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21EC62

Sixth Semester B.E. Degree Examination, June/July 2024
Microwave Theory and Antennas

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With the help of drift velocity graph and wave from, explain he constructional feature and working of n-type GaAs diode. **(10 Marks)**

b. A transmission line has the following primary constants R = 2 Ω/m, L = 8 nH/m, G = 0.5 m ℧/m, C = 0.23 p^F/m and f = 1 GHz. Find :

(i) Characteristic impedance Z_O.

(ii) Propagation constant γ

(iii) Wavelength λ.

(iv) Phase velocity V_P **(10 Marks)**

OR

2 a. Derive the expression for the voltage of current at any point on the transmission line equation and solution starting from the fundamentals. **(10 Marks)**

b. Explain the standing waves with neat waveforms. **(10 Marks)**

Module-2

3 a. Derive scattering parameters for a multiport network. **(10 Marks)**

b. The transmission lines of characteristic impedances Z_1 and Z_2 are joined at plane PP′ Express S-parameters in terms of impedances. **(10 Marks)**

OR

4 a. Derive S-matrix for a Magic Tee with neat diagram and its applications. **(10 Marks)**

b. Explain the working of precision Dielectric Rotary phase shifter. **(10 Marks)**

Module-3

5 a. Discuss the operation of micro strip lines with its structure. Compare strip line and microstrip line. **(10 Marks)**

b. Explain the operation of parallel strip line along with a neat diagram. Write down the expression for character impedance. **(10 Marks)**

OR

6 a. Explain the following terms as related to antenna system :

(i) Directivity and gain.

(ii) Beam area.

(iii) Effective height

(iv) Bandwidth **(10 Marks)**

b. A radio link has a 15 W transmitter connected to an antenna of 2.5 m^2 effective aperture at 5 GHz. The receiving antenna has an effective aperture 0.5 m^2 and is located 15 km line of sight distance from the transmitting antenna. Assuming loss less, matched antenna, find the power delivered to the receiver. **(10 Marks)**

Module-4

7 a. Explain the field pattern and phase pattern with a neat diagram. **(10 Marks)**

b. Derive an expression and draw the field pattern for an array of two isotropic point sources situated symmetrical with respect to origin with equal amplitude and phase spaced $\frac{\lambda}{2}$ apart. **(10 Marks)**

OR

8 a. Derive an expression for field of a dipole in general for the case of thin linear antenna. **(10 Marks)**

b. Find the directivity D for the sources with radiation intensity :

(i) $U = U_m \sin^2\theta,\ 0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi$

(ii) $U = U_m \cos^2\theta,\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le \phi \le 2\pi$ **(10 Marks)**

Module-5

9 a. Derive an expression for field strength E_ϕ and H_ϕ in case of small loop antenna. **(10 Marks)**

b. Derive an expression for radiation resistance of a small loop antenna. **(10 Marks)**

OR

10 a. Derive an expression for radiation resistance of a short dipole antenna. **(10 Marks)**

b. Explain the different types of horn antenna with a diagram. **(10 Marks)**

* * * * *

1. a

Gunn Diode's Working

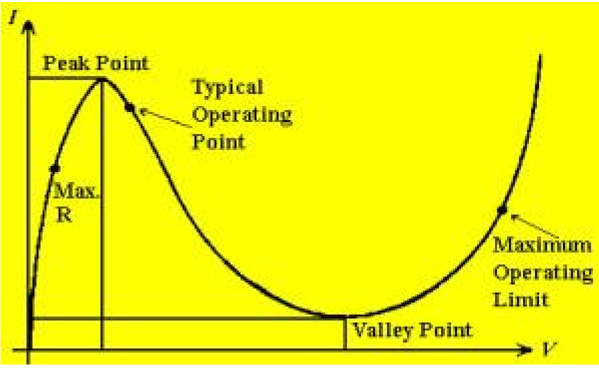
This diode is made of a single piece of N-type semiconductor such as Gallium Arsenide and InP (Indium Phosphide). GaAs and some other semiconductor materials have one extra-energy band in their electronic band structure instead of having only two energy bands, viz. valence band and conduction band like normal semiconductor materials. These GaAs and some other semiconductor materials consist of three energy bands, and this extra third band is empty at initial stage.

If a voltage is applied to this device, then most of the applied voltage appears across the active region. The electrons from the conduction band having negligible electrical resistivity are transferred into the third band because these electrons are scattered by the applied voltage. The third band of GaAs has mobility which is less than that of the conduction band.

Because of this, an increase in the forward voltage increases the field strength (for field strengths where applied voltage is greater than the threshold voltage value), then the number of electrons reaching the state at which the effective mass increases by decreasing their velocity, and thus, the current will decrease.

Thus, if the field strength is increased, then the drift velocity will decrease; this creates a negative incremental resistance region in V-I relationship. Thus, increase in the voltage will increase the resistance by creating a slice at the cathode and reaches the anode. But, to maintain a constant voltage, a new slice is created at the cathode. Similarly, if the voltage decreases, then the resistance will decrease by extinguishing any existing slice.

Gunn Diode's Characteristics



Gunn Diode Characterstics

The current-voltage relationship characteristics of a Gunn diode are shown in the above graph with its negative resistance region. These characteristics are similar to the characteristics of the tunnel diode.

As shown in the above graph, initially the current starts increasing in this diode, but after reaching a certain voltage level (at a specified voltage value called as threshold voltage value), the current decreases before increasing again. The region where the current falls is termed as a negative resistance region, and due to this it oscillates. In this negative resistance region, this diode acts as both oscillator *and amplifier, as in this region, the diode is enabled to amplify signals.*

1B)

A transmission line has the following parameters:

$R = 2\ \Omega/m$ (0.8 Ω/m) $G = 0.5$ mmho/m (0.8 mmho/m) $f = 1$ GHz (1.2 GHz)

$L = 8$ nH/m (0.01 μH/m) $C = 0.23$ pF/m (0.4 pF/m)

Calculate: (a) the characteristic impedance;

(b) the propagation constant.

(a)

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= \sqrt{\frac{2 + j\,2\pi \times 1 \times 10^{9} \times 8 \times 10^{-9}}{0.5 \times 10^{-3} + j\,2\pi \times 1 \times 10^{9} \times 0.23 \times 10^{-12}}}$$

$$= \sqrt{\frac{50.3 \angle 87.72^\circ}{1.529 \times 10^{-3} \angle 70.91^\circ}}$$

$$= 181.37 \angle 8.40^\circ$$

$$154.61 + j\,19.42\ \Omega$$

(b)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{(50.3 \angle 87.72^\circ)(1.529 \times 10^{-3} \angle 70.91^\circ)}$$

