive real part, zero real part and negative real part for a system $s^6 + 4s^5 + 3s^4 - 16s^2 - 64s - 48 = 0$.	6	L2	CO4
	b.	For a unity feedback system , $G(s) = \dfrac{K}{s(1+0.4s)(1+0.25s)}$, find range of values of K, Marginal value of K and frequency of sustained oscillations.	6	L2	CO4
	c.	Explain the angle condition in Root locus. Test the following points using angle condition for the system $G(s)\,H(s) = \dfrac{K}{s(s+2)(s+4)}$. i) $s = -0.75$ ii) $s = -1 + j4$.	8	L2	CO4
		OR			
Q.8	a.	Sketch the complete root locus and comment on the stability of the system $G(s)\,H(s) = \dfrac{K}{s(s+1)(s+2)(s+3)}$.	12	L2	CO4
	b.	Sketch the Bode plot for the transfer fl. Find value of 'K' for $W_{gc} = 5$ rad/sec. $G(s) = \dfrac{K s^2}{(1+0.2s)(1+0.02s)}$	8	L2	CO4
		Module – 5			
Q.9	a.	For a certain control system $G(s)\,H(s) = \dfrac{K}{s(s+2)(s+10)}$, sketch the Nyquist plot and hence calculate the range values of K for stability.	10	L2	CO5
	b.	Explain the Lag compensator and Lead compensator with the help of a circuit diagram.	10	L2	CO5
		OR			

Q.10	a.	Construct the state model using phase variables if the system is described by the differential equation $\frac{d^3y(t)}{dt^3}+4\frac{d^2y(t)}{dt^2}+7\frac{dy(t)}{dt}+2y(t)=5u(t)$. Also draw the state diagram.	6	L2	CO5
	b.	The transfer function of a control system is $\frac{Y(s)}{U(s)}=\frac{s^2+3s+4}{s^3+2s^2+3s+2}$. Obtain the State model using signal flow graph.	7	L2	CO5
	c.	Find the state transition matrix for $A=\begin{bmatrix}0 & -1\\ +2 & -3\end{bmatrix}$	7	L1	CO5

* * * * *

Q.1. Define Control Systems. Write down any four difference between open loop control systemsand closed loop control system.

Sol:

A **Control System** is a system of devices or a set of mechanisms that manages, commands, directs, or regulates the behavior of other devices or systems to achieve a desired output. Control systems are widely used in industries, engineering applications, and many automated processes.

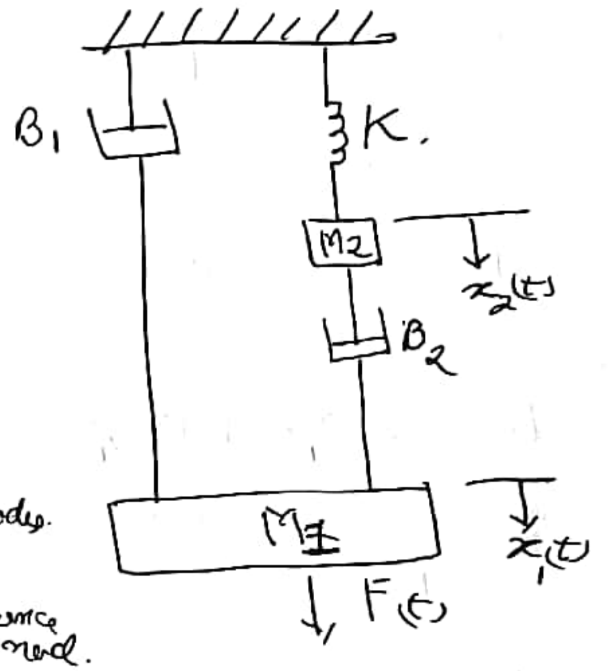
Differences between Open Loop and Closed Loop Control Systems

	Open Loop Control System	**Closed Loop Control System**
Feedback	No feedback mechanism is used	Feedback is used to compare output with the desired setpoint
Accuracy	Less accurate due to lack of feedback	More accurate as feedback helps correct deviations
Complexity	Simpler in design and generally easier to implement	More complex, requires sensors and additional components
Response to Disturbances	Cannot automatically correct for disturbances or changes	Automatically adjusts to disturbances to maintain desired output

Examples:

- **Open Loop**: Washing machine, toaster.
- **Closed Loop**: Thermostat-controlled heating system, cruise control in cars.

Q1-[b] For the mechanical system shown, Obtain the equivalent electrical system using Force-Voltage method.



Sol:

In a F-V analogy, we have

$F \rightarrow V$

$M \rightarrow L$

$B \rightarrow R.$

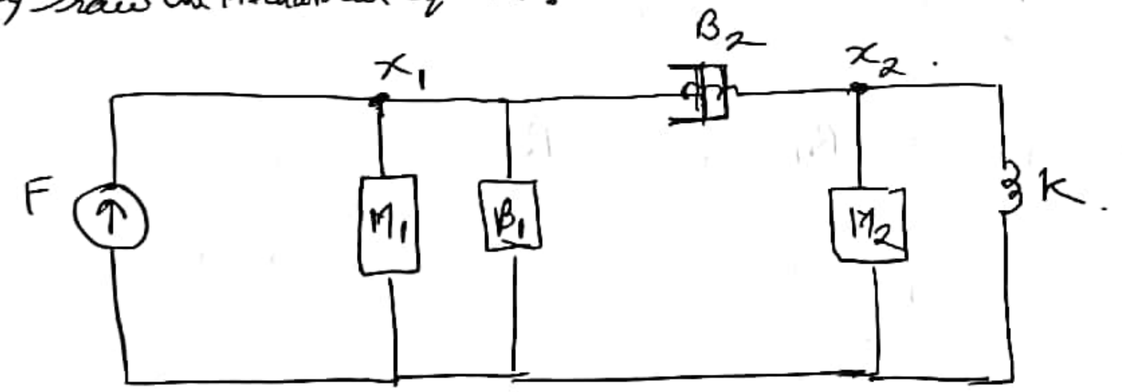
$K \rightarrow \frac{1}{C}$ & $I(x) \rightarrow \dot{q}$

$X \rightarrow q$

No of mass = no of displacements = no of nodes.

$\therefore$ Total nodes $\equiv$ no of mass + 1 reference node.

a) Draw the Mechanical eq. ckt:



b) Find the Force equation

Hence at node x_1, we get

$$F = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d}{dt}x_1 + B_2 \frac{d}{dt}(x_1 - x_2) \longrightarrow (A)$$

IIly for node 2, we get:

$$B_2 \frac{d}{dt}(x_1 - x_2) = M_2 \cdot \frac{d^2x_2}{dt^2} + K x_2 \longrightarrow (B)$$

Taking Laplace Transform of eq (A) & (B) we get

$$F(s) = s^2 M_1 X_1(s) + s B_1 X_1(s) + B_2 s [X_1(s) - X_2(s)] \longrightarrow (C)$$

& $$s B_2 [X_1(s) - X_2(s)] = s^2 M_2 X_2(s) + K X_2(s) \longrightarrow (D)$$

Now using F V analogy we have eq (C) & (D) as

$[F \to V, M \to L, B \to R, K \to \frac{1}{C}, X \to q, s q(s) \to I(s)]$

$$\Longrightarrow V(s) = s^2 L_1 q_1(s) + s B_1 q_1(s) + s B_2 [q_1(s) - q_2(s)]$$

& $\Rightarrow$ using $q(s) = \frac{I(s)}{s}$ we get

$$V(s) = s^2 L_1 \frac{I_1(s)}{s} + s \cdot R_1 \cdot \frac{I_1(s)}{s} + s R_2 \frac{[I_1(s) - I_2(s)]}{s}$$

$$V(s) = s L_1 I_1(s) + R_1 I_1(s) + R_2 [I_1(s) - I_2(s)] \longrightarrow (1)$$

2lly

$$s \cdot B_2 [q_1(s) - q_2(s)] = s^2 M_2 q_2(s) + K q_2(s)$$

$$\Rightarrow B_2 [I_1(s) - I_2(s)] = s M_2 I_2(s) + \frac{K}{s} I_2(s)$$

$\because B \to R, M \to L, K \to \frac{1}{C}$.

$$\Rightarrow R_2 [I_1(s) - I_2(s)] = s L_2 I_2(s) + \frac{1}{sC} I_2(s) \longrightarrow (2)$$

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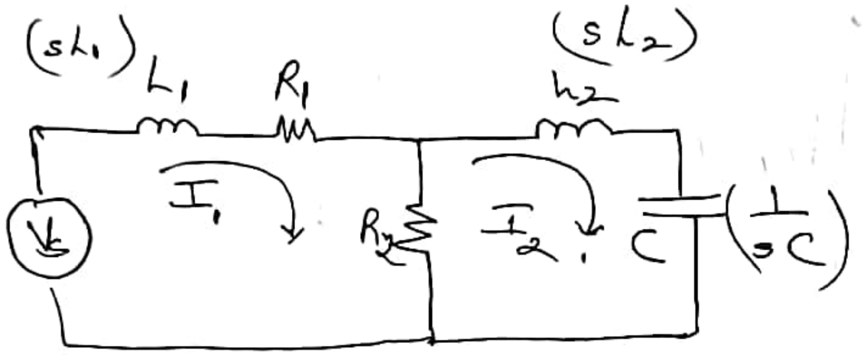
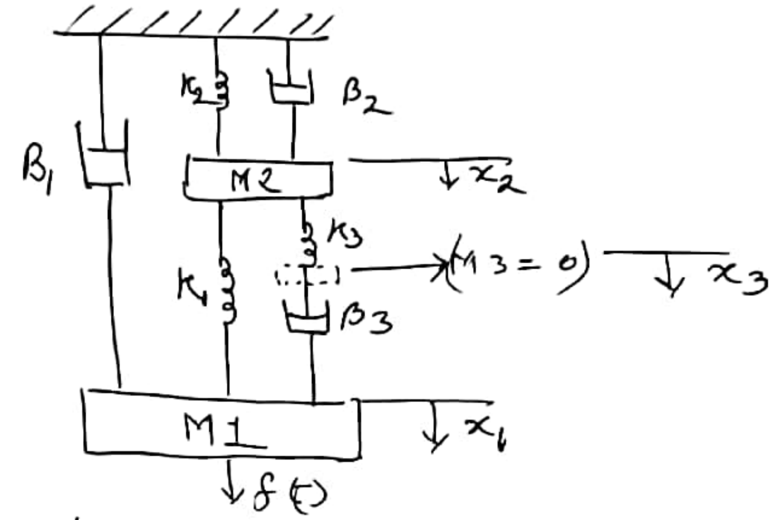
(Q1-(b) Continued)

∴ From eq. (1) & (2) we can draw the combined ckt as:



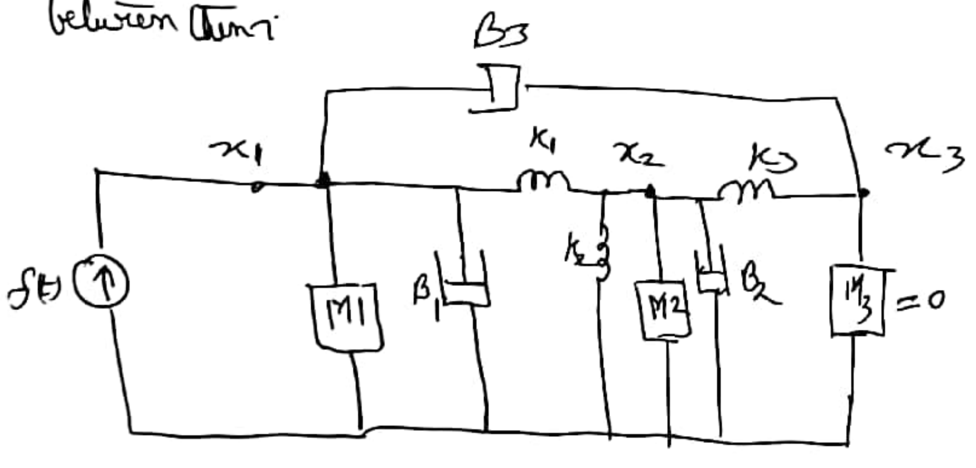
Q1-(c): For the mechanical system, shown, obtain the equivalent electrical system using Force - Current Method



Sol:

Total no of masses present = 02.

And since K_3 & B_3 are in sequence we can consider a Mass $M3 = 0$ between them.



Applying force equations at x_1, x_2 & x_3 we get

$\Longrightarrow$

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1(x_1 - x_2) + B_3 \frac{d}{dt}(x_1 - x_3) \longrightarrow (A)$$

$$K_1(x_1 - x_2) = K_2 x_2 + M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_3(x_2 - x_3) \longrightarrow (B)$$

$$K_3(x_2 - x_3) + B_3 \frac{d}{dt}(x_1 - x_3) = M_3 \frac{d^2 x_3}{dt^2} \longrightarrow (C)$$

Taking L.T. of (A), (B) & (C) we get

$$F(s) = s^2 M_1 X_1(s) + s B_1 X_1(s) + K_1 [X_1(s) - X_2(s)] + s \cdot B_3 [X_1(s) - X_3(s)] \longrightarrow (a)$$

$$K_1 [X_1(s) - X_2(s)] = K_2 X_2(s) + s^2 M_2 X_2(s) + s B_2 X_2(s) + K_3 [X_2(s) - X_3(s)] \longrightarrow (b)$$

$$K_3 [X_2(s) - X_3(s)] + s B_3 [X_1(s) - X_3(s)] = s^2 M_3 X_3(s) \longrightarrow (c)$$

using F–I analogy we have,

$F \to I$, $M \to C$, $B \to \frac{1}{R}$, $K \to \frac{1}{L}$, $X \to \phi$, $V(s) = s \cdot \phi(s)$

$\Longrightarrow$

$$I(s) = s^2 C_1 \phi_1(s) + s \times \frac{1}{R_1} \times \phi_1(s) + \frac{1}{L_1} [\phi_1(s) - \phi_2(s)] \longrightarrow (1)$$

$$\frac{1}{L_1} [\phi_1(s) - \phi_2(s)] = \frac{1}{\cancel{R_2}} \phi_2(s) + s^2 C_2 \phi_2(s) + s \times \frac{1}{R_2} \times \phi_2(s) + \frac{1}{L_3} [\phi_2(s) - \phi_3(s)] \longrightarrow (2)$$

$$\frac{1}{L_3} [\phi_2(s) - \phi_3(s)] + s \frac{1}{R_3} [\phi_1(s) - \phi_3(s)] = s^2 C_3 \phi_3(s). \longrightarrow (3)$$