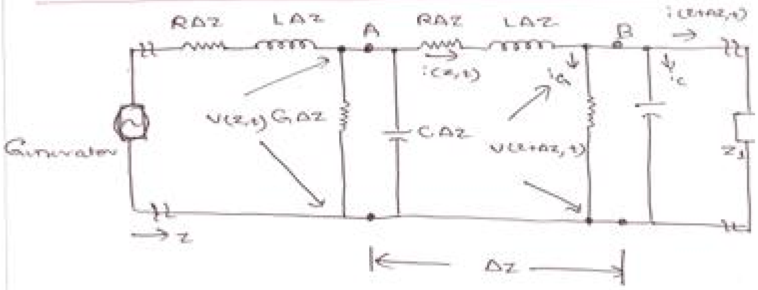
$$= 0.2774 \angle 79.31^\circ$$

$$= 0.051 + j\,0.272$$

$$= 0.0654 + j\,0.48$$

2.a

Transmission Lines Equations and Solutio



Elementary section of a transmission line

By Kirchoff's Voltage law, the summation of the voltage drops around the central loop is given by

$$v(z,t) = R\Delta z\, i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z+\Delta z, t)$$

$$= R\Delta z\, i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z \quad (1)$$

Rearranging this equation, dividing it by Δz, and then omitting the argument (z,t) which is understood, we get

LMK

$$\boxed{-\frac{\partial v}{\partial z} = R i + L\frac{\partial i}{\partial t}} \qquad (5)$$

Using Kirchoff's current law, the summation of the currents at point B can be expressed as

$$i(z,t) = G\,\Delta z\, v(z+\Delta z, t) + C\,\Delta z\,\frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z, t)$$

$$= G\,\Delta z\left[v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z\right] + C\,\Delta z\,\frac{\partial}{\partial t}\left[v(z,t) + \frac{\partial v(z,t)}{\partial z}\Delta z\right] + i(z,t) + \frac{\partial i(z,t)}{\partial z}\Delta z \qquad (3)$$

$$\cancel{= G\Delta z\, v(z,t) + G}$$

$$0 = G\,v(z,t) + G\,\cancel{\frac{\partial v(z,t)}{\partial z}\Delta z} + C\frac{\partial}{\partial t}v(z,t) + C\frac{\partial}{\partial t}\left[\cancel{\frac{\partial v(z,t)}{\partial z}\Delta z}\right] + \frac{\partial i(z,t)}{\partial z}$$

$$\Rightarrow \boxed{-\frac{\partial i}{\partial z} = G v + C\frac{\partial v}{\partial t}} \qquad (4)$$

Differentiating Eq. 2 w.r.t z we get

$$-\frac{\partial^2 V}{\partial z^2} = R\frac{\partial i}{\partial z} + L\frac{\partial}{\partial z}\left(\frac{\partial i}{\partial t}\right) \quad (5)$$

Differentiating Eq. 4 w.r.t t we get

$$-\frac{\partial}{\partial t}\left(\frac{\partial i}{\partial z}\right) = G\frac{\partial V}{\partial t} + C\frac{\partial^2 V}{\partial t^2} \quad (6)$$

Substituting equations 4 & 6 in Eq. 5 we get

$$-\frac{\partial^2 V}{\partial z^2} = R\left(-GV - C\frac{\partial V}{\partial t}\right) + L\left(-G\frac{\partial V}{\partial t} - C\frac{\partial^2 V}{\partial t^2}\right)$$

$$= -RGV - RC\frac{\partial V}{\partial t} - LG\frac{\partial V}{\partial t} - LC\frac{\partial^2 V}{\partial t^2}$$

$$\boxed{\frac{\partial^2 V}{\partial z^2} = RGV + (RC+LG)\frac{\partial V}{\partial t} + LC\frac{\partial^2 V}{\partial t^2}} \quad (6)$$

~~Also~~ ~~By~~ Differentiating Eq. 2 w.r.t t we get

$$-\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial z}\right) = R\frac{\partial i}{\partial t} + L\frac{\partial^2 i}{\partial t^2} \quad (7)$$

CMR

Differentiating Eq. (4) w.r.t. z we get

$$-\frac{\partial^2 i}{\partial z^2} = G\frac{\partial v}{\partial z} + C\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial t}\right) \quad (8)$$

Substituting Eqs (2), (7) in Eq. (8) we get

$$-\frac{\partial^2 i}{\partial z^2} = G\left(-Ri - L\frac{\partial i}{\partial t}\right) + C\left[-R\frac{\partial i}{\partial t} - L\frac{\partial^2 i}{\partial t^2}\right]$$

$$= -RGi - LG\frac{\partial i}{\partial t} - RC\frac{\partial i}{\partial t} - LC\frac{\partial^2 i}{\partial t^2}$$

$$\therefore \boxed{\frac{\partial^2 i}{\partial z^2} = RGi + (LG+RC)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}} \quad (9)$$

The voltage and current on the line are the functions of both position z and time t.

The instantaneous line voltage and current can be expressed as

$$v(z,t) = \operatorname{Re} V(z)\, e^{j\omega t} \quad (10)$$

$$i(z,t) = \operatorname{Re} I(z)\, e^{j\omega t} \quad (11)$$

where Re stands for "real part of".

The factors V(z) and I(z) are complex quantities of the sinusoidal functions of position z on the line and are known as phasors

CMR

If we substitute $j\omega$ for $\frac{\partial}{\partial t}$ in equations ②, ④, ⑥ and ⑨ and divide each equation by $e^{j\omega t}$, the transmission-line equations in phasor form of the frequency domain become

$$\frac{dV}{dz} = -(R + j\omega L)\,I = -ZI \qquad (12)$$

$$\frac{dI}{dz} = -(G + j\omega C)\,V = -YV \qquad (13)$$

$$\frac{d^2V}{dz^2} e^{j\omega t} = RGVe^{j\omega t} + (RC + LG)\,j\omega V e^{j\omega t} + LC\,(j\omega)^2 V e^{j\omega t}$$

$$\frac{d^2V}{dz^2} = RGV + (RC + LG)\,j\omega V + LC(j\omega)^2 V$$

$$= \left[RG + RCj\omega + LGj\omega + LC(j\omega)^2\right]V$$

$$= \left[R(G + j\omega C) + j\omega L(G + j\omega C)\right]V$$

$$= (R + j\omega L)(G + j\omega C)V$$

$$= ZYV$$

CMR

$$\frac{d^2V}{dz^2} = \gamma^2 V \qquad (14)$$

$$\frac{d^2I}{dz^2} = \gamma^2 I \qquad (15)$$

in which the following substitutions have been made:

$$Z = R + j\omega L \quad \text{(ohms per unit length)} \qquad (16)$$

$$Y = G + j\omega C \quad \text{(mhos per unit length)}$$

$$\gamma = \sqrt{ZY} = \alpha + j\beta \quad \text{(propagation constant)}$$

α is the attenuation constant in nepers per unit length

β is the phase constant in radians per unit length

2b

Standing Wave and Standing Wave Ratio

→ Standing Wave

The general solutions of the transmission line equation consist of two waves travelling in opposite directions with unequal amplitude as given by

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z} = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z} \quad (1)$$

$$I = Y_0 V_+ e^{-\gamma z} - Y_0 V_- e^{\gamma z}$$

$$= Y_0 V_+ e^{-\alpha z} e^{-j\beta z} - Y_0 V_- e^{\alpha z} e^{j\beta z} \quad (2)$$

Eq. (1) can be written as,

$$V = V_+ e^{-\alpha z}[\cos(\beta z) - j\sin(\beta z)] + V_- e^{\alpha z}[\cos(\beta z) + j\sin(\beta z)] \quad (3)$$

$$= (V_+ e^{-\alpha z} + V_- e^{\alpha z})\cos(\beta z) - j(V_+ e^{-\alpha z} - V_- e^{\alpha z})\sin(\beta z) \quad (4)$$

$$= X + jY$$

Equation of the voltage standing wave

→ The magnitude of the Eq. (4) is

$$|V| = \left[(V_+ e^{-\alpha z} + V_- e^{\alpha z})^2 \cos^2\beta z + (V_+ e^{-\alpha z} - V_- e^{\alpha z})^2 \sin^2\beta z\right]^{1/2} \quad (5)$$

Called the amplitude of the standing wave

1. The maximum amplitude is

$$V_{max} = V_+ e^{-\alpha z} + V_- e^{\alpha z} = V_+ e^{-\alpha z}(1 + |\Gamma|) \quad (6)$$

and this occurs at $\beta z = n\pi$, where $n = 0, \pm1, \pm2, \cdots$

2. The minimum amplitude is

$$V_{min} = V_+ e^{-\alpha z} - V_- e^{\alpha z} = V_+ e^{-\alpha z}(1 - |\Gamma|) \quad (7)$$

and this occurs at $\beta z = (2n+1)\pi/2$, where $n = 0, \pm1, \pm2 \cdots$

3. The distance between any two successive maxima or minima is one-half wave-length, since

$$\beta z = n\pi$$

$$z = \frac{n\pi}{\beta} \Rightarrow z = \frac{n\pi}{2\pi/\lambda} = n\frac{\lambda}{2}$$

$(n = 0, \pm1, \pm2 \cdots)$

Then $z_1 = \frac{\lambda}{2}$

→ The standing-wave patterns of two oppositely travelling waves with unequal amplitude in lossy and lossless line are shown in figs 1 and 2.

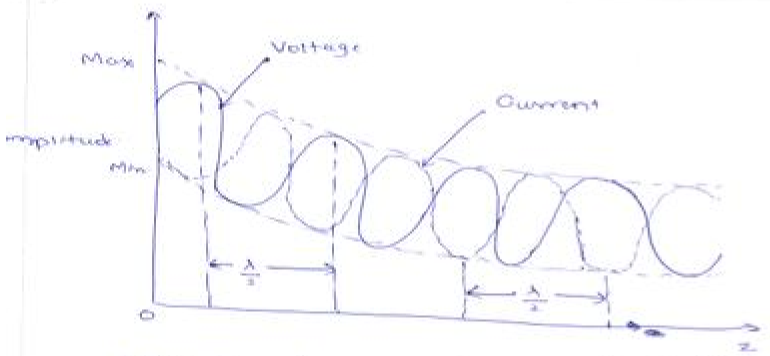


Fig 1: Standing-wave pattern in a lossy line

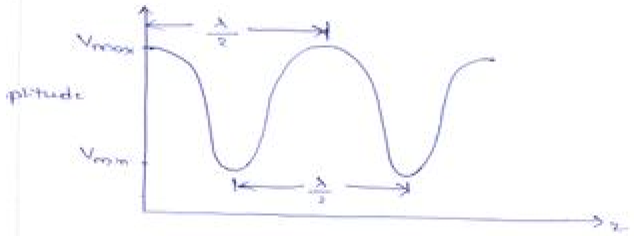
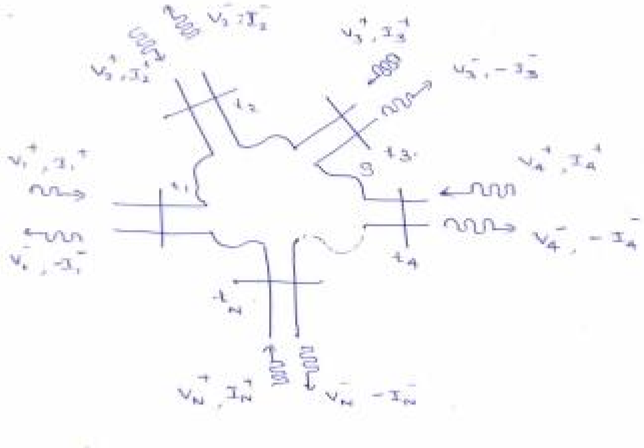


Fig 2: Voltage standing-wave pattern in a lossless line

3a

Consider an arbitrary N-port microwave network, as shown in the figure.



An arbitrary N-port microwave network

At a specific point on the nth port, a terminal plane, t_n is defined along with equivalent voltages and currents for the incident (V_n^+, I_n^+) and reflected (V_n^-, I_n^-) waves.

→ The terminal planes are important in providing a phase reference for the voltage and current phasors.

→ Now at the nth terminal plane, the voltage and current is given by

$$V_n = V_n^+ + V_n^- \quad (1)$$

$$I_n = I_n^+ - I_n^- \quad (2)$$

[Note: These are written from

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I = I_+ e^{-\gamma z} - I_- e^{\gamma z}$$

when $z = 0$]

The impedance matrix $[Z]$ of the microwave network then relates these voltages and currents:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & & & \vdots \\ \vdots & & & \\ Z_{N1} & \cdots & & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

or in matrix form as

$$[V] = [Z][I] \qquad (3)$$

Similarly, we can define an admittance matrix $[Y]$ as

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & & & \vdots \\ \vdots & & & \vdots \\ Y_{N1} & & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

or in matrix form as

$$[I] = [Y][V] \qquad (4)$$

The $[Z]$ and $[Y]$ matrices are the inverses of each other

$$[Y] = [Z]^{-1} \qquad (5)$$

3b

Assuming the characteristic impedances (Z_{0n}) of all the ports are identical. (← ~~$Z_{0n}=1$~~)
Also, setting $Z_{0n} = 1$ (for convenience)
The total voltage and current at the nth port can be written as

$$V_n = V_n^+ + V_n^- \qquad (1)$$

$$I_n = V_n^+ - V_n^- \qquad (2)$$

Adding equations (1) and (2) we obtain

$$2V_n^+ = V_n + I_n$$

$$V_n^+ = \frac{1}{2}(V_n + I_n)$$

or $$[V_n^+] = \frac{1}{2}([V] + [I]) = \frac{1}{2}([Z][I] + [I])$$

$$[V^+] = \frac{1}{2}\{[Z]+[U]\}[I] \quad (3)$$

~~$[V^-]$~~

Subtracting equation (3) from equation (1) we obtain

$$2V_n^- = V_n - I_n$$

$$V_n^- = \frac{1}{2}(V_n - I_n)$$

$$[V^-] = \frac{1}{2}([V]-[I])$$

$$= \frac{1}{2}\{[Z][I]-[I]\}$$

$$= \frac{1}{2}\{[Z]-[U]\}[I] \quad (4)$$

Dividing equation (4) by equation (3),

$$\frac{[V^-]}{[V^+]} = \{[Z]-[U]\}\{[Z]+[U]\}^{-1}$$

$$[V^-] = \{[Z]-[U]\}\{[Z]+[U]\}^{-1}[V^+] \quad (5)$$

So that

$$[S] = \{[Z]-[U]\}\{[Z]+[U]\}^{-1} \quad (6)$$

Taking transpose of (6) gives

$$[S]^t = \{([Z]-[U])^t\}\{([Z]+[U])^t\}^{-1}$$

Now $[U]$ is diagonal, so $[U]^t = [U]$

4a

Magic Tee (E-H Tee)

→ Magic Tee is a combination of E-pla[ne] Tee and H-plane Tee.