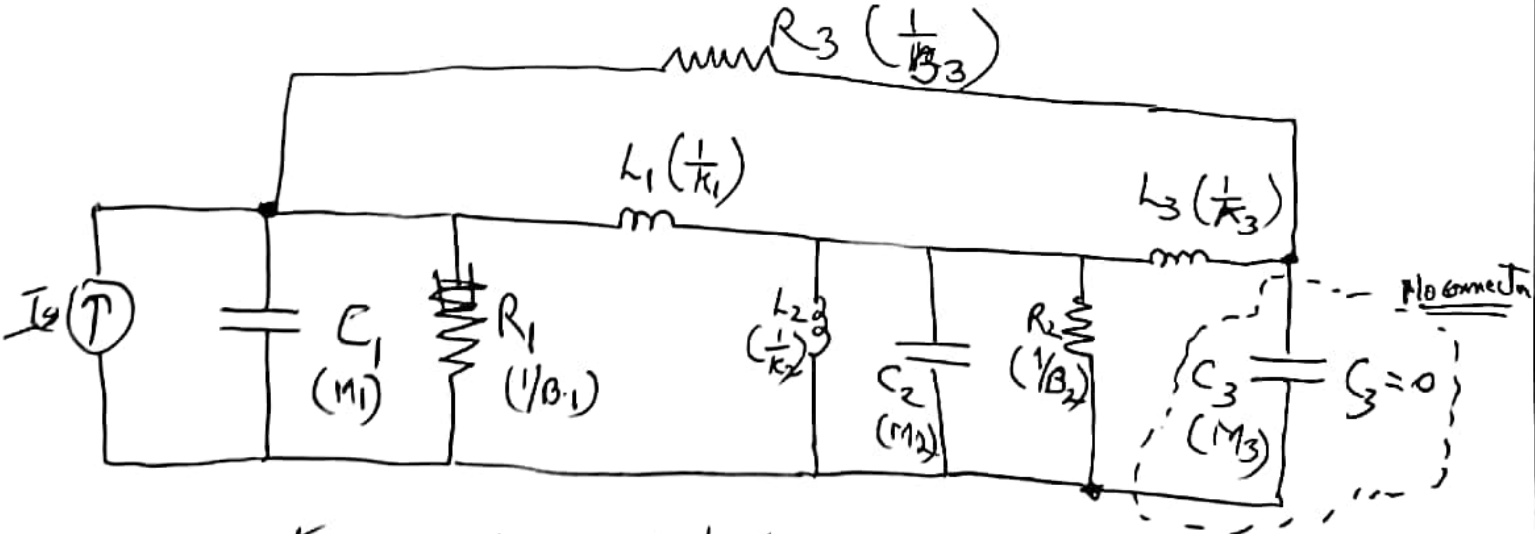
Putting $V(s) = s\cdot\phi(s)$ we get

$\Longrightarrow$

$$I(s) = sC_1 V_1(s) + \frac{V_1(s)}{R_1} + \frac{1}{sL_1}\left[V_1(s) - V_2(s)\right] \longrightarrow \text{(I)}$$

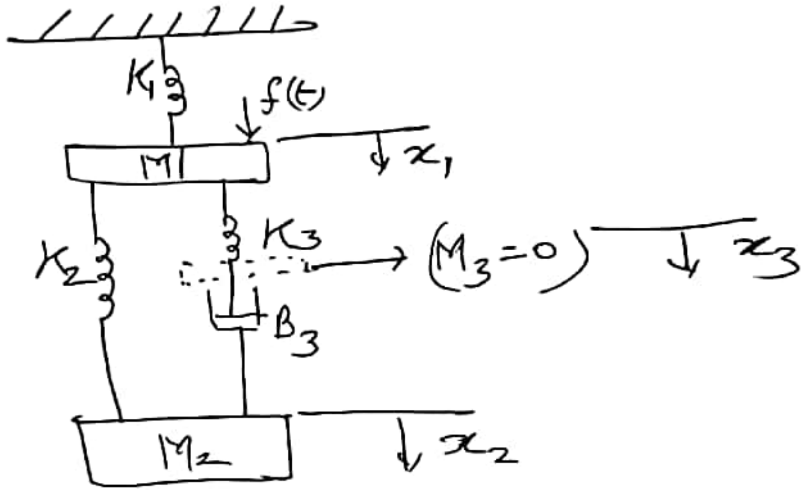
$$\frac{1}{sL_1}\left[V_1(s) - V_2(s)\right] = \frac{1}{sL_2}V_2(s) + sC_2 V_2(s) + \frac{V_2(s)}{R_2} + \frac{1}{sL_3}\left[V_2(s) - V_3(s)\right] \longrightarrow \text{(II)}$$

$$\frac{1}{sL_3}\left[V_2(s) - V_3(s)\right] + \frac{\left[V_1(s) - V_3(s)\right]}{R_3} = sC_3 V_3(s) \longrightarrow \text{(III)}$$



Equivalent Electrical ckt using F–I Analogy

Q2.(a) : For the mechanical system shown, obtain the equivalent electrical system using Force-Voltage method.

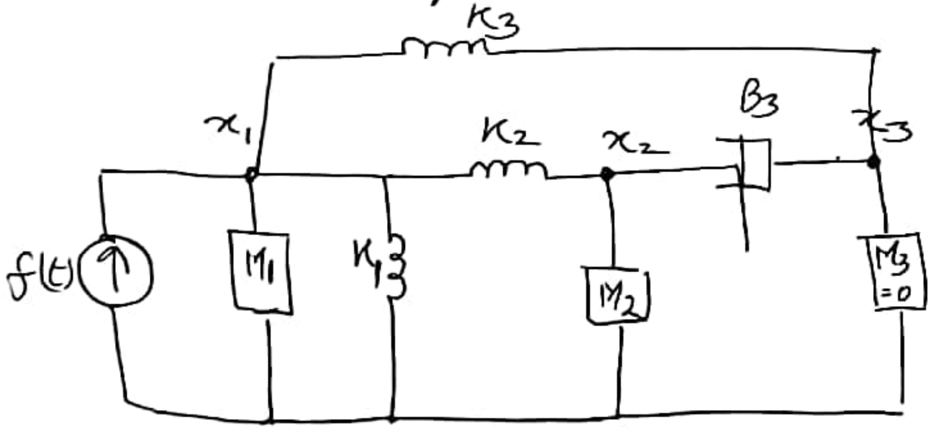


Sol:

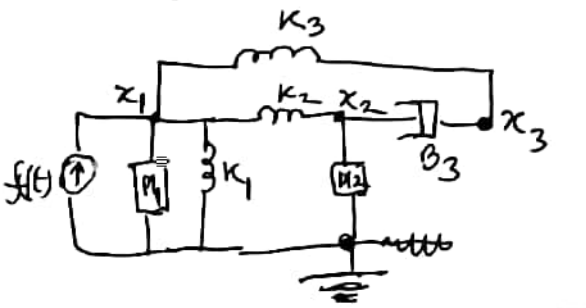
Since K_3 & B_3 are in sequence, we consider a mass $M_3=0$ between them at node 'x_3'.

∴ Total no of Masses = 03

∴ Total Nodes req = 03+1 = 04.



Applying force equations at x_1, x_2 & x_3 we get



∵ $M_3 = 0$.

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + K_2(x_1 - x_2) + K_3(x_1 - x_3) \longrightarrow (A)$$

IIly
$$K_2(x_1 - x_2) = M_2 \frac{d^2 x_2}{dt^2} + B_3 \frac{d(x_2 - x_3)}{dt} \longrightarrow (B)$$

IIly
$$B_3 \frac{d}{dx}(x_2 - x_3) + K_3(x_1 - x_3) = M_3 \frac{d^2 x_3}{dt^2} \longrightarrow (C)$$

Here $M_3 = 0$

$\Longrightarrow$

$$B_3 \frac{d}{dx}(x_2 - x_3) = -K_3(x_1 - x_3) \longrightarrow (D)$$

Taking L.T. of (A), (B) & (D) we get

$$F(s) = s^2 M_1 X_1(s) + K_1 X_1(s) + K_2[X_1(s) - X_2(s)] + K_3[X_1(s) - X_3(s)] \longrightarrow (a)$$

$$K_2[X_1(s) - X_2(s)] = s^2 M_2 X_2(s) + sB_3[X_2(s) - X_3(s)] \longrightarrow (b)$$

$$s \cdot B_3[X_2(s) - X_3(s)] = -K_3[X_1(s) - X_3(s)] \longrightarrow (d)$$

using F-V-analogy we have:

$F \to V$, $M \to L$, $B \to R$, $K \to \frac{1}{C}$, $X \to q$,

& $s \cdot q(s) = I(s)$.

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$$\Rightarrow V(s) = s^2 L_1 q_1(s) + \frac{1}{C_1} q_1(s) + \frac{1}{C_2}\left[q_1(s) - q_2(s)\right] + \frac{1}{C_3}\left[q_1(s) - q_3(s)\right]$$

using $s \cdot q(s) = I(s)$

$$\Rightarrow V(s) = s L_1 I_1(s) + \frac{1}{sC_1} I_1(s) + \frac{1}{sC_2}\left[I_1(s) - I_2(s)\right] + \frac{1}{sC_3}\left[I_1(s) - I_3(s)\right] \longrightarrow \mathrm{I}$$

||ly,

$$\frac{1}{C_2}\left[q_1(s) - q_2(s)\right] = s^2 L_2 q_2(s) + s R_3\left[q_2(s) - q_3(s)\right]$$

putting $s \cdot q(s) = I(s)$, we get

$$\frac{1}{sC_2}\left[I_1(s) - I_2(s)\right] = s L_2 I_2(s) + R_3\left[I_2(s) - I_3(s)\right] \longrightarrow \mathrm{II}$$

$$s R_3\left[q_2(s) - q_3(s)\right] = -\frac{1}{C_3}\left[q_1(s) - q_3(s)\right]$$

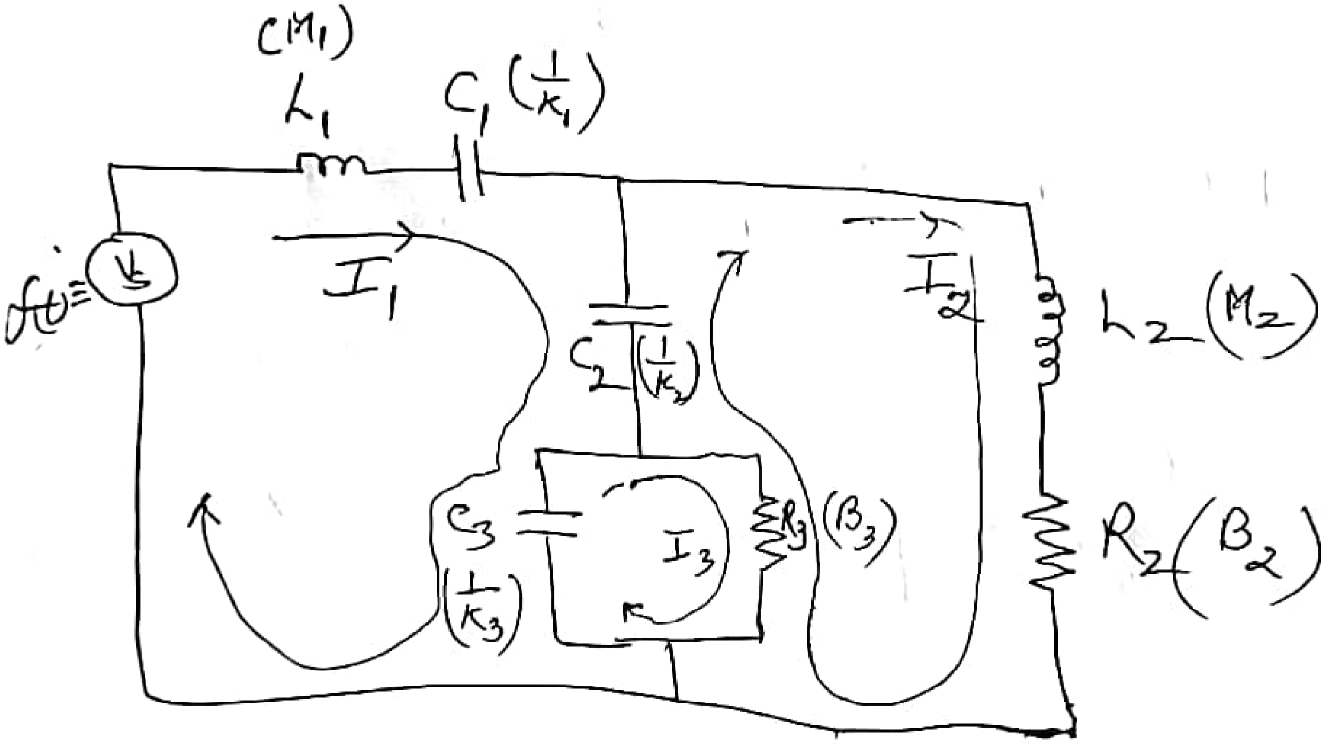
||ly putting $s \cdot q(s) = I(s) \Rightarrow$

$$R_3\left[I_2(s) - I_3(s)\right] = -\frac{1}{sC_3}\left[I_1(s) - I_3(s)\right]$$

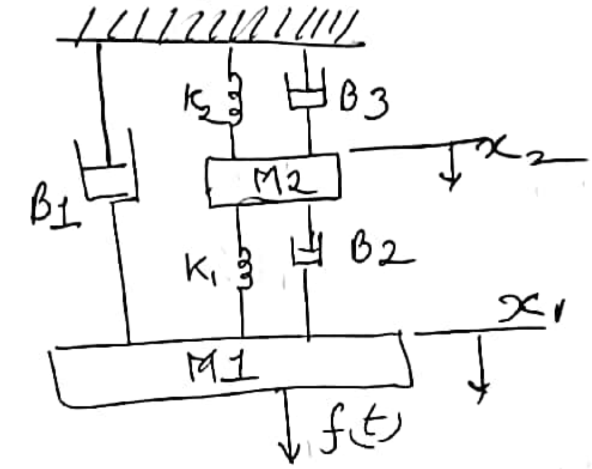
$$= \frac{1}{sC_3}\left[I_3(s) - I_1(s)\right] \longrightarrow \mathrm{III}$$

Q 2. (a) Continued (Page 4)

Hence from I, II & III we get the electrical equivalent as :



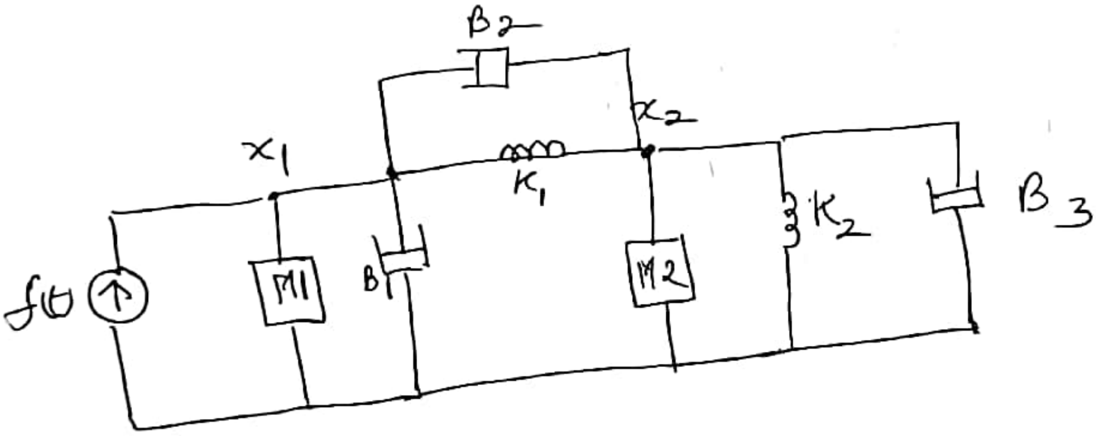
2.2 (b): For the mechanical systems shown, obtain the equivalent electrical system using Force-Voltage Method



Sol: No of masses = 02.