→ In this waveguide is cut at both widt[h] and breadth and side arms in the direction of magnetic field and electr[ic] field are inserted, respectively.

→ Ports (1) and (2) form collinear ports, port (3) is called H-arm and port (4) is called E-arm.

S - matrix of Magic Tee

$$[S]_{MT} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (1)$$

$S_{13} = S_{23}$ (2) (From H-plane tee action)

$S_{14} = -S_{24}$ (3) (From E-plane tee action)

Due to the geometry, power fed at port 3 cannot come out of port 4. and vice versa.

$$S_{34} = S_{43} = 0 \quad (4)$$

Assume ports 3 and 4 are matched

$$S_{33} = S_{44} = 0 \quad (5)$$

Magic Tee is reciprocal. Hence, Symmetric property also holds good.

$$S_{ij} = S_{ji} \quad (6)$$

Using equations (2) to (6) in equation (1), the S-matrix simplifies to the following matrix

$$[S]_{MT} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad (7)$$

$$[S][S]^{*} = [U] \quad (8)$$

(from unitary property)

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^{*} & S_{12}^{*} & S_{13}^{*} & S_{14}^{*} \\ S_{12}^{*} & S_{22}^{*} & S_{13}^{*} & -S_{14}^{*} \\ S_{13}^{*} & S_{13}^{*} & 0 & 0 \\ S_{14}^{*} & -S_{14}^{*} & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From (13), $2|S_{14}|^2 = 1$

$$|S_{14}|^2 = \frac{1}{2}$$

$$S_{14} = \frac{1}{\sqrt{2}} \quad (16)$$

Using (15) and (16) in (10) and (11),

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0 \quad (17)$$

$$|S_{12}|^2 + |S_{22}|^2 = 0 \quad (18)$$

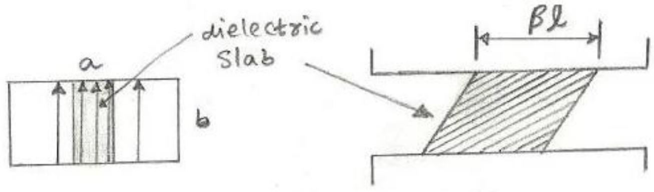
$$\Rightarrow \quad S_{11} = S_{22} \quad (19)$$

Equation (17) is possible only when $S_{11} = S_{12} = 0$

$$[S]_{MT} = \begin{bmatrix} 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

4b

A phase shifter is two port passive device. It produces a variable change in phase of transmitted wave passing through it. It is realized by placing a lossless dielectric slab within a waveguide parallel to and at the maximum E field position. The dielectric slab is tapered at both the ends to reduce reflections. There are various types of phase shifters which include Precision **phase shifter**, MIC phase shifter,Reciprocal and Non-reciprocal phase shifter.



Phase shifter

A differential phase change is produced due to change of wave velocity through slab. As shown in the figure, by adjusting the length L, different phase shifts can be produced. S matrix of the ideal phase shifter is as follows.

[S] =

$$\begin{vmatrix} 0 & e_{-j\Delta\Phi} \\ & \\ e_{-j\Delta\Phi} & 0 \end{vmatrix}$$

Precision Phase Shifter

Precision Phase shifter can be designed as rotary type. It is made of following parts:

- Both the ends circular to rectangular waveguide transitions
- Circular waveguide with lossless dielectric plate of half wave length (180 degree) at the Middle
- Between waveguide transition and above half wave plate, there are plates of quarter lenth(90 degree)

TE10 mode of rectangular waveguide becomes TE11 after passing through the transition at the end. Appropriate phase shift is obtained after wave goes through the two quarter sections provided at the ends and also rotary half wave section available in the middle.

Q no. 5a. Ans :

6.2 MICROSTRIP LINES

Figure 6.1 shows a microstrip line consisting of a single ground plane and a thin strip conductor on a low loss dielectric substrate above the ground plate. The microwave solid state active devices can be easily mounted on top and easily connected to these microstrip lines.

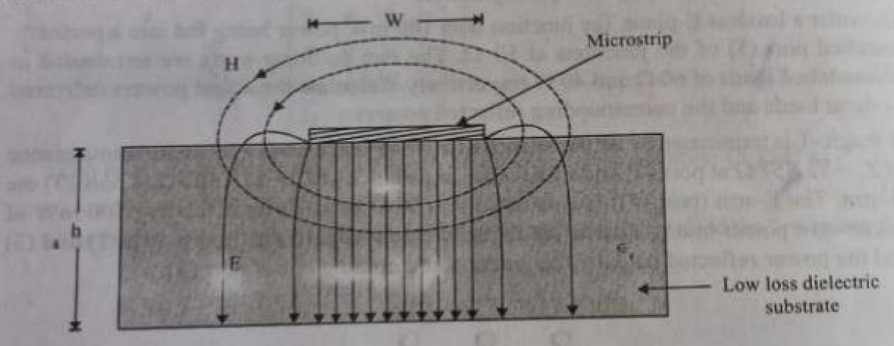


Figure 6.1 · ·‥ustrating micro strip line

The electric field lines remain partially in air and partially in the lower dielectric substrate as shown in figure 6.1. This makes the mode of propagation not pure TEM but quasi-TEM. Since it is an open structure, microstrip lines radiate electro-magnetic energy. The radiation loss is proportional to the square of the frequency. By using thin and high dielectric materials, the radiation losses can be reduced.

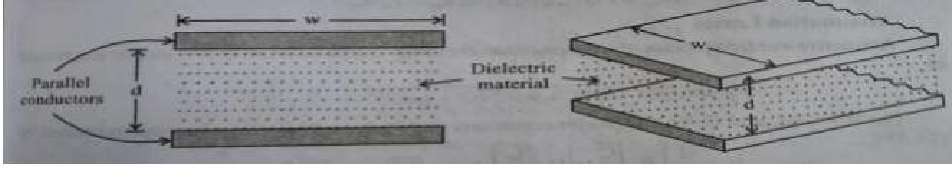
Comparison of Stripline and Microstrip

Sr. No.	Stripline	Microstrip
1.	Dielectrics used are teflon, polystyrene.	Alumina, quartz, silica
2.	Suitable for design of only passive circuits.	Suitable for the design of passive circuits and series mounting of active components across a gap in strip.
3.	Strip line losses are mainly in conductor.	Microstrip losses are i) In dielectric ii) Ohmic loss in the strip and ground plane due to finite conductivity.
4.	3 conductor transmission system i.e. two ground planes and a stripline	2-Conductors i.e. one ground plane and a microstrip.
5.	Propagation mode is pure TEM.	Propagation mode is quasi TEM.