∴ No of nodes = 02 + 01 (Reference Node)

∴



Applying force equations at x_1 & x_2 we get:

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B_2 \frac{d(x_1 - x_2)}{dt} + K_1 (x_1 - x_2) \longrightarrow (A)$$

$$B_2 \frac{d}{dt}(x_1 - x_2) + K_1 (x_1 - x_2) = M_2 \cdot \frac{d^2 x_2}{dt^2} + B_3 \cdot \frac{dx_2}{dt} + K_2 x_2 . \longrightarrow (B)$$

Taking L.T. of (A) & (B) we get

$$F(s) = s^2 M_1 X_1(s) + s \cdot B_1 X_1(s) + s \cdot B_2 (X_1(s) - X_2(s)) + K_1 (X_1(s) - X_2(s)) \longrightarrow (C)$$

$$s B_2 (X_1(s) - X_2(s)) + K_1 (X_1(s) - X_2(s)) = s^2 M_2 X_2(s) + s B_3 X_2 + K_2 X_2(s) \longrightarrow (D)$$

using F-V-analogy, we have:

$F \to V$, $M \to L$, $B \to R$, $K \to \frac{1}{C}$, $X \to q$, $s\,q(s) = I(s)$

$\therefore$ we have eq (C) & (D) as:

$$V(s) = s^2 L_1 q_1(s) + s.R_1 q_1(s) + s.R_2(q_1(s) - q_2(s)) + K_1(q_1(s) - q_2(s))$$

Also $s.q(s) = I_s$,

$\Longrightarrow$

$$V(s) = sL_1 I_1(s) + R_1 I_1(s) + R_2\{I_1(s) - I_2(s)] + \frac{1}{sC_1}(I_1(s) - I_2(s)) \longrightarrow (1)$$

IIly

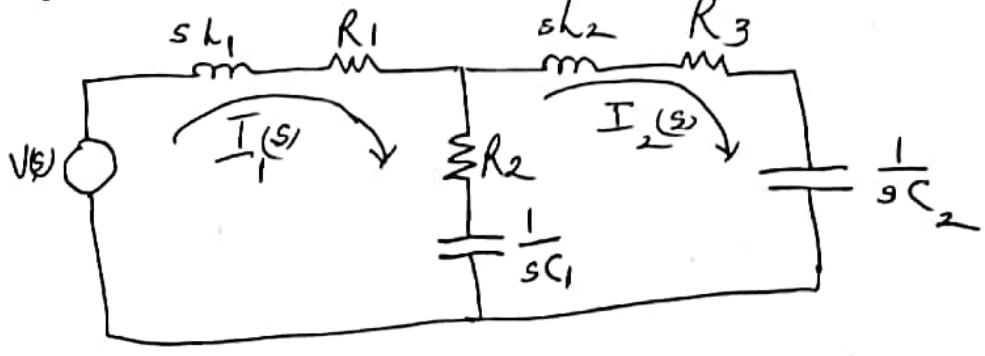
$$s R_2[q_1(s) - q_2(s)] + \frac{1}{C_1}[\cancel{q}_1(s) - q_2(s)] = s^2 L_2 q_2(s) + s R_3 q_2(s) + \frac{1}{C_2} q_2(s).$$

using $s.q(s) = I(s)$

$\Longrightarrow$

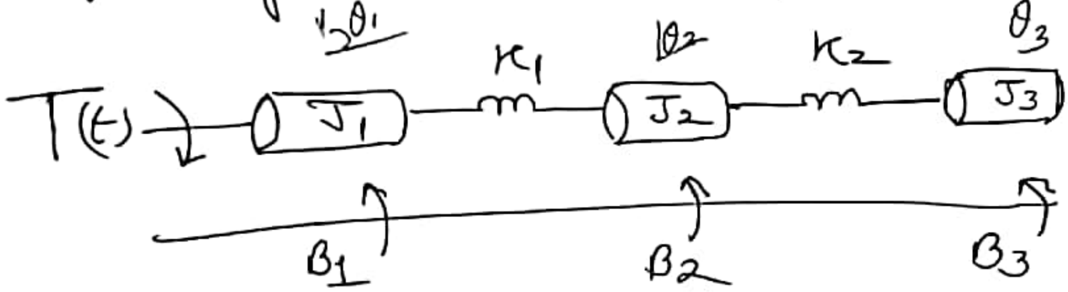
$$R_2[I_1(s) - I_2(s)] + \frac{1}{sC_1}[I_1(s) - I_2(s)] = sL_2 I_2(s) + R_3 I_2(s) + \frac{1}{sC_2} I_2(s) \longrightarrow (2)$$

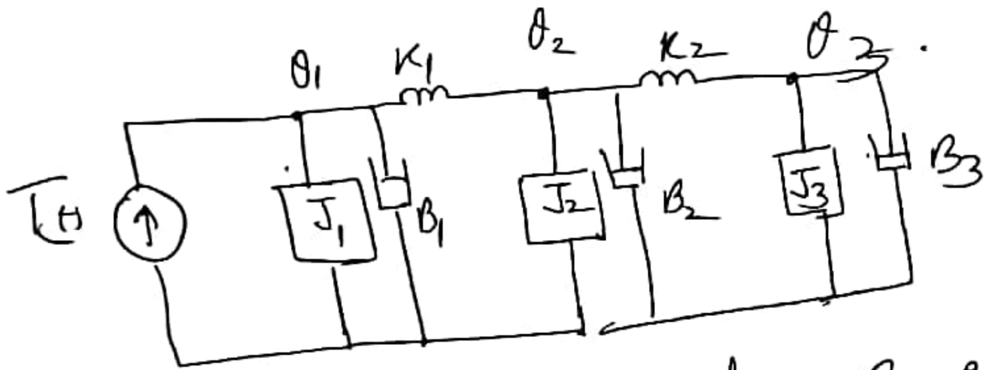
Thus from (1) & (2) we can have the electrical equivalent ckt as:



//.

Q2(c): Draw the electrical network based on torque - current analogy and write the performance equation for the mechanical system shown below:





We have the Torque equation at displacement θ_1, θ_2 & θ_3 as:

a. $$\boxed{T(t) = J_1 \frac{d\theta_1^2}{dt} + B_1 \frac{d\theta_1}{dt} + K_1 \cancel{\frac{d}{dt}} (\theta_1 - \theta_2)} \cancel{+ J_2 \frac{d^2\theta_2}{dt^2}}$$

$$\cancel{+ B_2 \frac{d\theta_2}{dt} + K_2(\theta_2 - \theta_3) + J_3 \frac{d^2\theta_3}{dt^2} + B_3 \frac{d\theta_3}{dt}}.$$

b. $$\boxed{K_1(\theta_1 - \theta_2) = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + K_2(\theta_2 - \theta_3)}$$

c. $$\boxed{K_2(\theta_2 - \theta_3) = J_3 \frac{d^2\theta_3}{dt^2} + B_3 \frac{d\theta_3}{dt}}$$

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Q.2.(c) Continued

Taking L.T. of a, b, & c resp.

$$T(s) = s^2 J_1 \theta_1(s) + s B_1 \theta_1(s) + K_1\left(\theta_1(s) - \theta_2(s)\right) \longrightarrow (A)$$

$$K_1\left(\theta_1(s) - \theta_2(s)\right) = s^2 J_2 \theta_2(s) + s \cdot B_2 \theta_2(s) + K_2\left(\theta_2(s) - \theta_3(s)\right) \longrightarrow (B)$$

$$K_2\left(\theta_2(s) - \theta_3(s)\right) = s^2 J_3 \theta_3(s) + s B_3 \theta_3(s) \longrightarrow (C)$$

using T–V equivalent we have

$T \to V$, $J \to L$, $B \to R$, $K \to \frac{1}{C}$, $\theta \to q$,

$s \cdot q(s) = I(s)$

$$V(s) = s^2 L_1 q_1(s) + s R_1 q_1(s) + \frac{1}{C}\left(q_1(s) - q_2(s)\right)$$

Putting $s \cdot q(s) = I(s) \Rightarrow$

$$V(s) = s L_1 I_1(s) + R_1 I_1(s) + \frac{1}{sC}\left(I_1(s) - I_2(s)\right) \longrightarrow (1)$$

$$\frac{1}{C_1}\left(q_1(s) - q_2(s)\right) = s^2 L_2 q_2(s) + s \cdot R_2 q_2(s) + K_2\left(q_1(s) - q_2(s)\right)$$

Putting $s \cdot q(s) = I(s)$ we have.

$$\frac{1}{sC_1}\left(I_1(s) - I_2(s)\right) = s L_2 I_2(s) + R_2 I_2(s) + \frac{1}{sC_2}\left(I_1(s) - I_2(s)\right) \longrightarrow (2)$$

~~Conclusion~~: Equations (A), (B), (C) represents the Mechanical ~~equations~~

~~while equation (1)~~,

IIly from eq (C) we have:

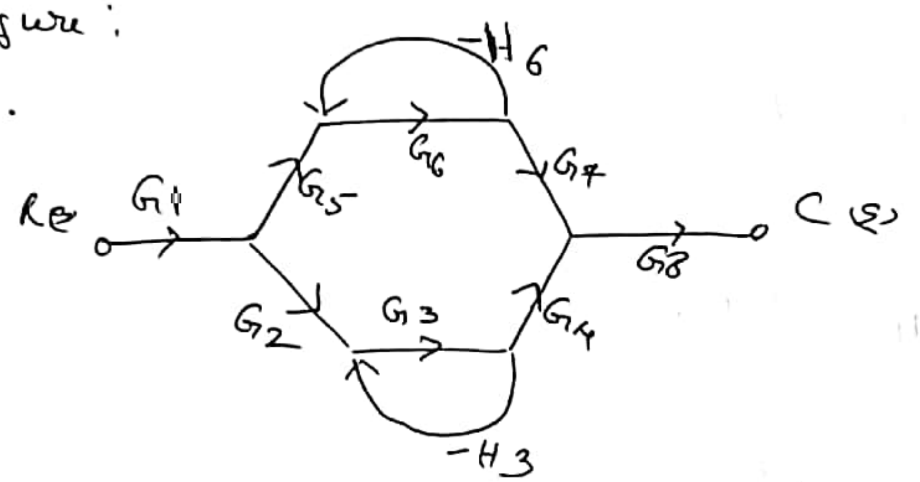
$$\frac{1}{C_2}\left[q_2(s) - q_3(s)\right] = s^2 L_3 q_3(s) + s R_3 q_3(s)$$

Putting $s \cdot q(s) = I(s)$ in above equation we get

$$\Rightarrow \frac{1}{sC_2}\left[I_2(s) - I_3(s)\right] = s L_3 I_3(s) - R_3 I_3(s) \longrightarrow (3)$$

Thus equation (A), (B), (C) represents the Mechanical performance equation while equations (1), (2) & (3) represents the Performance equation in electrical terms.

Q3. (a)]] Find $\frac{C(s)}{R(s)}$ by Masons gain formula for the given figure:



Sol: No of forward paths : 02.

$F_1 = G_1 G_5 G_6 G_7 G_8$ $\qquad$ $F_2 = G_1 G_2 G_3 G_4 G_8$

Step 2: Total no of single loops:

$L_{11} = -G_3 H_3$; $L_{21} = -G_6 H_6$

Step 3: Total no of two non-Touching Loops:

$L_{12} = L_{11} \times L_{21} = G_3 G_6 H_3 H_6$

Step 4: There are no 3-non-touching loops.

Step 5:

$$\Delta = 1 - (L_{11} + L_{21}) + L_{12}$$

$$= 1 + G_3 H_3 + G_6 H_6 + G_3 G_6 H_3 H_6$$

Step 6: find the values Δ_1 & Δ_2 ;

For F_1, loop (L_{21}) does not touch it.

$$\therefore \Delta_1 = 1 - (L_{21}) = 1 + G_3 H_3$$

For F_2, loop (L_{11}) does not touch.

$$\therefore \Delta_2 = 1 - (L_{11}) = 1 + G_6 H_6.$$

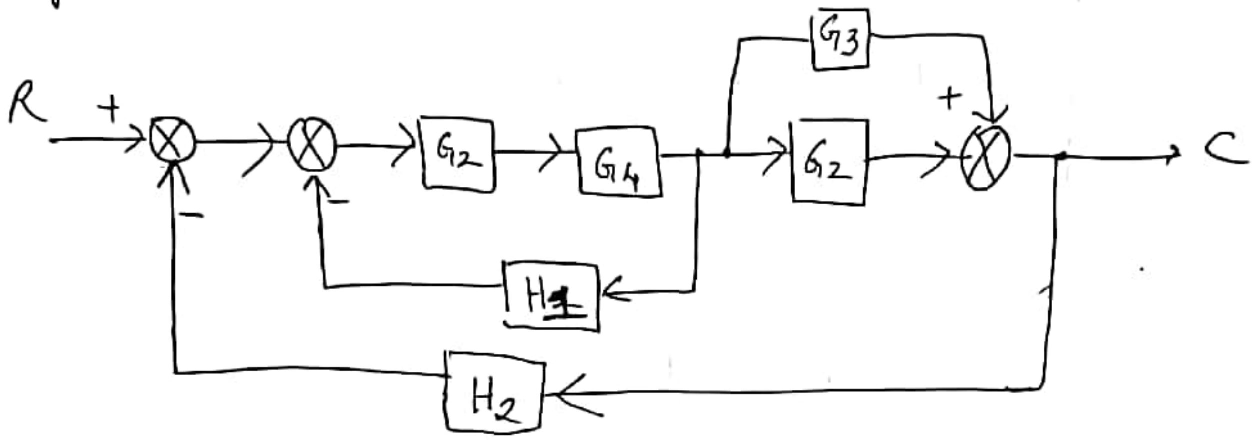
Step 7: Obtain T.F.:

$\Longrightarrow$

$$T.F. = \frac{C}{R} = \frac{F_1 \Delta_1 + F_2 \Delta_2}{\Delta}$$

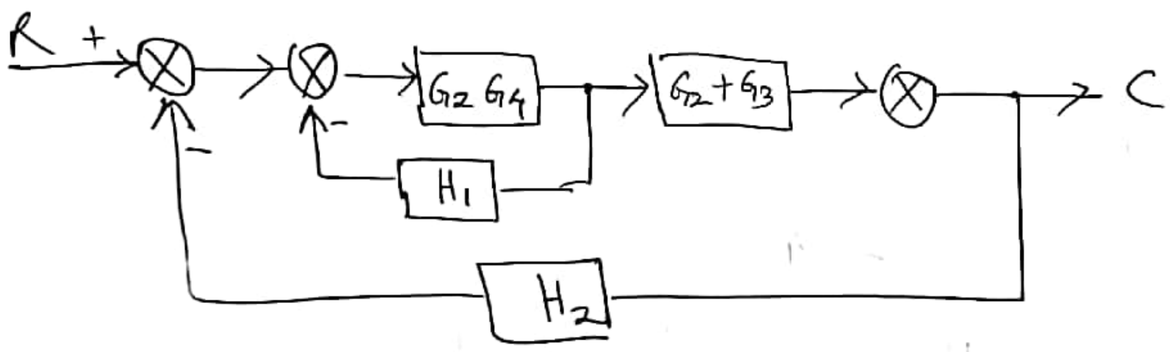
$$T.F = \frac{G_1 G_5 G_6 G_7 G_8 (1 + G_3 H_3) + G_1 G_2 G_3 G_4 G_8 (1 + G_6 H_6)}{1 + G_3 H_3 + G_6 H_6 + G_3 G_6 H_3 H_6}.$$

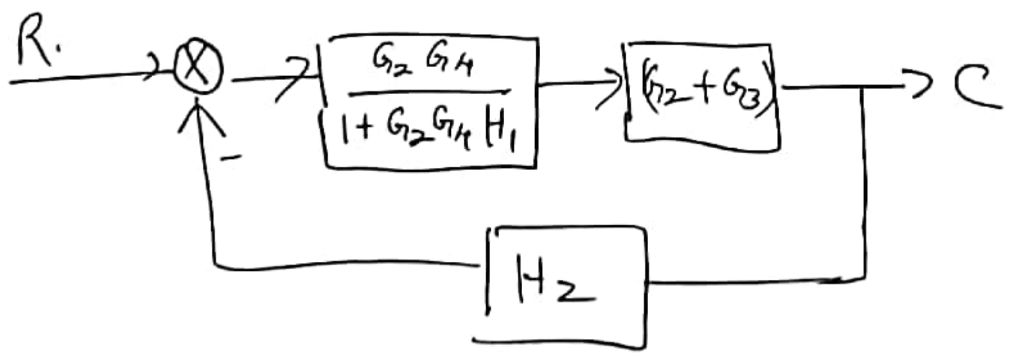
Q3.(b)]]: Determine the transfer function $\frac{C(s)}{R(s)}$ of the system shown :

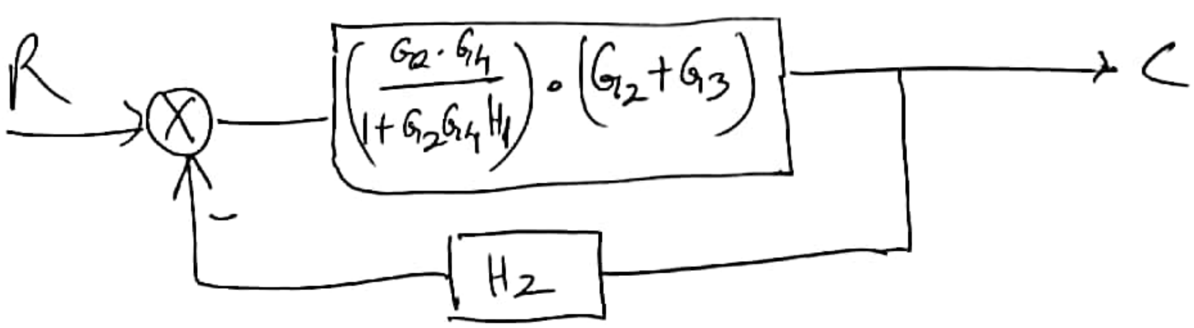


Solution : using Block diagram reduction technique we have.

$\Longrightarrow$







$$\Rightarrow$$

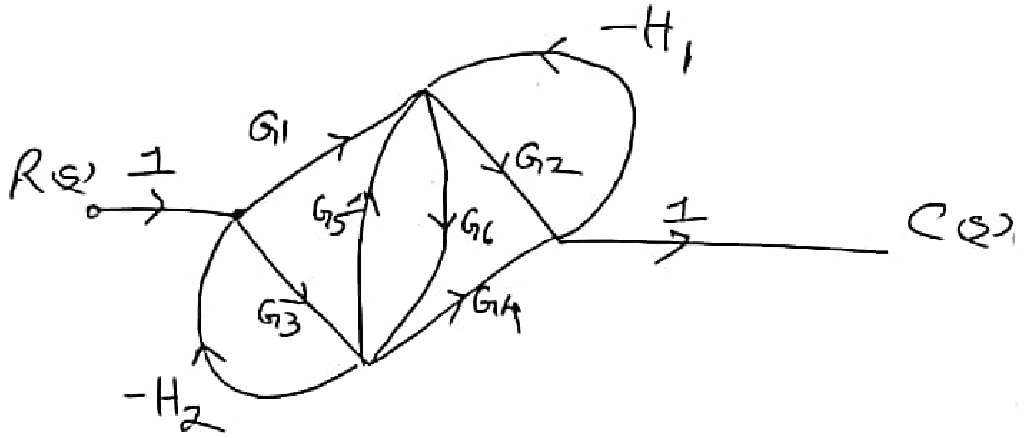
$$R \longrightarrow \boxed{\dfrac{\left(\dfrac{G_2 \cdot G_4}{1+G_2G_4H_1}\right)(G_2+G_3)}{1+\dfrac{(G_2 \cdot G_4)(G_2+G_3)}{(1+G_2G_4H_1)} \cdot H_2}} \longrightarrow C$$

$$R \longrightarrow \boxed{\dfrac{G_2G_4 \cdot (G_2+G_3)}{(1+G_2G_4H_1)+(G_2G_4)(G_2+G_3) \cdot H_2}} \longrightarrow C$$

$\therefore$ the T.F. is

$$\boxed{\frac{C}{R} = \frac{G_2G_4 \cdot (G_2+G_3)}{(1+G_2G_4H_1) + G_2G_4(G_2+G_3) \cdot H_2}}$$

Q3. (c) For the signal flow graph shown below, find the transfer function using Mason's Gain formula.



$F_1 = G_1 G_2$ $\qquad L_{11} = -G_2 H_1$

$F_2 = G_3 G_4.$ $\qquad L_{21} = -G_3 H_2$

$F_3 = G_1 G_6 G_4$ $\qquad L_{31} = G_5 G_6$

$F_4 = G_3 G_5 G_2$ $\qquad L_{41} = G_4 H_1 G_6 = -G_4 G_6 H_1$

$L_{51} = -H_2 G_1 G_6 = -G_1 G_6 H_2.$

$L_{12} = G_2 H_1 G_3 H_2 = G_2 G_3 H_1 H_2.$

There are no 3-non touching Loops.

$$T.F. = \frac{C(s)}{R(s)} = \frac{\sum_i F_i \cdot \Delta_i}{\Delta}$$

Ans

$$\Delta = 1 - (L_{11} + L_{21} + L_{31} + L_{41} + L_{51}) + (L_{12})$$

$$\Delta = 1 - (-G_2 H_1 - G_3 H_2 + G_5 G_6 - G_4 G_6 H_1 - G_1 G_6 H_2)$$

$$\Delta = 1 + G_2 H_1 + G_3 H_2 - G_5 G_6 + G_4 G_6 H_1 + G G_6 H_2$$

$$\Rightarrow \Delta = 1 + G_2H_1 + G_3H_2 - G_5G_6 + G_4G_6H_1 + G_1G_6H_2 .$$

$\therefore$ ~~$\Delta_1 =$~~

1) For F_1 $(G_1 G_2)$

$$\Delta_1 = 1 \cancel{+ G_3H_2}$$

2) For F_2 $(G_3 G_4)$

$$\Delta_2 = 1$$

3) For F_3 $(G_1 G_6 G_4)$

$$\Delta_3 = 1$$

4) For F_4 $(G_3 G_5 G_2)$

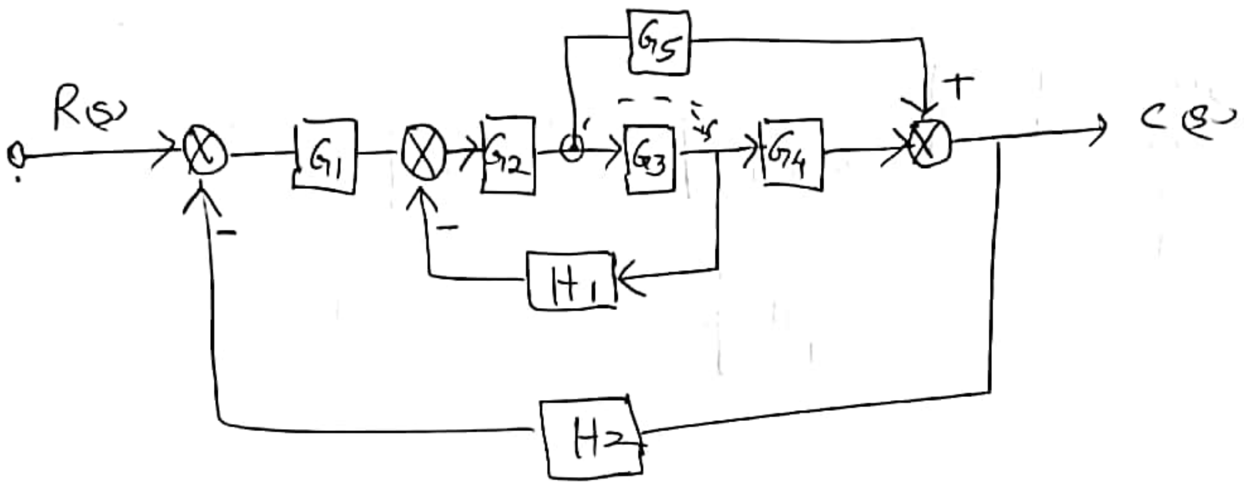
$$\Delta_4 = 1 .$$

$$\therefore T.F = \frac{\sum_i F_i \Delta_i}{\Delta} = \frac{G_1G_2 + G_3G_4 + G_1G_6G_4 \cancel{+G_3} + G_3G_5G_2}{1 + G_2H_1 + G_3H_2 - G_5G_6 + G_4G_6H_1 + G_1G_6H_2}$$

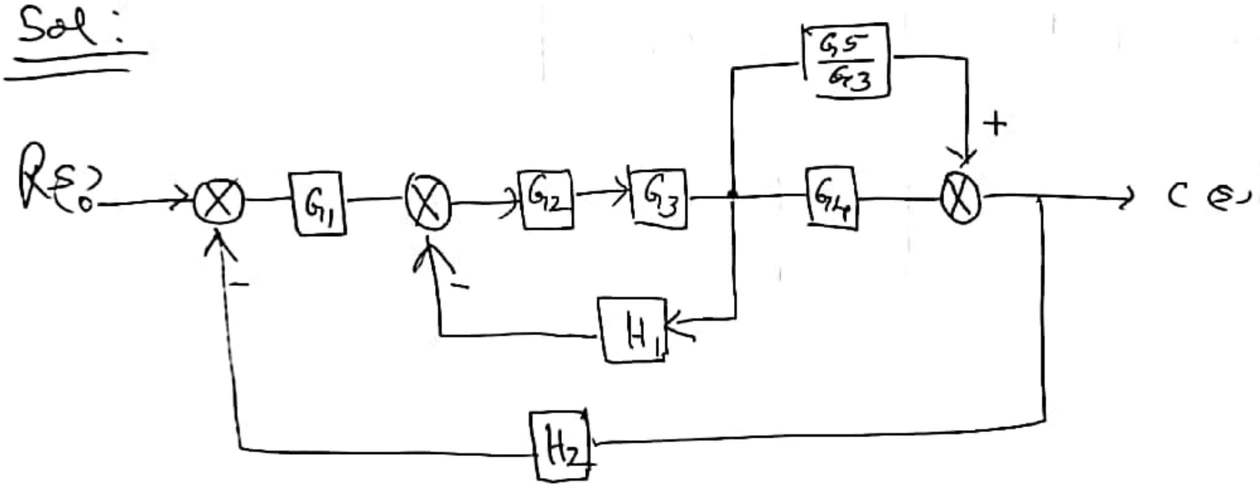
$$\boxed{T.F = \frac{G_1G_2 + G_3G_4 + G_1G_6G_4 + G_3G_5G_2}{1 + G_2H_1 + G_3H_2 - G_5G_6 + G_4G_6H_1 + G_1G_6H_2}}$$

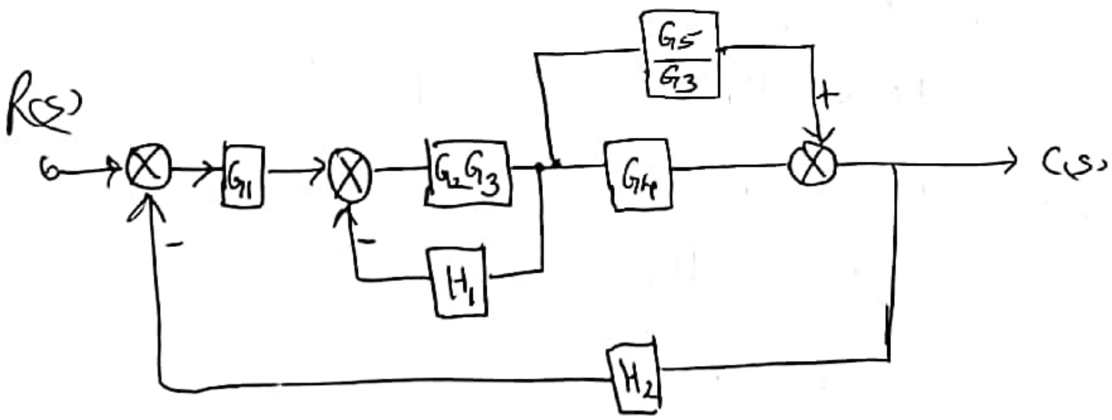
//.

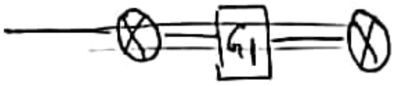
Q4.(a): Reduce the block diagram to its canonical form and obtain C(s)/R(s) of the system given below.