Q no. 5b. Ans :

6.3 PARALLEL STRIP LINES

Figure 6.4 shows a parallel strip lines consisting of two perfectly parallel strips made of copper. The space between the parallel strips is filled with a dielectric material having uniform thickness. The width of each strip is "w" and the thickness of the dielectric material is "d" and has a dielectric constant of ϵ_{rd}.



Characteristic Impedance Z_0

Attenuation Losses

Q.no.6a i) Directivity and gain:

ANTENNA & WAVE PROPAGATION

544

DIRECTIVITY (D)

ANTENNA TERMINOLOGY

545

Therefore, the directivity of an antenna is nothing but solid angle of a sphere (i.e. 4π steradian) divided by the antenna beam solid angle Ω_A.

From the definition of directivity eqn. 6.36 and eqn. 6.48

$$D = \frac{\Phi_{max}}{\Phi_{av}} = \frac{4\pi}{\Omega_A} \qquad \because \quad \Phi_{av} = \frac{W_r}{4\pi}$$

or

$$4\pi\,\Phi_{av} = \Omega_A\,\Phi_{max}$$

or

$$\cancel{4\pi}\cdot\frac{W_r}{\cancel{4\pi}} = \Omega_A\,\Phi_{max} \qquad \ldots(6.50)$$

$$\boxed{W_r = \Omega_A\,\Phi_{max}}$$

where W_r = Total power radiated.

Thus beam solid angle is the solid angle through which all the power radiated would stream if the power per unit solid angle equal to the maximum value of radiation intensity Φ_{max} over the beam solid angle.

Directivity is a dimensionless (*i.e.* constant) quantity which indicates the effectiveness of concentrating power into a limited solid angle. The narrower the solid angle, the higher the directivity.

It entirely depends on the far field pattern. Unlike gain or power gain, antenna efficiency is not involved in the directivity. *If an antenna has not any losses like ohmic, dielectric mismatch i.e. 100% efficient, then directivity and gain are same.* However, for an antenna with losses, gain will be less than directivity by a factor which corresponds to efficiency. The directivity and gain are related as

$$\boxed{G_0 = kD} \qquad \ldots(6.51)$$

where

G = gain

k = efficiency factor = 1 for 100% efficiency

< 1 if losses are present

D = Directivity

This is derived as follows. Let us assume that the maximum radiation intensity from the test antenna is Φ'_{max} and let this maximum radiation intensity Φ'_{max} be related to the maximum radiation intensity Φ_{max} of 100% efficient test antenna by a factor k known as radiation efficiency factor as

$$\Phi'_{max} = k\,\Phi_{max} \qquad \ldots(6.52)$$

where the values of k lies between 0 and 1 *i.e.* $0 \le k \le 1$

But the gain w.r.t. an isotropic antenna is

$$G_0 = \frac{\Phi'_{max}}{\Phi_0} \qquad \text{from Eqn. 6.24 (b)}$$

where Φ_0 = R.I. from a lossless isotropic antenna

or

$$G_0 = \frac{k\,\Phi_{max}}{\Phi_0} \qquad \text{from Eqn. 6.52}$$

From Eqn. 6.~~49~~ 51

$$\boxed{G_0 = kD} \qquad \ldots 6.52\ (a)$$

$$D = \frac{\Phi_{max}}{\Phi_0}$$

Since, in many antennas the antenna losses are extremely small and hence the value of gain is almost equal to the directivity. That is why, gain and directivity are interchangeably used.

ii) Beam area:

Oct 99 In

6.28. ANTENNA BEAM AREA OR BEAM SOLID ANGLE Ω (Roorkee Univ. 1981-82)

An area ds of the surface of a sphere as seen from the centre of the sphere subtends a solid angle Ω (Fig. 6.22). The total solid angle subtended by the sphere is 4π steradians (or square radians)

abbreviated as S_r.

From Fig. 6.22 incremental area ds of the surface of sphere is given by

$$ds = (r\sin\theta\, d\varphi)(r\, d\theta)$$
$$= r^2 \sin\theta\, d\theta\, d\varphi$$
$$\boxed{ds = r^2 \cdot d\Omega \ \ \mathrm{m}^2} \quad \text{... (6.132)}$$

where $d\Omega$ = Solid angle subtended by area ds

or $$d\Omega = \frac{ds}{r^2} \quad S_r$$

or $$d\Omega = \frac{4\pi r^2}{r^2} \quad S_r$$

$\because$ $ds = 4\pi r^2$ = Area of sphere

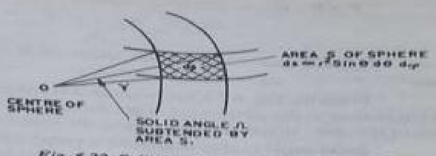


Fig. 6.22. Solid angle Ω subtended by area S.

$$\boxed{d\Omega = 4\pi} \quad \text{... (6.133)}$$

$$1 \text{ Steradian} = \frac{d\Omega}{4\pi}$$

$$1 \text{ Steradian} = \frac{\text{Solid angle of sphere}}{4\pi}$$

$$= 1\,(\text{radian})^2 = \left(\frac{180^\circ}{\pi}\right)^2 \ (\text{degrees})^2$$

$$= \frac{180 \times 180}{3.1416 \times 3.1416}\ (\text{deg.})^2 = \frac{32{,}400}{3.1416 \times 3.1416}\ (\text{deg.})^2$$

$$= \frac{10313.216}{3.1416}\ (\text{deg.})^2$$

$$\boxed{1 \text{ Sr.} = 3282.7909\ (\text{deg.})^2} \quad \text{...(6.134)}$$

since π radian $= 180^\circ$

$$1 \text{ radian} = \left(\frac{180^\circ}{\pi}\right)$$

Therefore,

$$4\pi \text{ Steradians} = 3282.7909 \times 4\pi\ (\text{deg.})^2$$
$$= 13131.163 \times 3.1416\ (\text{deg.})^2 = 41252.861\ (\text{deg.})^2$$

$$\boxed{4\pi S_r \cong 41253\ (\text{deg.})^2 = \text{Solid angle in a sphere}} \quad \text{... (6.135)}$$

The beam area (or beam solid angle) Ω_A for an antenna is therefore, given by the integral of the normalised power pattern over a sphere ($4\pi S_r$) or

$$\Omega_A = \int_0 \int_0 P_n(\theta, \varphi) \cdot \sin\theta\, d\theta\, d\varphi \quad S_r$$

iii) Effective height:

condition.

For Receiving Antenna. The effective length of a receiving antenna may be defined as the ratio of open circuit voltage developed at the terminals of antenna and the received field strength *i.e.*

$$l_{er} = \frac{\text{Open circuit voltage } (V_0)}{\text{Electric field strength } (E)}$$

or $$\boxed{V_0 = l_{er} E} \quad \text{... (6.83)}$$

... for receiving case also but with terminal no. 1 is short

iii) Band width:

6.25. ANTENNA BAND-WIDTH (AMIETE, Nov. 1971, 72, 73)

Like some of the other properties of antenna, there is no unique definition of band-width of an antenna or antenna system. It is because for the operation of antenna many factors like gain, side-lobe-level SWR or Front-to-Back-ratio, pattern, impedance and Polarization characteristics etc. are considered and thes requirements may change when the antenna operates. Therefore, the functional band-width of an antenna i limited by one (or more) of these factors and accordingly antenna band width may be specified in many differe ways as

(i) Band-width over which the gain is higher than some acceptable value, or

(ii) Band-width over which atleast a given front to back ratio is achieved, or

(iii) Band-width over which the SWR on the transmission line can be maintained below a chosen value.

In other words, it can be said that *antenna band-width is a width (i.e. range) of frequency over which the antenna maintains certain required characteristics like gain, front to back ratio or S.W.R. pattern (shape or direction), polarization and impedance.* In practice, however, these requirements change with the operation of the antenna, therefore, specifications are set to meet the needs of a particular application. For antennas, where increase in side lobe-level decrease in gain and change in impedance value, pattern and polarization characteristics are important, then one of these factors (*e.g.* gain or impedance) determines the low frequency limit and the other factor (*e.g.* change of pattern-shape or direction) the high frequency limit. Hence the band-width of a particular antenna, in general, can then, be defined as *"the band-width within which the antenna maintains a given set of specifications"*.

In general, the band-width of an antenna, as said, mainly depends on its two characteristics *e.g. impedance and pattern.* At low frequency of relatively small dimension ($\lambda/2$ or less) the band-width is usually determined by *impedance variation* because the pattern characteristic is insensitive to frequency i.e. pattern changes less rapidly. Under this condition antenna performance is limited by impedance variation and band-width depends on its reciprocal (*i.e.* "Q" of the antenna). A considerable mathematical analysis will show (Example 6.6) that two frequency limits (*i.e.* ω_2 and ω_1) or band width is given by

$$\Delta\omega = \omega_2 - \omega_1 = \frac{\omega_r}{Q} = \text{Band-width}$$

or
$$\boxed{\Delta\omega = \frac{\omega_r}{Q}} \qquad \because\ \omega_r = 2\pi f_r \qquad \ldots 6.125\ (a)$$

or
$$\boxed{\Delta f = \frac{f_r}{Q}} \qquad \Delta\omega = 2\pi\Delta f \qquad \ldots 6.125\ (b)$$

or
$$\Delta f \propto \frac{1}{Q} \qquad \ldots 6.125\ (c)$$

where f_r = Centre or resonant or design frequency.

$$\boxed{Q = 2\pi\,\frac{(\text{Total energy stored by antenna})}{(\text{Energy dissipated or radiated per cycle})}} \qquad \ldots\ 6.125\ (d)$$

Thus, the lower the "Q" of the antenna the higher the band-width and vice-versa.

For antennas of larger dimensions in wavelength (like thick cylindrical antennas or biconical antennas or antennas arrays except super-directive arrays *i.e.* arrays designed to give supergain) impedance characteristic may be satisfactory over a wide band and it is *pattern characteristic that determines the limits of frequencies. In this case the design statements are formulated in terms of beam-width and side-lobe-level requirements.*

For the antennas of about one wavelength dimension (*i.e.* neither larger nor small) band-width is limited by either on impedance or on pattern characteristic depending on the particular application.

Now the recent researches have led to the development of *Frequency-Independent antennas like log-periodic antennas etc. which have unlimited band-width where lower and upper frequencies limits are specified independently. In such cases, the band width is represented by a ratio of highest to lowest operating frequency.* For example, band width of broad band antennas, like those of log Periodic, 20 : 1 is attained with ease and 100 : 1 with careful design. Band width generally of low and moderate values are expressed *in terms of Percentage of centre frequency*

$$\text{B.W.\%} = \frac{\text{Operating range}}{\text{Centre frequency}} \times 100$$

Q.no.6b

Formula → 4M
Solution → 6M

$$P_r = P_t \frac{A_e A_e}{r^2 \lambda^2} = 15 \frac{2.5 \times 0.5}{15^2 \times 10^6 \times 0.06^2} = 23\,\mu W$$

Q.no.7a

Feb.-11

2.4 Field Patterns

The power patterns discussed in the preceding sections, were based on power considerations. The discussion could be simplified by considering the power flow as a scalar quantity since it has only a radial component. However, to describe the field of a

point source more completely, one needs to consider the electric field E and/or the magnetic field H which are vectors. Since we deal only with the far fields, for point sources, the E and H bear following relations :

i) Both are entirely transverse to the direction of propagation.

ii) They are perpendicular to each other.

point source more completely, one needs to consider the electric field E and/or the magnetic field H which are vectors. Since we deal only with the far fields, for point sources, the E and H bear following relations :

i) Both are entirely transverse to the direction of propagation.

ii) They are perpendicular to each other.

iii) They are in phase.

iv) They are related in magnitude by the intrinsic impedance of the medium (which is $\eta = 377\ \Omega$ for free space).

Note that the poynting vector around a point source is radial everywhere, it follows that the electric field is entirely transverse, having only E_θ and E_ϕ components. The space relations of the radial poynting vector component, P_r, and the electric field components is illustrated in Fig. 2.4.1 using a spherical co-ordinate system. From this the far field is seen to be characterized by

1. Poynting vector radial (P_r component only).

2. Transverse electric field (E_ϕ and E_θ components only).

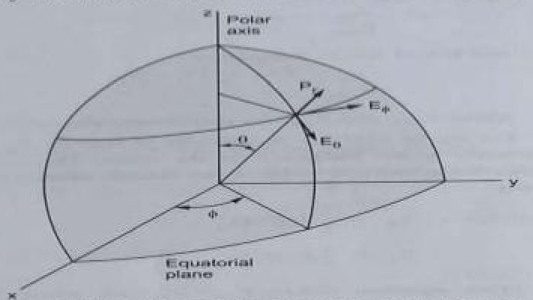


Fig. 2.4.1 Relation of the poynting vector and the electric

At large values of r, a small section of the spherical wave front may be considered as a plane. Hence the poynting vector and the field at a point of the far field are related in the same manner as they are in a plane wave which is given by

$$P_r = \frac{1}{2}\frac{E^2}{\eta} \qquad \dots (2.4.1)$$

where η is the intrinsic impedance of the medium and $E = \sqrt{E_\phi^2 + E_\theta^2}$. The field may be linearly, elliptically or circularly polarized. A 'field pattern' is a pattern showing the variation of the electric field intensity of a constant radius r as a function of angle (θ, ϕ). The far field patterns of an antenna can be completely specified through the field patterns for the two components, E_θ and E_ϕ, of the electric field since the total electric

2.5 Phase Pattern

Feb.-11

For a given frequency assuming that the field has a harmonic time variation, the far field due to a source, in all direction, can be completely specified knowing the following quantities :