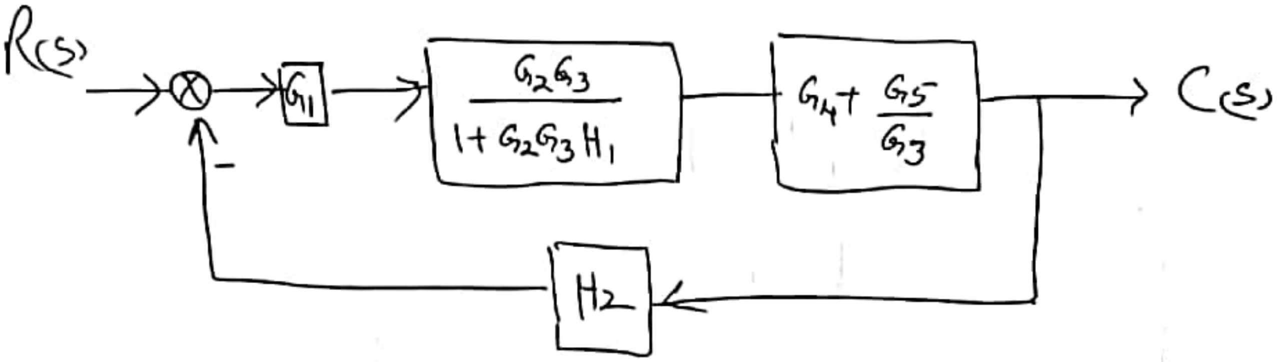


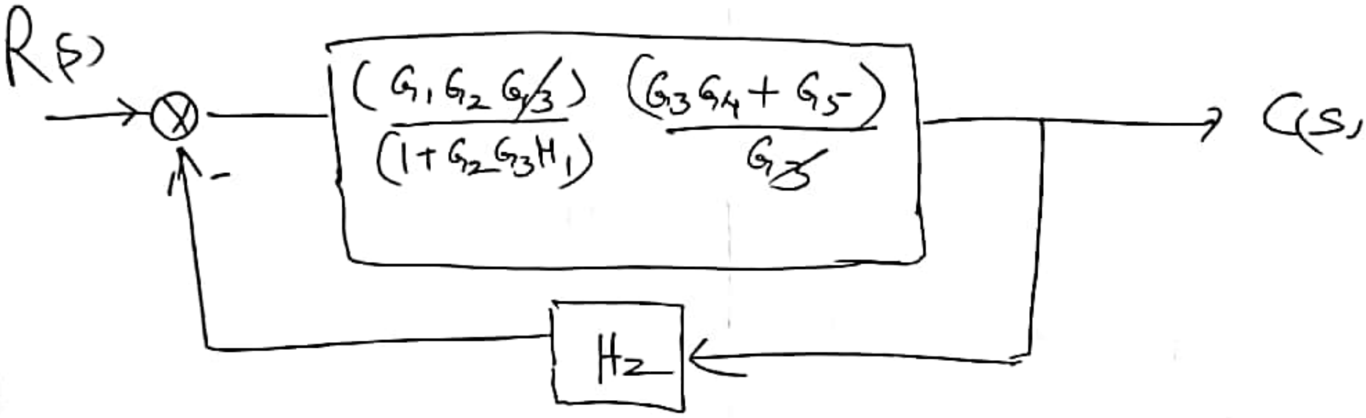
Sol:

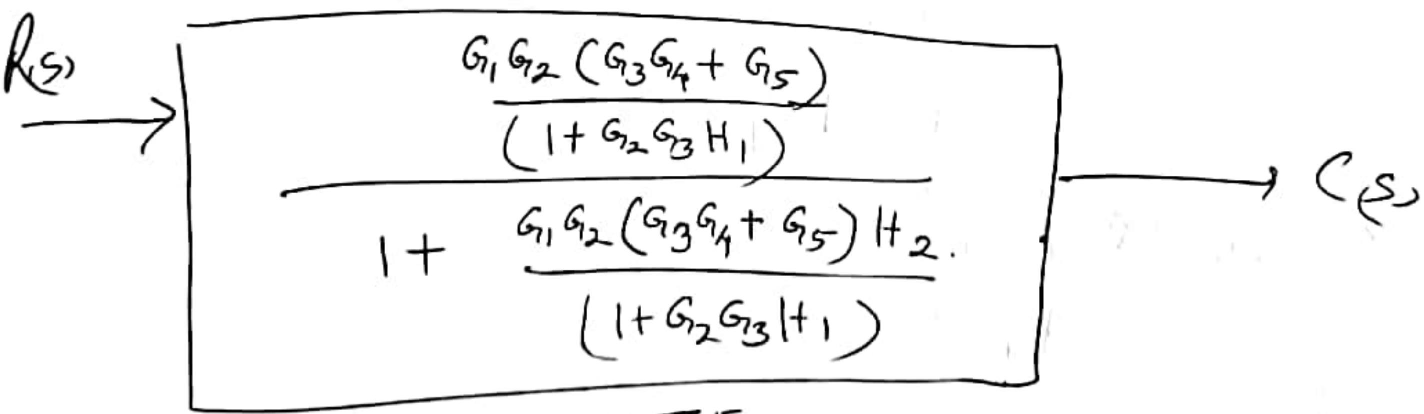




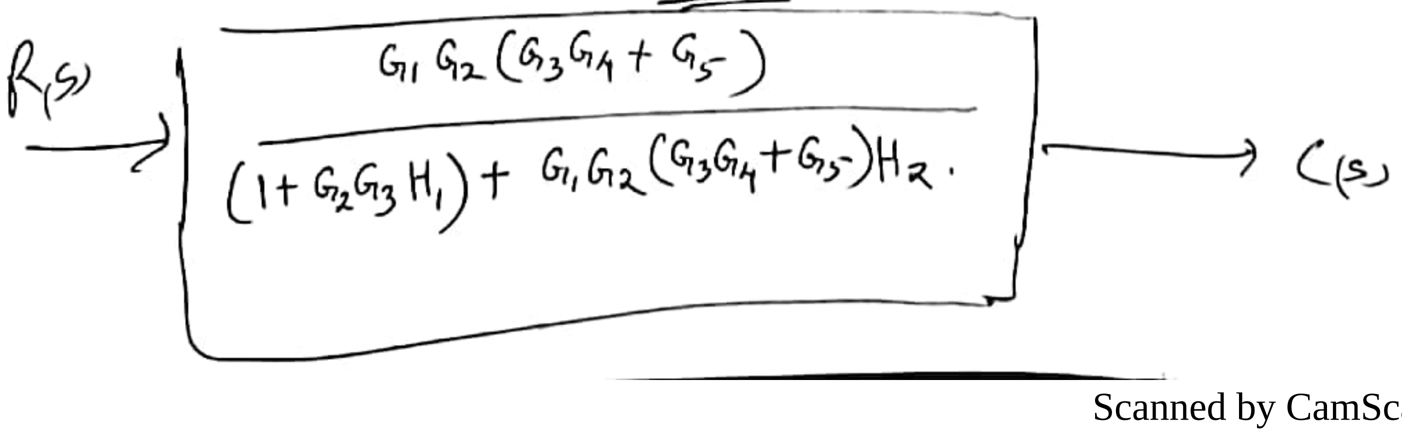

G1


R(s)
G1
G2G3 / 1+G2G3H1
G4+ G5/G3
C(s)
-
H2

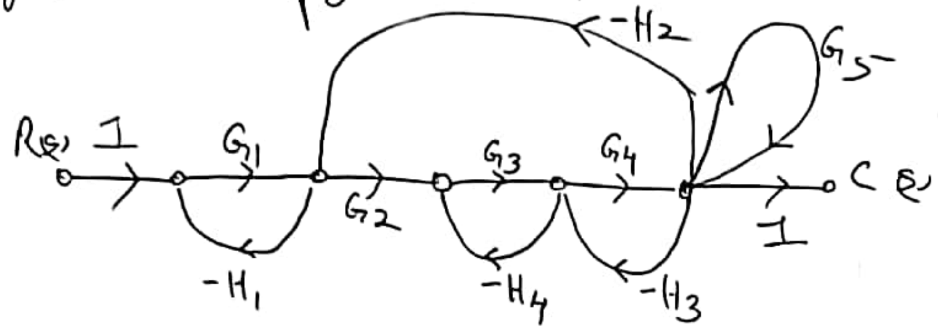

R(s)
(G1G2G3) / (1+G2G3H1)   (G3G4+G5) / G3
C(s)
-
H2


R(s)
[G1G2(G3G4+G5) / (1+G2G3H1)] / [1+ G1G2(G3G4+G5)H2 / (1+G2G3H1)]
C(s)

T.F.


R(s)
G1G2(G3G4+G5) / (1+G2G3H1)+ G1G2(G3G4+G5)H2
C(s)

Q4.(b):→ Obtain the transfer function of the signal flow graph shown in figure below:



Sol: No of forward paths = 1.

$$T.F = \frac{F_1 \Delta_1}{\Delta}$$

$F_1 = G_1 G_2 G_3 G_4$.

By Mason's gain formula.

Individual feedback loops:

1) $L_{11} = -G_1 H_1$

2) $L_{21} = -G_3 H_4$

3) $L_{31} = -G_4 H_3$

4) $L_{41} = -G_2 G_3 G_4 H_2$

5) $L_{51} = G_5$.

Two Non-Touching Loops:

1) $L_{12} = G_1 G_3 H_1 H_4$

2) $L_{22} = G_1 G_4 H_1 H_3$

3) $L_{32} = -G_1 H_1 G_5$

4) $L_{42} = -G_3 G_5 H_4$

Three Non-Touching Loops.

$L_{13} = G_1 G_3 G_5 H_1 H_4$

$$\therefore \Delta = 1 - \left[L_{11} + L_{21} + L_{31} + L_{41} + L_{51}\right] + \left[L_{12} + L_{22} + L_{32} + L_{42}\right]$$

$$- L_{13}.$$

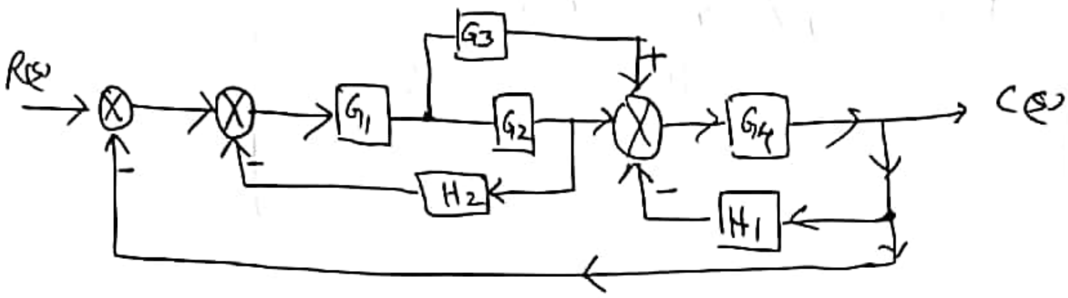
$$\Delta_1 = 1.$$

$$\Rightarrow \Delta = 1 + G_1H_1 + G_3H_4 + G_4H_3 + G_2G_3G_4H_2 + G_5 + G_1G_3H_1H_4$$
$$+ G_1G_4H_1H_3 - G_1G_5H_1 - G_3G_5H_4 - G_1G_3G_5H_1H_4.$$

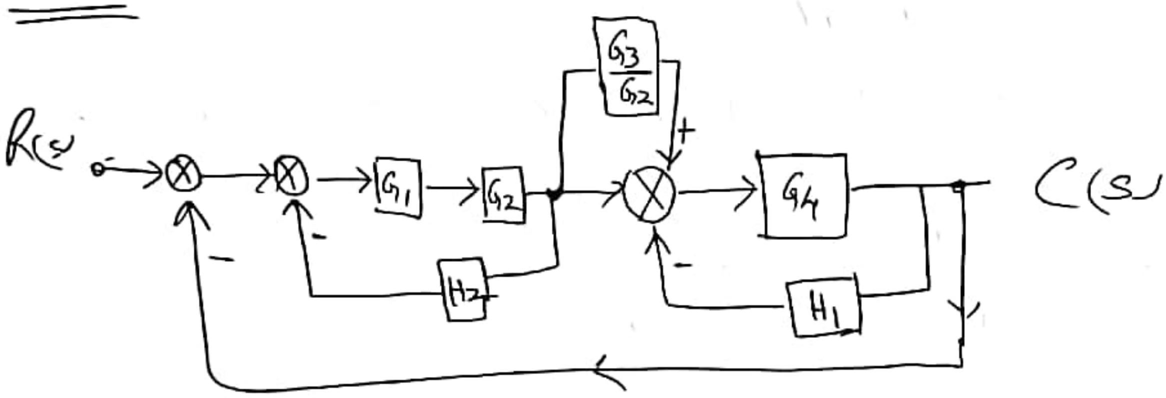
$$\Rightarrow \frac{C(s)}{R(s)} = \frac{F_1\Delta_1}{\Delta} =$$

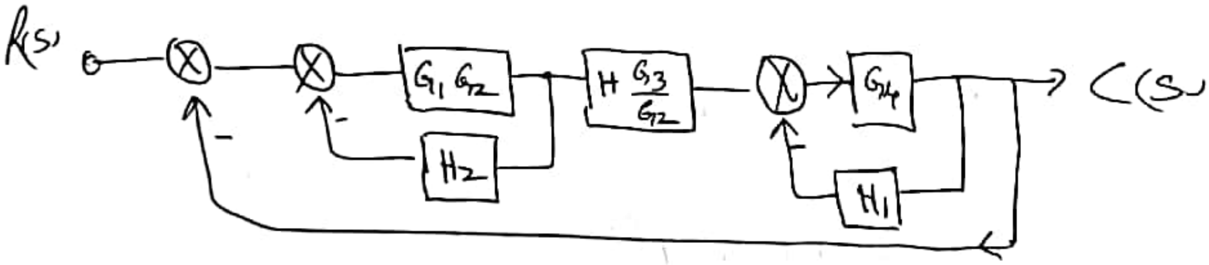
$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{\begin{array}{l}1 + G_1H_1 + G_3H_4 + G_4H_3 + G_2G_3G_4H_2 + G_5 + G_1G_3H_1H_4 \\ \quad + G_1G_4H_1H_3 - G_1G_5H_1 - G_3G_5H_4 - G_1G_3G_5H_1H_4\end{array}}}$$

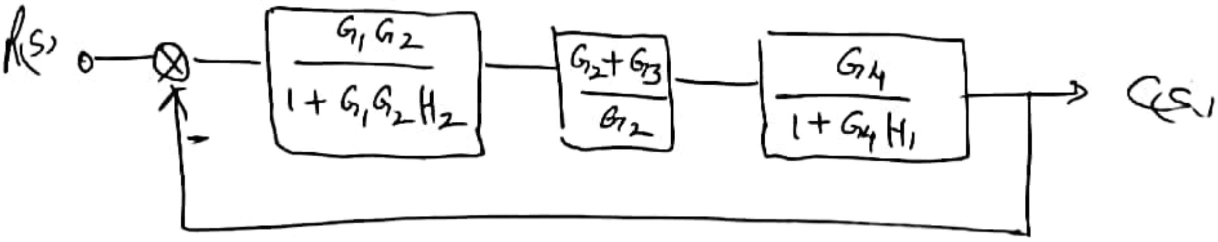
Q4.(c.):→ Reduce the block diagram of fig below to its simple form and obtain $C(s)/R(s)$.:

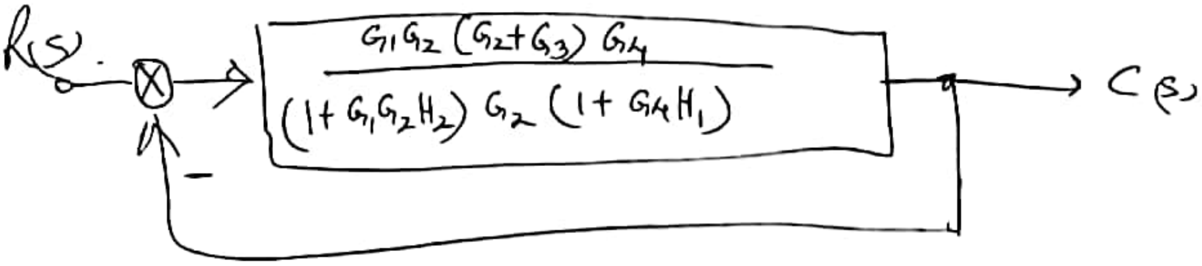


Sol:









$$R(s) \longrightarrow \boxed{\dfrac{\dfrac{G_1 \cancel{G_2} G_4 (G_2 + G_3)}{(1+G_1G_2H_2)(1+G_4H_1)\cancel{G_2}}}{1 + \dfrac{G_1 \cancel{G_2} G_4 (G_2 + G_3)}{(1+G_1G_2H_2)(1+G_4H_1)\cancel{G_2}}}} \longrightarrow C(s)$$

$$R(s) \longrightarrow \boxed{\dfrac{G_1 G_4 (G_2 + G_3)}{(1+G_1G_2H_2)(1+G_4H_1) + G_1G_4(G_2+G_3)}} \longrightarrow C(s)$$

$$\boxed{\therefore T.F = \frac{C(s)}{R(s)} = \frac{G_1 G_4 (G_2 + G_3)}{(1+G_1G_2H_2)(1+G_4H_1) + G_1G_4(G_2+G_3)}}$$

Some of the standard test signals that are used are :

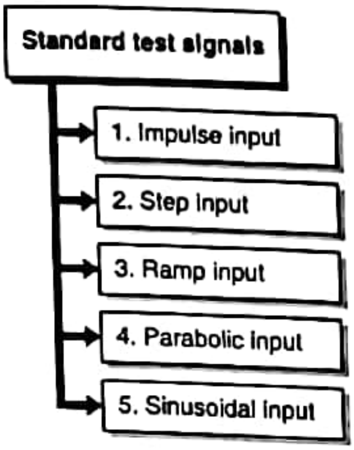


Fig. C5.1 : Standard test signals

1. Impulse input

Impulse represents a sudden change in input. An impulse is infinite at t = 0 and zero everywhere else. The area under the curve is 1. A unit impulse has magnitude 1 at t = 0

$$r(t) = \delta(t) = 1 ; \quad t = 0$$
$$= 0 ; \quad t \neq 0$$

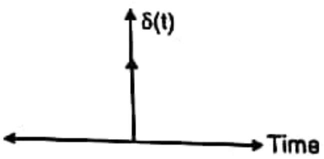


Fig. 5.2.1

In the Laplace domain we have

$$L[r(t)] = L[\delta(t)] = 1$$

Impulse inputs are used to derive a mathematical model of the system.

2. Step input

A step input represents a constant command such as position. The input given to an elevator is a step input. Another example of a step input is setting the temperature of an air conditioner.

A step signal is given by the formula.

$$r(t) = u(t) = A ; \quad t \geq 0$$
$$= 0 ; \quad \text{otherwise}$$

If A = 1, it is called a unit step.

In the Laplace domain, we have

$$L[r(t)] = R(s) = \frac{A}{s}$$

In case of a unit step, we get $L[r(t)] = R(s) = \frac{1}{s}$

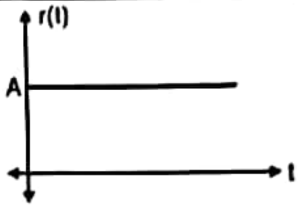


Fig. 5.2.2

3. Ramp input

The ramp input represents a linearly increasing input command. It is given by the formula.

$$r(t) = At \quad t \geq 0 \text{ ; Here A is the slope.}$$
$$= 0 \quad t < 0$$

If A = 1, it is called a unit ramp.

In the Laplace domain we have,

$$L[r(t)] = R(s) = \frac{A}{s^2}$$

In case of unit ramp, we have $R(s) = \frac{1}{s^2}$

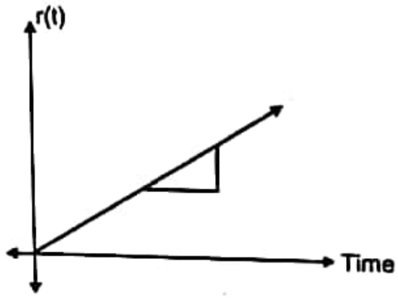


Fig. 5.2.3

Systems are subjected to Ramp inputs when we need to study the system behaviour for linearly increasing functions like velocity.

4. Parabolic input

Rate of change of velocity is acceleration. Acceleration is a parabolic function. It is given by the formula,

$$r(t) = \frac{A}{2}t^2 ; \quad t \geq 0$$
$$= 0 ; \quad t < 0$$

If A = 1, it is called a unit parabola. In the Laplace domain we have,

$$L[r(t)] = R(s) = \frac{A}{s^3}$$

In case of unit parabola, we have

$$R(s) = \frac{1}{s^3}$$

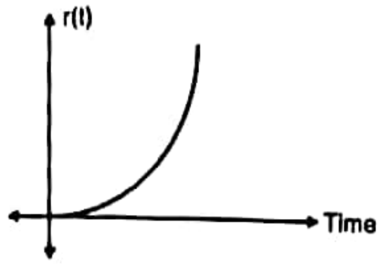


Fig. 5.2.4

5. Sinusoidal Input

- There are applications where we need to subject the control system to sinusoidal inputs of varying frequencies and study the system frequency response. A typical example is when we want to check the quality of speakers of a music system. In this we play different frequencies (sinusoidal waves) and study their attenuations.
- It is given by the equation, $r(t) = A \sin(\omega t)$
- This chapter involves using inputs that are a function of time and hence sinusoidal inputs which are functions of frequencies will not be discussed here. We will discuss it in the chapter Frequency Response Analysis. Hence the inputs required for time response analysis are,

Input	Laplace domain
Impulse	1
Step	A/s
Ramp	A/s^2
Parabola	A/s^3

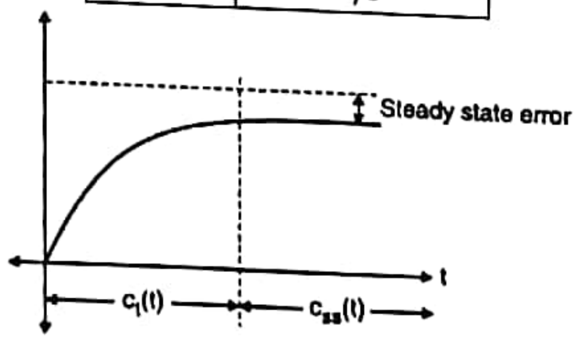


Fig. 5.2.5

We had stated in the earlier section that the entire time response is made up of two parts viz; transient response $c_t(t)$ and steady state response $c_{ss}(t)$.

i.e. $c(t) = c_t(t) + c_{ss}(t)$

We shall discuss the steady state response first as it is easier and then follow it up by transient response.

Q5.(b): Find K_p, K_v, K_a and steady state error for a system with Open Loop T.F. $G(s) \cdot H(s) = \frac{10(s+2)(s+3)}{s(s+1)(s+4)(s+5)}$

where $r(t) = 3 + t + t^2$.

Sol:

Static error coefficients are.

(1) $K_p = \lim_{s\to 0} G(s) H(s) = \lim_{s\to 0} \frac{10(s+2)(s+3)}{s(s+1)(s+4)(s+5)}$

$\Rightarrow K_p = \infty$

(2) $K_v = \lim_{s\to 0} s \cdot G(s) \cdot H(s) = \lim_{s\to 0} \frac{\cancel{s}\, 10(s+2)(s+3)}{\cancel{s}(s+1)(s+4)(s+5)}$

$= \frac{10\times 2\times 3}{1\times 4\times 5} = \frac{3\cancel{0}}{2\cancel{0}} = 1.5$

$\Rightarrow K_v = 1.5$

(3) $K_a = \lim_{s\to 0} s^2 G(s) H(s) = \lim_{s\to 0} \frac{s^{\cancel{2}} \times 10(s+2)(s+3)}{\cancel{s}(s+1)(s+4)(s+5)}$

$\Rightarrow K_a = 0$

(4) Steady state error $= e_{ss} = \lim_{s\to 0} \frac{s \times R(s)}{1 + G(s) \cdot H(s)}$

$\Rightarrow$ ~~$e_{ss} = \lim_{s\to 0} s \times$~~ $r(t) = 3 + t + t^2 = 3 + t + \frac{2t^2}{2}$

$\therefore R(s) = 3 \times \frac{1}{s} + \frac{1}{s^2} + \frac{2}{\cancel{2}} \times \cancel{2} \times \frac{1}{s^3}$

$R(s) = \left[\frac{3}{s} + \frac{1}{s^2} + \frac{2}{s^3} \right]$

$\Rightarrow$

$$R(s) = \frac{3}{s} + \frac{1}{s^2} + \frac{2}{s^3}$$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s \times R(s)}{1 + G(s) \cdot H(s)}$$

$$= \lim_{s \to 0} \frac{s \times \left[\frac{3}{s} + \frac{1}{s^2} + \frac{2}{s^3}\right]}{1 + G(s) \cdot H(s)} = \lim_{s \to 0} \frac{\cancel{s} \times \left[\frac{3s^2 + s + 2}{s^{\cancel{3}2}}\right]}{1 + G(s) H(s)}$$

$$= \lim_{s \to 0} \frac{\left(\frac{3s^2 + s + 2}{s^2}\right)}{1 + G(s) H(s)}$$

$$= \lim_{s \to 0} \frac{\left(\frac{3s^2 + s + 2}{s^2}\right)}{1 + \frac{10(s+2)(s+3)}{s(s+1)(s+4)(s+5)}} = \lim_{s \to 0} \frac{\frac{3s^2 + s + 2}{s^{\cancel{2}}}}{\frac{s(s+1)(s+4)(s+5) + 10(s+2)(s+3)}{\cancel{s}(s+1)(s+4)(s+5)}}$$

$$= \lim_{s \to 0} \frac{(3s^2 + s + 2)}{s} \times \frac{(s+1)(s+4)(s+5)}{\left[s(s+1)(s+4)(s+5) + 10(s+2)(s+3)\right]}$$

$$= \lim_{s \to 0} \frac{(3s^2 + s + 2)}{s} \times \frac{(s+1)(s+4)(s+5)}{\left[s(s+1)(s+4)(s+5) + 10(s+2)(s+3)\right]}$$

$$\boxed{e_{ss} = \infty}$$

//

Q5.(c): The open loop T.F. of a servo system with unity feedback is given as $G(s) = \frac{10}{s(0.1s+1)}$. Find out static error constants and obtain steady state error when an input $r(t) = A_0 + A_1 t + \frac{A_2}{2} t^2$ is applied.

Sol:

$$O.L.T.F. = \frac{10}{s(0.1s+1)} := G(s).$$

Error constants are:

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{10}{s(0.1s+1)}$$

a) $\boxed{K_p = \infty}$

b) $$K_v = \lim_{s\to 0} s.G(s).H(s) = \lim_{s\to 0} s \times \frac{10}{s(0.1s+1)} = \lim_{s\to 0} \frac{10}{(0.1s+1)}$$

b) $\boxed{K_v = 10}$

c) $$K_a = \lim_{s\to 0} s^2 G(s)H(s) = \lim_{s\to 0} s^2 \times \frac{10}{\cancel{s}(0.1s+1)}$$

c) $\boxed{K_a = 0}$

Now i/p: $r(t) = A_0 + A_1 t + \frac{A_2}{2} t^2$

$$\therefore R(s) = L\{r(t)\} = \left[\frac{A_0}{s} + \frac{A_1}{s^2} + \frac{A_2}{s^3}\right]$$

$$\therefore e_{ss} = \lim_{s\to 0} \frac{s \times R(s)}{1 + G(s)H(s)}.$$

$$\Rightarrow e_{ss} = \lim_{s \to 0} \frac{s \times \left[\frac{A_0}{s} + \frac{A_1}{s^2} + \frac{A_2}{s^3} \right]}{1 + \frac{10}{s(0.1s+1)}}$$

$$= \lim_{s \to 0} \frac{\cancel{s} \times \left[\frac{A_0 \cdot s^2 + A_1 \cdot s + A_2}{s^{\cancel{3}2}} \right]}{1 + \frac{10}{s(0.1s+1)}}$$

$$= \lim_{s \to 0} \frac{\frac{(A_0 \cdot s^2 + A_1 \cdot s + A_2)}{s^2}}{\frac{s(0.1s+1) + 10}{\cancel{s}(0.1s+1)}}$$

$$= \lim_{s \to 0} \frac{(A_0 s^2 + A_1 \cdot s + A_2)}{s} \times \frac{(0.1s + 1)}{\{s(0.1s+1) + 10\}}$$

$$= \infty$$

$$\Rightarrow \boxed{e_{ss} = \infty}$$

Q6. a] : For a unity feedback control system with

$G(s) = \frac{64}{s(s+9.6)}$, write the output response

to a unit step i/p. Determine

(1) The response at t = 0.1 sec

(2) Maximum value of response and the time at which it occurs.

(3) Settling time.

Sol:

$G(s) = \frac{64}{s(s+9.6)}$, unity f/b is given.

$$\therefore T(s) = \frac{G(s)}{1+G(s)} = \frac{\frac{64}{s(s+9.6)}}{1+\frac{64}{s(s+9.6)}} = \frac{64}{s(s+9.6)+64}$$

$$T(s) = \frac{64}{s(s+9.6)+64}.$$

$$T(s) = \frac{64}{s^2 + 9.6s + 64} = \frac{C(s)}{R(s)} \longrightarrow (I)$$

We compare the equation with the standard 2nd order equation i.e. $\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

Comparing the denominator of eq (I) with the denominator of standard expression we get

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 9.6s + 64.$$

$$\Rightarrow \omega_n^2 = 64$$

$$\Rightarrow \boxed{\omega_n = 8 \text{ rad/sec}}$$

|||ly $2\xi\omega_n = 9.6$

$$\Rightarrow 2\xi \times 8 = 9.6$$

$$\Rightarrow \xi = \frac{9.6}{2 \times 8} = 0.6$$

$$\boxed{\therefore \xi = 0.6}$$

Q6.(a) – Continued Page 3

Maximum Overshoot $= M_p$:

$$\% M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \times 100$$

$$= e^{\frac{-0.6\times\pi}{\sqrt{1-0.6^2}}} \times 100$$

$$= 9.47\%$$

Peak Time T_p :

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{\pi}{8\sqrt{1-0.6^2}}$$

$$\boxed{T_p = 0.49087 \text{ Sec}}$$

Transient response for a unit step i/p :

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \times \sin(\omega_d t + \theta)$$

Q6(a) - Continued Page-4

$\Rightarrow \quad \omega_d = \omega_n\sqrt{1-\xi^2} = 6.4 \text{ rad/sec}$

$$\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]$$

$$\theta = 53.13^\circ$$

$$\therefore \quad C(t) = 1 - e^{\frac{-0.6\times 8\times t}{\sqrt{1-0.6^2}}} \times \sin(6.4t + 53.13^\circ)$$

$$\boxed{C(t) = 1 - e^{-6t} \times \sin(6.4t + 53.13^\circ)}$$

Settling time $T_{(s)}$:

$$T_s = \frac{4}{\xi\omega_n} \text{ sec } [\text{for a } 2\% \text{ tolerance band}]$$

$$= \frac{4}{0.6\times 8} = \frac{1}{1.2} = 0.833 \text{ sec}$$

$$\Rightarrow \boxed{T_s = 0.833 \text{ sec}}$$

Response at time $t = 0.1$ sec : $\rightarrow$

$$C(t)\Big|_{t=0.1\,sec} = 1 - e^{-6\times 0.1}\left[\sin(.64 + 53.13^\circ)\right]$$

$$= 1 - e^{-0.6} \times 0.806652 = 1 - 0.4427 = \underline{\underline{0.55729}}$$

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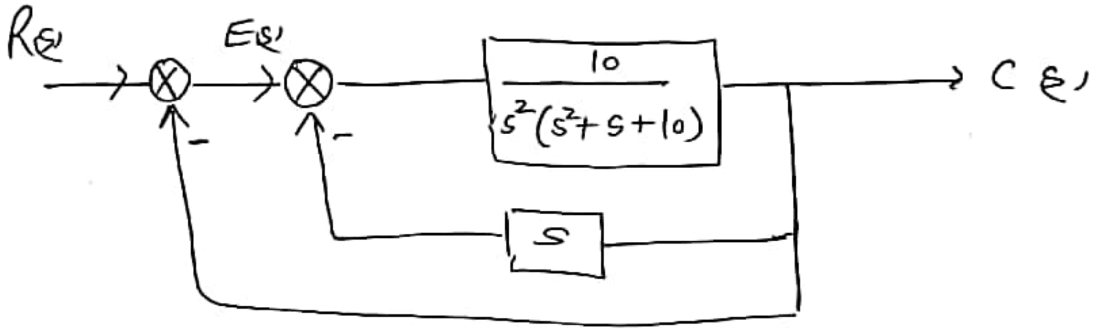
Q6.(b)]] For the system given below

1) Identify the type of $C(s)/E(s)$.