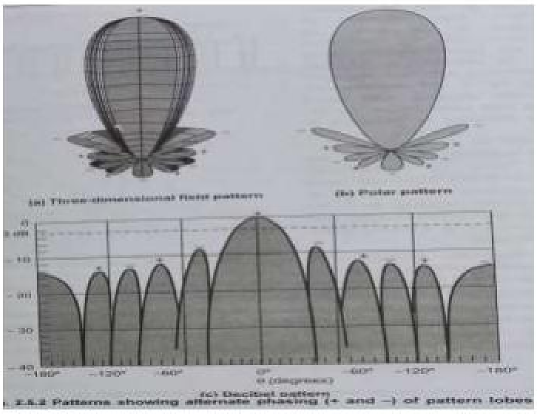
1. Magnitude of the azimuthal component E_ϕ of the electric field as a function of r, θ and φ.
2. Magnitude of the polar component E_θ of the electric field as a function of r, θ and φ.
3. Phase lag of E_ϕ behind E_θ as a function of φ and θ.
4. Phase lag of either field component behind its value of a reference point as a function of r, θ and φ.

In the far field considerations, we consider every source of radiation as a point source. Hence the above four quantities completely specify the far field of a point source. Also, the amplitudes of the field components at any distance can be obtained from the knowledge of their amplitude of a particular radius, using the inverse distance law.

Consider the pattern of a directional antenna with maximum radiation in z-direction at $\theta = 0°$. The three dimensional field pattern of such a radiator is shown in Fig. 2.5.1, where there is a major lobe containing most of the radiation and minor lobes contributing for radiation in other directions. This pattern is symmetrized in φ and is a function of θ alone. (See Fig. 2.5.1 on next page)

The three dimensional version of this radiation pattern is shown in Fig. 2.5.1 (a). Also shown are the polar and decibel forms in Fig. 2.5.1 (b) and (c) respectively.
The polarity of the lobe is also indicated.

It is observed that the polarity of the side lobes alternate (– and +). A null is formed when the magnitude of the field of one lobe (–) and the adjacent lobe (+) are equal. (See Fig. 2.5.1 (a), (b) and (c) on page 2-15)



. 2.5.2 Patterns showing alternate phasing (+ and –) of pattern lobes

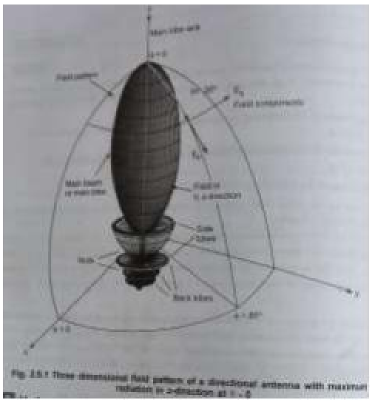


Fig. 2.5.1 Three dimensional field pattern of a directional antenna with maximum radiation in z-direction at θ = 0

Q.no.7b.

Minima directions. E is minimum when $\cos(\pi/2 \cos\theta)$ is minimum and its minimum value is 0.

$\therefore$ E is minimum. When $\cos(\pi/2 \cos\theta) = 0$

$$(\pi/2 \cos\theta_{min}) = \pm(2n+1)\pi/2 \quad \text{where } n = 0, 1, 2, \ldots$$

or $$(\pi/2 \cos\theta_{min}) = \pi/2$$

or $$\cos\theta_{min} = \pm 1$$

or $$\boxed{\theta_{min} = 0^\circ \text{ and } 180^\circ} \quad \ldots 7.5\text{ (b)}$$

Half power point direction. At half power points power is $\frac{1}{2}$ or voltage or current is $1/\sqrt{2}$ times the maximum value of voltage or current.

$$\cos(\pi/2 \cos\theta) = \pm 1/\sqrt{2}$$

$\therefore$ $$\pi/2 \cos\theta_{HPPD} = \pm(2n+1)\pi/4. \quad \text{where } n = 0, 1, 2, \ldots$$

or $$\pi/2 \cos\theta_{HPPD} = \pm \pi/4$$

or $$\cos\theta_{HPPD} = \pm \frac{1}{2}$$

or $$\boxed{\theta_{HPPD} = 60^\circ, 120^\circ} \quad \ldots 7.5\text{ (c)}$$

If now the field pattern bet. E versus θ is drawn for the case $d = \lambda/2$, then the Fig. 7.9 (b) is obtained which is a bidirectional, figure of eight. 360° rotation of this figure around x-axis will generate the 3-dimensional space pattern — **a doughnut shape.**

This is the simplest type of "**broad-side array**" and is also known as "**broad-side couplet**" as two isotropic radiators radiates in phase.

As an alternative, if reference point in Fig. 7.9 (a) is shifted to say source 1 (instead midway of the array then amplitude of the field pattern remains the same (e.g. $2E_0$) but the phase pattern changes as shown below.

Fig. 7.10. Two point source of equal amplitude and phase, separated by a distance d.

The resultant far field pattern, in this case, is the vector sum of the fields of individual sources at the distant point P.

$\therefore$ $$E = E_1 e^{j0} + E_2 e^{j\psi} = E_1 + E_2 e^{j\psi} \quad | \because e^{j0} = e^0 = 1 \quad \ldots 7.6\text{ (a)}$$

where $$\psi = \beta d \cos\theta$$

Applying the condition 7.4, we have

$$E = E_0(1 + e^{j\psi}) = E_0 e^{j\psi/2}(e^{-j\psi/2} + e^{j\psi/2}) \quad | \text{ Taking } e^{+j\psi/2} \text{ common}$$

or $$= 2E_0 e^{j\psi/2}\left(\frac{e^{-j\psi/2} + e^{j\psi/2}}{2}\right) = 2E_0 e^{j\psi/2}\cos\psi/2$$

$$\boxed{\begin{array}{l} E = \underset{\text{Amp.}}{(2E_0)}\ \underset{\text{Phase}}{(\cos\psi/2\, e^{j\psi/2})} \\ E_{norm} = (\cos\psi/2\ e^{j\psi/2}) \end{array}} \quad \ldots 7.7\text{ (a)}$$

or $| \because 2E_0 = 1 \quad \ldots 7.7\text{ (b)}$

Thus, comparison of 7.7. (b) and 7.4 (a) indicates that phases are not the same. Eqn. 7.7 (b) may be rewritten as

$$E_{norm} = \cos\psi\left\{\cos\frac{\psi}{2} + j\sin\frac{\psi}{2}\right\} \quad \ldots 7.8\text{ (a)}$$