2) Find the values of K_p, K_v, K_a.

3) If $r(t) = 10\,u(t)$, find steady state value of the output.



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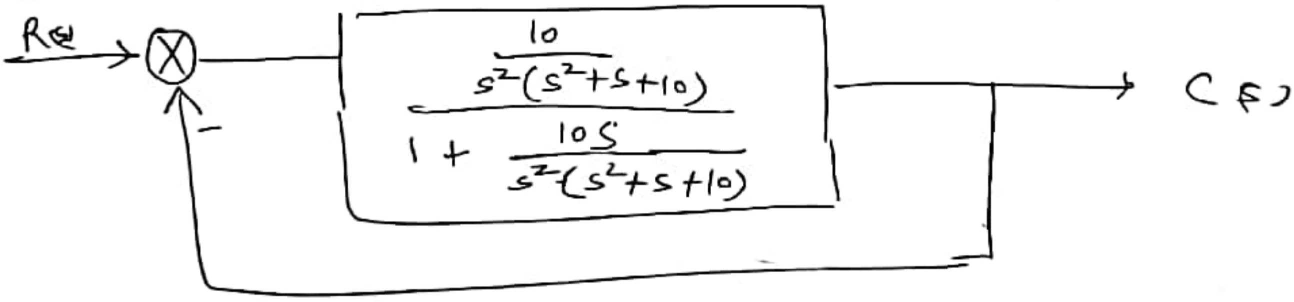

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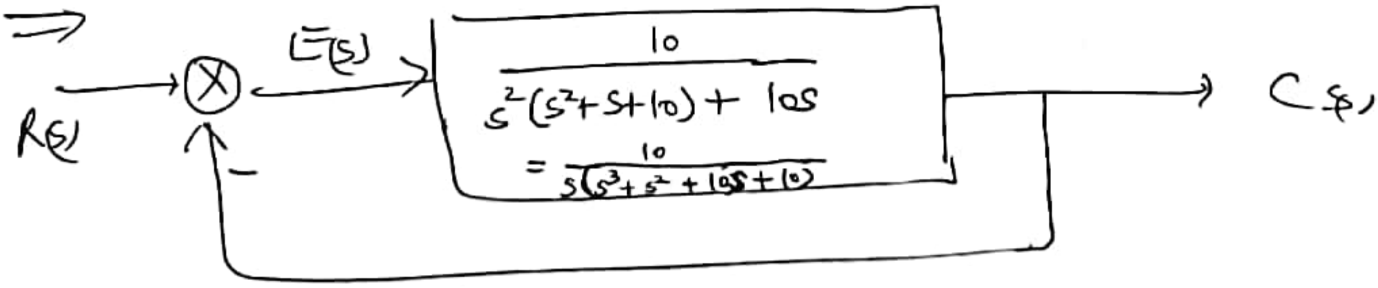


For $C(s)/E(s)$, we have $G(s)H(s) = \frac{10\,\cancel{s}}{s^{\cancel{2}}(s^2+s+10)}$

$$\Longrightarrow \quad G(s)H(s) = \frac{10}{s(s^2+s+10)}.$$

$\therefore$ the Type of System for $\frac{C(s)}{E(s)}$ is <u>Type-1</u>, since there is one pole located at <u>$s=0$.</u>

<u>Static Error Constants:</u>

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{10}{s(s^3+s^2+10s+10)}$$

a) $\boxed{K_p = \infty}$

b) $K_v = \lim_{s\to 0} s\cdot G(s)H(s) = \lim_{s\to 0} \cancel{s} \times \frac{10}{\cancel{s}(s^3+s^2+10s+10)}$

$$= \frac{10}{10} = 1$$

$\boxed{K_v = 1}$

c) $K_a = \lim_{s\to 0} s^2 G(s)H(s) = \lim_{s\to 0} s^{\cancel{2}} \times \frac{10}{\cancel{s}(s^3+s^2+10s+10)}$

$$= \lim_{s\to 0} s \times \frac{10}{(s^3+s^2+10s+10)} = 0$$

$\Longrightarrow \boxed{K_a = 0}.$

$e_{ss} = \lim_{s \to 0}$

Q6. (b) — Continued Page - 2.

We know that the system is a Type-1 system & hence

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+\infty} = 0$$

$\therefore$ for a step i/p $r(t) = 10\,u(t)$

$$\boxed{C(t) = 10\,\mu(t)}$$

Q-7(a):→ Find the no of roots with positive real part, zero real part, and negative real part for a system

$$s^6 + 4s^5 + 3s^4 - 16s^2 - 64s - 48 = 0$$

s^6	1	3	-16	-48
s^5	4	0	-64	0
s^4	3	0	-48	0
s^3	0	0	0	0
s^2				
s^1				
s^0				

s^3 row is zero.

$\therefore A(s) = 3s^4 - 48 = 0$

$\Rightarrow \frac{dA}{ds} = 12s^3$

⇒

s^6	1	3	-16	-48
s^5	4	0	-64	0
s^4	3	0	-48	0
s^3	12	0	0	0
s^2	$0(\varepsilon)$	-48	0	0
s^1	$\frac{576}{\varepsilon}$	0	0	
s^0	-48			

$$\lim_{\varepsilon \to 0} \left(\frac{576}{\varepsilon}\right) = +\infty$$

Only one sign change & hence one root in right-half of s-Plane.

Now get details about the roots represented by auxiliary equation:

Solve $A(s) = 3s^4 - 48 = 0$

$\Rightarrow 3s^4 = 48 \Rightarrow s^4 = 16$

$s^2 = 4$ i.e. $s = \pm 2$ & $s^2 = -4$ i.e. $s = \pm 2j$

Hence two roots $(\pm 2j)$ are on $j\omega$ axis & remaining in left-half of s-plane.

$\therefore$ Roots with +ve Real Part = 01 (One)
Roots with −ve Real Part = 03 (Three)

Roots with zero real part = Two (02).

Example 8.9.1 *For unity feedback system,*

$$G(s) = \frac{K}{s(1+0.4s)(1+0.25s)}$$ *, find range of values of K, marginal value of K and frequency of sustained oscillations.*

Solution : Characteristic equation, $1 + G(s)H(s) = 0$ and $H(s) = 1$

$$\therefore \quad 1+\frac{K}{s(1+0.4s)(1+0.25s)} = 0$$

$$s[1 + 0.65s + 0.1s^2] + K = 0$$

$$\therefore \quad 0.1s^3 + 0.65s^2 + s + K = 0$$

s^3	0.1	1	From s^0, $K > 0$
s^2	0.65	K	From s^1,
s^1	$\frac{0.65-0.1K}{0.65}$	0	$0.65 - 0.1K > 0$ $\therefore 0.65 > 0.1K$
s^0	K		$\therefore 6.5 > K$

$\therefore$ Range of values of K, $0 < K < 6.5$

The marginal value of 'K' is a value which makes any row other than s^0 as row of zeros.

$$\therefore \quad 0.65 - 0.1\, K_{mar} = 0$$

$$\therefore \quad \boxed{K_{mar} = 6.5}$$

To find frequency, find out roots of auxiliary equation at marginal value of 'K'.

$$A(s) = 0.65\, s^2 + K = 0$$

$$\therefore \quad 0.65\, s^2 + 6.5 = 0 \qquad \because \; K_{mar} = 6.5$$

$$s^2 = -10$$

$$\mathbf{s = \pm j\, 3.162}$$

Comparing with $s = \pm j\omega$, ω = Frequency of oscillations = **3.162 rad/sec**

9.3 Angle and Magnitude Condition

For a general closed loop system the characteristic equation is,

$$1 + G(s)H(s) = 0$$

i.e. $G(s)H(s) = -1$

As s-plane is complex we can write above equation as,

$$\boxed{G(s)H(s) = -1 + j0}$$

All s-values can be expressed as '$\sigma + j\omega$' i.e. G(s)H(s) term is also complex one. **So for any value of 's' if it has to be on the root locus, it must satisfy the above equation.**

As both sides of the above equations are in rectangular form, we can convert both sides in polar form and then we can equate angle and magnitude of both sides. This gives us two conditions of root locus called i) Magnitude condition and ii) Angle condition.

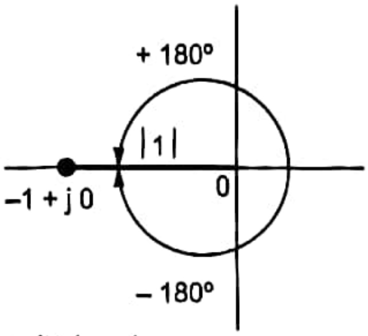
9.3.1 Angle Condition

$$G(s)H(s) = -1 + j0$$

Equating angles of both sides,

$$\angle G(s)H(s) = \pm(2q+1)\,180^\circ \quad q = 0, 1, 2 \,\ldots\ldots$$

Key Point *$-1 + j0 = 1\angle\pm 180^\circ$ but the point $-1 + j0$ is a point on negative real axis which can be traced as magnitude 1 at an angle $\pm 180^\circ$, $\pm 540^\circ$, $\pm 900^\circ$ ..... $\pm(2q+1)\,180^\circ$.*



Magnitude = 1
Angle = $\pm 180^\circ$, $\pm 540^\circ$, - - - - - $\pm(2q+1)\,180^\circ$

Fig. 9.3.1

∴ Angle condition can be stated as,

$\angle G(s)H(s)\,|$ for any value of 's' which is the root of equation $[1 + G(s)H(s) = 0]$ is

$$= \pm(2q+1)\,180^\circ \quad \text{where } q = 0, 1, 2 \,\ldots\ldots$$

$$= \text{Odd multiple of } 180^\circ$$

If any point in s-plane has to be on the root locus then it has to satisfy above angle condition. The angle of G(s)H(s) calculated at that point must be an odd multiple of $\pm 180^\circ$.

Key Point *Any point in s-plane which satisfies the angle condition has to be on the root locus of the corresponding system.*

9.3.2 Use of Angle Condition

As all the points on the root locus must satisfy the angle condition, we can use the angle condition to test any point in s-plane for its existence on the root locus of the given system. This can be explained by taking an example.

Example 9.3.1 *Consider the system with* $G(s)H(s) = \dfrac{K}{s(s+2)(s+4)}$. *Find whether* $s = -0.75$ *is on the root locus or not, using angle condition.*

Solution : Let us test whether s = – 0.75 is located on the root locus of above system i.e. whether s = – 0.75 is a root of the characteristic equation 1 + G(s)H(s) = 0 or not. Use Angle condition,

$$\angle G(s)H(s)\Big|_{\text{at point } s=-0.75} = \pm(2q+1)\,180° \qquad q = 0, 1, 2, \ldots..$$

Substituting s = – 0.75 in all the terms of G(s)H(s),

$$\angle G(s)H(s)\Big|_{\text{at } s=-0.75} = \frac{\angle K+j0}{\angle -0.75+j0 \cdot \angle\ 1.25+j0 \cdot \angle\ 3.25+j0}$$

Converting to polar form and considering angles, (**use calculator to obtain polar form from rectangular form** and consider angle.)

$$= \frac{0°}{180° \cdot 0° \cdot 0°} = -180°$$

That is ∠ G(s)H(s) = – 180° at s = – 0.75 which satisfies angle condition and we can conclude that point s = – 0.75 is on the root locus of the given system.

Let us test, s = – 1 + j4 for its existence on the root locus of the same system,

$$\angle G(s)H(s)\Big|_{\text{at } s=-1+j4} = \frac{\angle K+j0}{\angle -1+j4 \cdot \angle\ 1+j4 \cdot \angle\ 3+j4}$$

$$= \frac{0°}{104.03° \cdot 75.963° \cdot 53.13°} = -233.123°$$

$$\angle G(s)H(s)\Big|_{\text{at } s=-1+j4} = -233.123°$$

As this is not satisfying the angle condition, the point (–1 + j4) cannot be on the root locus of the given system.

Q8.(a) :→ Sketch the complete root locus and comment on the stability of the system

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)(s+3)}$$

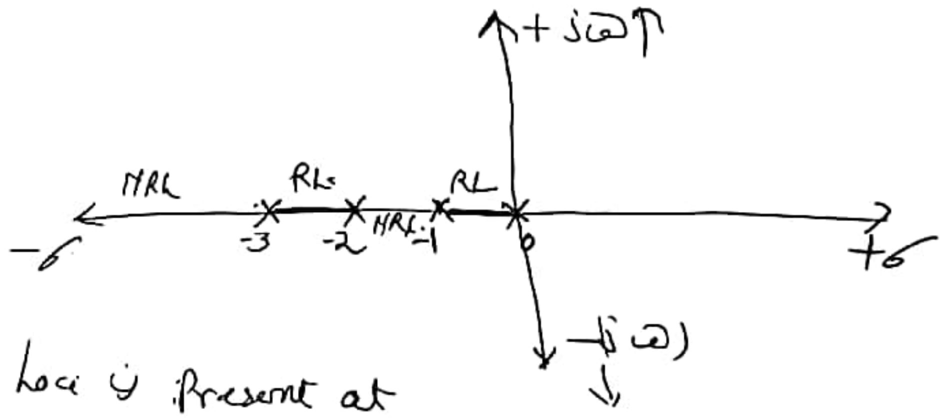
Sol:

Step 1: No of Loci ?

Here $p = 4$ & Zeros $(Z) = 0$

$\therefore$ No of loci ending at $\infty = p - Z = 4$.

Step 2: Draw the Poles & Zeros to scale:



Step 3: The real axis loci is present at

1) $-1 < \sigma < 0$ (2) $-3 < \sigma < -2$

Step 4: No of asymptotes $= p - Z = 4$.

Angle of asymptote $= \beta_x = \frac{(2x+1)180^\circ}{p - Z}$; $[x = 0, 1, 2, 3]$

$[\because x = 0, 1, \cdots (p - Z - 1)]$

$\therefore$ $\beta_0 = 45^\circ$, $\beta_1 = 135^\circ$

$\beta_2 = 225^\circ$, $\beta_3 = 315^\circ$.

Step 5: Centroid (σ_c)

$$\sigma_c = \frac{\sum \text{Real parts of Poles} - \sum \text{Real parts of zero}}{\text{Poles} - \text{Zeros}}$$

$$= \frac{\sum -(0+1+2+3) - (0)}{4} = \frac{\cancel{6}\,3}{\cancel{4}\,2} = -1.5$$

$$\boxed{\sigma_c = -1.5}$$

Draw the asymptots passing through Centroid

Step 6: Breakaway point:

Take the char-equation

$\Rightarrow 1 + G(s)H(s) = 0$

$$\Rightarrow 1 + \frac{K}{s(s+1)(s+2)(s+3)} = 0$$

$\Rightarrow s(s+1)(s+2)(s+3) + K = 0$

$\Rightarrow K = -\left[(s^2+s)(s^2+5s+6)\right]$

$= -\left[s^4 + 5s^3 + 6s^2 + s^3 + 5s^2 + 6s\right]$

$K = -\left[s^4 + 6s^3 + 11s^2 + 6s\right]$

$$\frac{dK}{ds} = \left(4s^3 + 18s^2 + 22s + 6\right) = 0.$$

Solving the above equation we get three roots

$\Rightarrow$ $s_1 = -1.5$, $s_2 = -2.618$, $s_3 = -0.382$

Now $s_1 = -1.5$ does not lie on the loci.

While $s_2 = -2.618$ & $s_3 = -0.382$ lies on the Root Loci.

& hence are valid breakaway points.

$\therefore$ $\boxed{\sigma_b = -2.618 \ \& \ -0.382}$

Step 7: Not required as there are no complex poles and zeros.

Step 8: Intersection of Imaginary Axis:

Char equation is: $1 + G(s) H(s) = 0$

$\Rightarrow s^4 + 6s^3 + 11s^2 + 6s + K = 0$

$\therefore$ The Routh array:

s^4	1	11	K
s^3	6	6	0
s^2	10	K.	0
s^1	$\frac{60-6K}{10}$	0	0
s^0	K.		

Hence for marginal stability $\frac{60-6K}{10} = 0 \Rightarrow 60 = 6K$

$\Rightarrow \boxed{K_{mar} = \frac{60}{6} = 10}$

Hence at K_{max}, s^1 row is all zeros.

Hence we consider the Auxilar equation for s^2 row

$$\Rightarrow A(s) = 10s^2 + k_{max} = 0.$$

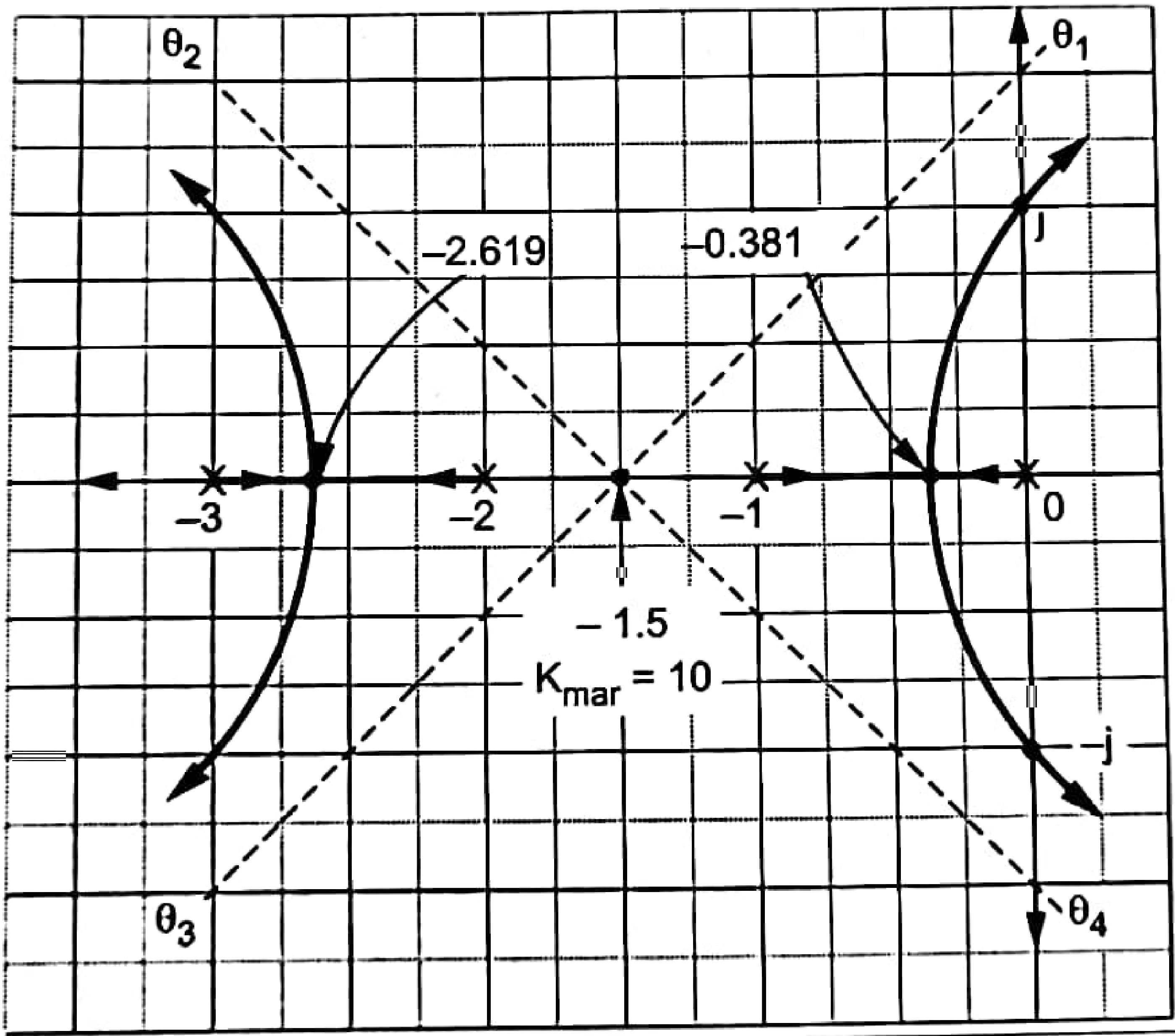
$$\Rightarrow -K_{max} = 10s^2$$

$$\Rightarrow -10 = 10s^2$$

$$\Rightarrow s^2 = -1 \Rightarrow s = \pm\sqrt{-1} = \pm j$$

$$\Rightarrow s = (+j) \& (-j)$$

Hence the root locus intersect the imaginary axis at $s = \pm j$ and the value of K at crossover is 10.


θ2
θ1
–2.619
–0.381
j
–3
–2
–1
0
– 1.5
Kmar = 10
j
θ3
θ4

Example 11.7.3 *Sketch the bode plot for the transfer function*

$$G(s) = \frac{K s^2}{(1+0.2\,s)(1+0.02\,s)}$$

Determine the value of K for the gain cross-over frequency to be 5 rad/sec.

Solution : Step 1 : G(s) is in the time constant form.

Step 2 : Analysis of factors

1. K is unknown and its effect is to shift the entire magnitude plot by 20 Log K dB.
2. Two zeros at origin, so straight line of slope + 40 dB/decade passing through intersection point of $\omega = 1$ and 0 dB.
3. Simple pole, $1/1 + 0.2\,s$, $T_1 = 0.2$

$$\therefore \quad \omega_{C1} = \frac{1}{T_1} = \frac{1}{0.2} = 5 \text{ rad/sec}$$

So straight line of slope – 20 dB/dec for $\omega \geq 5$.

4. Simple pole, $1/1 + 0.02s$, $T_2 = 0.02$

$$\therefore \quad \omega_{C2} = \frac{1}{T_2} = \frac{1}{0.02} = 50 \text{ rad/sec}$$

Resultant slope table :

Starting slope	+ 40 dB/dec
$0 < \omega < 5$	+ 40 dB/dec
$5 < \omega < 50$	+ 20 dB/dec
$50 < \omega < \infty$	0 dB/dec

Step 3 : Phase angle table

$$G(j\omega) = \frac{K(j\omega)^2}{(1+0.2\,j\omega)\,(1+0.02\,j\omega)}$$

ω	$(j\omega)^2$	$-\tan^{-1} 0.2\omega$	$-\tan^{-1} 0.02\omega$	ϕ_R
0.1	+ 180°	– 1.14°	– 0.11	+ 178.74°
1	+ 180°	– 11.3°	– 1.14	+ 167.55°
10	+ 180°	+ 63.4°	– 11.3°	+ 105.29°
100	+ 180°	– 87.1°	– 63.43°	+ 29.46°
∞	+ 180°	– 90°	– 90°	0°

Step 4 : The Bode plot is shown in the Fig. 11.7.6.

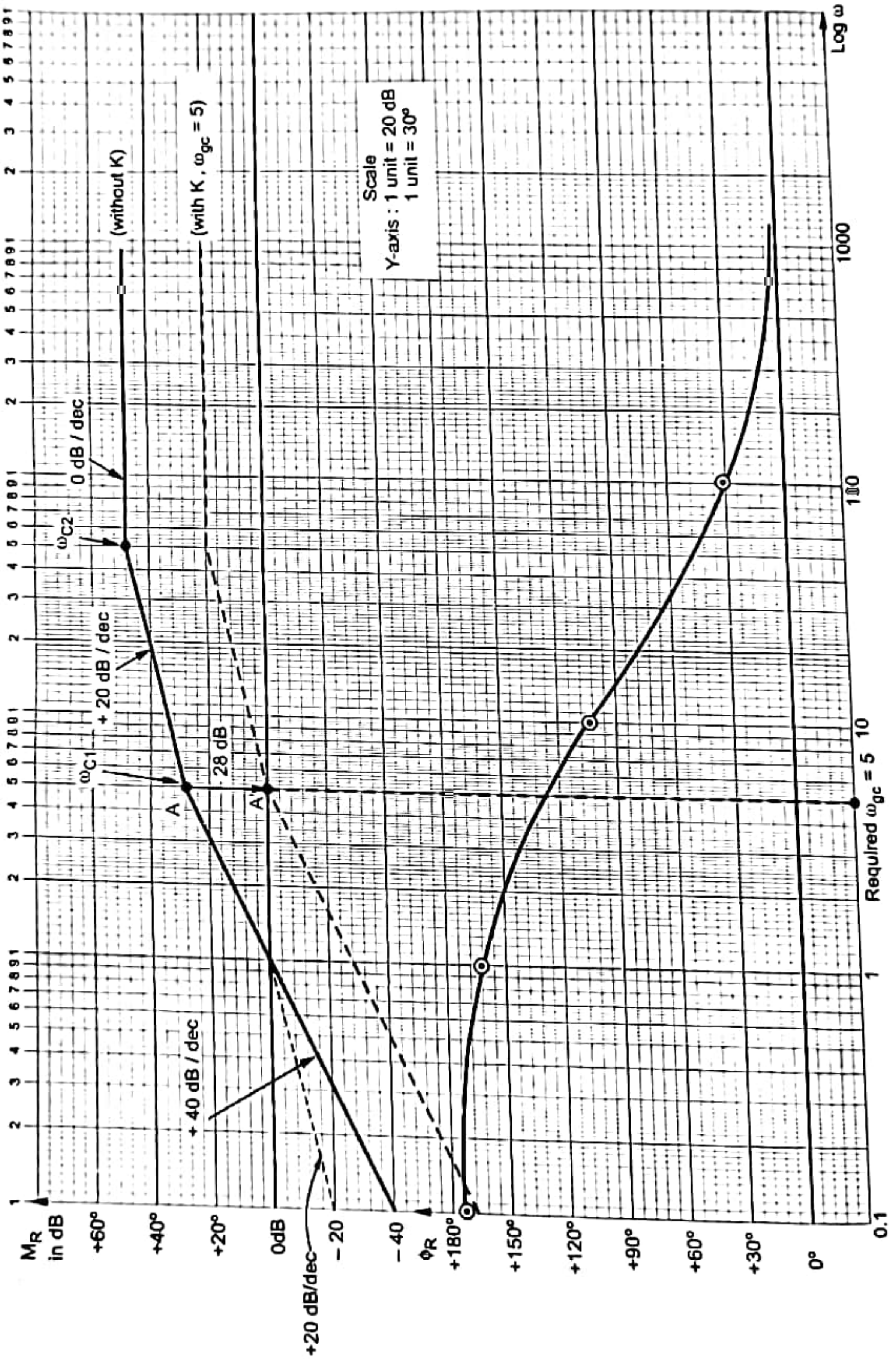


Fig. 11.7.6

From the Bode plot to get $\omega_{gc} = 5$, the plot must intersect 0 dB line at $\omega = 5$ rad/sec.

But at $\omega = 5$, the point on plot without K is 28 dB away from the 0 dB line. This is point A as shown. It should be on 0 dB at A′ as shown for $\omega_{gc} = 5$. So shift A to A′ must be contributed by 20 Log K, which remains constant for all the frequencies, to get $\omega_{gc} = 5$. The shift must be treated negative as it is downwards.

$$\therefore \quad 20 \text{ Log K} = -28 \text{ dB}$$

$$\therefore \quad \text{K} = 0.0399 \approx 0.04 \text{ to get}$$

$$\omega_{gc} = 5 \text{ rad/sec}$$

Module 5

9. a) $G(s)H(s) = \dfrac{K}{s(s+2)(s+10)}$

step 1 $\rightarrow$ $P = 0$

step 2 $\rightarrow$ $N = -P = 0$ critical point $-1+j0$ should not get encircled by Nyquist plot.

Step 3 — pole at origin

Section 1 — $s = +j\infty$ to $s = +j0$

$\omega \rightarrow \infty$ to $\omega \rightarrow +0$

Starting point $\omega \rightarrow \infty$ $0\angle -90^0$

Terminating point $\omega \rightarrow +0$ $0\angle +270$

Section 2 $s = +j0$ to $s = -j0$

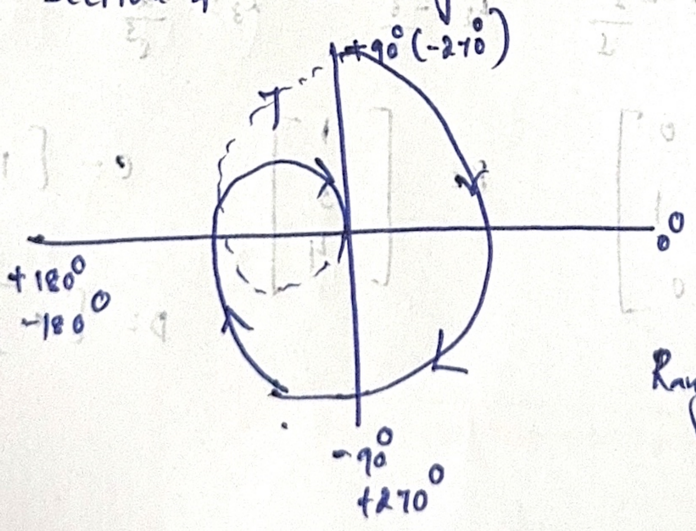
Starting point $\omega \rightarrow +0$ $\infty\angle -90^0$

Terminating point $\omega \rightarrow -0$ $\infty \angle +90^0$

5 M

section 3 is mirror image about real axis

Section 4 is origin



3 M

Range & stability — 2M

b) Working of lag Compensator with ckt diagram – 5

Working of lead Compensator with ckt diagram – 5

10. a) $\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -2x_1 - 7x_2 - 4x_3 + 5u(t)$ } 2M

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -7 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ } 4M

10 b) $P_1 = \frac{1}{s}$ $P_2 = \frac{3}{s^2}$ $P_3 = \frac{4}{s^3}$

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{s} + \frac{3}{s^2} + \frac{4}{s^3}}{1 - \left(-\frac{2}{3} - \frac{3}{s^2} - \frac{2}{s^3}\right)}$$ } 4M

$\Delta_1 = \Delta_2 = \Delta_3 = 1$

Loops $L_1 = -\frac{2}{s}$ $L_2 = -\frac{3}{s^2}$ $L_3 = -\frac{2}{s^3}$

$A = \begin{bmatrix} -2 & 1 & 0 \\ -3 & 0 & 1 \\ -2 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ $D = \begin{bmatrix} 0 \end{bmatrix}$ } 3M

10 c). State transition matrix

$$A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

07 M

$$(sI - A)^{-1} = \frac{\begin{vmatrix} s+3 & -1 \\ 2 & s \end{vmatrix}}{s(s+3) + 2}$$

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & -e^{-t} + e^{-2t} \\ 2e^{-t} - 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$