$$= \cos\psi\left\{e^{j\frac{\psi}{2}}\right\} \quad \ldots 7.8\text{ (b)}$$

8a

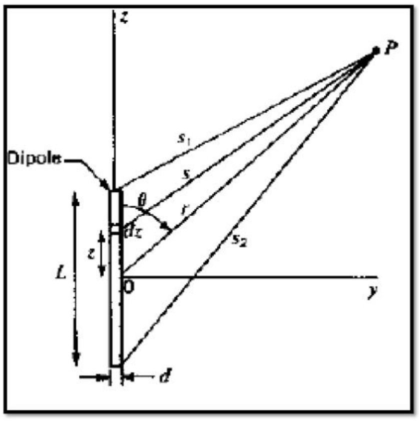


Fig 3a: Geometry for short dipole

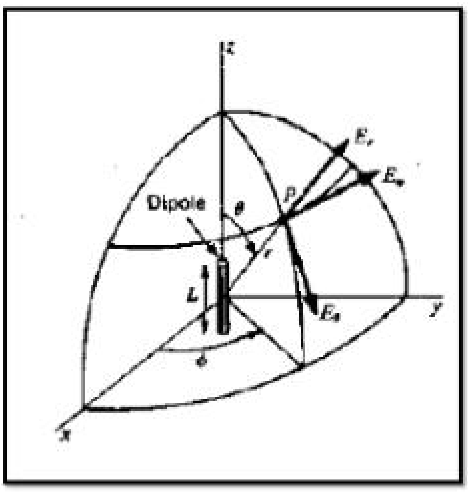


Fig 2: Relation of dipoles to coordinates

Fields of a short dipole

- The fields from dipole have only three components E_r, E_θ and H_φ.
- When r is very large, the terms in $1/r^2$ and $1/r^3$ in (29), (30) and (33) can be neglected in favour of the terms in $1/r$.
- In the far-field E_r is negligible and we have effectively only two field components E_θ and H_φ given by
- $E_\theta = \frac{I_0 L \sin\theta \, e^{j\omega(t-(r/c))}}{4\pi\varepsilon_0} (\frac{j\omega}{c^2 r})$ (34)
- $H_\varphi = \frac{I_0 L \sin\theta \, e^{j\omega(t-(r/c))}}{4\pi} (\frac{j\omega}{cr})$ (35)

10a

Radiation Resistance of a Short Dipole

- $S_r = \frac{1}{2} Re(E_\theta \text{ x } H_\varphi{}^*)$ (2)
- Where E_θ and H_φ are complex.
- The far-field components are related by the intrinsic impedance of the medium.
- $E_\theta = H_\varphi Z = H_\varphi \sqrt{\frac{\mu}{\varepsilon}}$ (3)
- Therefore (2) now becomes
- $S_r = \frac{1}{2} Re \; ZH_\varphi \; H_\varphi{}^* = \frac{1}{2} |H_\varphi|^2 \sqrt{\frac{\mu}{\varepsilon}}$ (4)

Radiation Resistance of a Short Dipole

- The total power P radiated is then
- $P = \iint S_r \, ds = \frac{1}{2} \sqrt{\frac{\mu}{\varepsilon}} \int_0^{2\pi} \int_0^{\pi} |H_\varphi|^2 r^2 \sin\theta \, d\theta d\varphi$ (5)
- $|H_\varphi| = \frac{\omega I_0 L \sin\theta}{4\pi c r}$ (6)
- Substituting this into (5), we have
- $P = \frac{1}{32} \sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{\pi^2} \int_0^{2\pi} \int_0^{\pi} \sin^3\theta \, d\theta d\varphi$ (7)

Radiation Resistance of a Short Dipole

- $\sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi} = (\frac{I_0}{\sqrt{2}})^2 R_r$ (9)
- Solving for R_r,
- $R_r = \sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 L^2}{6\pi}$ (10)
- For air or vacuum $\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 = 120\pi\Omega$, so that (10) becomes
- Dipole with uniform current :
- $R_r = 80\pi^2 (\frac{L}{\lambda})^2$ (11)

9a

Small loop

- The field pattern of a small circular loop of radius "***a***" may be determined by considering a square loop of the same area, that is,
- $d^2 = \pi a^2$ (1)
- Where d is side length of square loop as shown in Fig 1
-

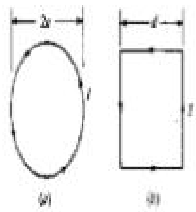


Fig1: Circular loop (a) and square loop (b) of equal area

Small loop

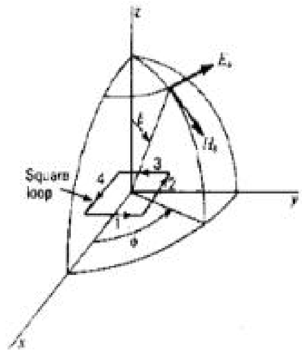


Fig2: Relation of square loop to coordinates

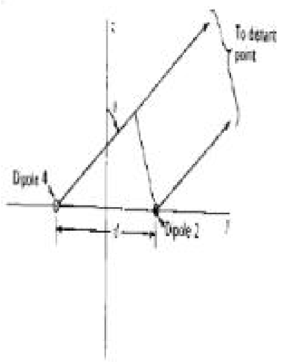


Fig3: Construction for finding far field of dipoles 2 and 4 of square loop

Small loop

- A cross section through the loop in the yz plane is presented in Fig 3.
- Since the individual small dipoles 2 and 4 are non-directional in the yz plane, the field pattern of the loop in this plane is the same as that for two isotropic point sources as treated earlier.
- $E_\varphi = -E_{\varphi 0}e^{j\psi/2} + E_{\varphi 0}e^{-j\psi/2}$ (2)
- Where $E_{\varphi 0}$ is electric field from individual dipole and
- $\psi = d_r sin\theta = \frac{2\pi d}{\lambda} sin\theta$ (3)

Small loop

- Substituting (6) in (5) then gives
- $E_\phi = \frac{60\pi[I]Ld_r sin\theta}{r\lambda}$ (7)
- However, the length L of the short dipole is the same as d, that is, L=d.
- Noting also that $d_r = \frac{2\pi d}{\lambda}$ and that the area A of the loop is d^2, (7) becomes
- $E_\phi = \frac{120\pi^2[I]sin\theta}{r}\frac{A}{\lambda^2}$ (8)

Small loop

- This is the instantaneous value of the E_ϕ component of the field of a small loop of area A.
- The peak value of the field is obtained by replacing [I] by I_0, where I_0 is the peak current in time on the loop.
- The other component of the far field of the loop is H_θ, which is obtained by the intrinsic impedance of the medium, in this case, free space.
- $H_\theta = \frac{E_\phi}{120\pi} = \frac{\pi[I]\ sin\theta}{r}\frac{A}{\lambda^2}$ (9)

Radiation Resistance of Loops

- $P = \frac{I_m^2}{2} R_r$
- $P = \iint S_r \, ds,$
- $S_r = \frac{1}{2}|H|^2\eta,$
- $ds = r^2 sin\theta d\theta d\phi$
- $R_r = 31{,}171(\frac{A}{\lambda^2})^2 = 197 C_\lambda^4$

Where
- R_r is the radiation resistance of the loop antenna
- P is power radiated,
- I_m is peak value of current from loop,
- S_r is the radial component of the Poynting vector,
- ds is the area of small region in the sphere,
- η is the intrinsic impedance of free space equal to 120π Ω,
- A is the area of the loop,
- C_λ is the circumference of the loop = $\frac{2\pi a}{\lambda} = \beta a$

10 b

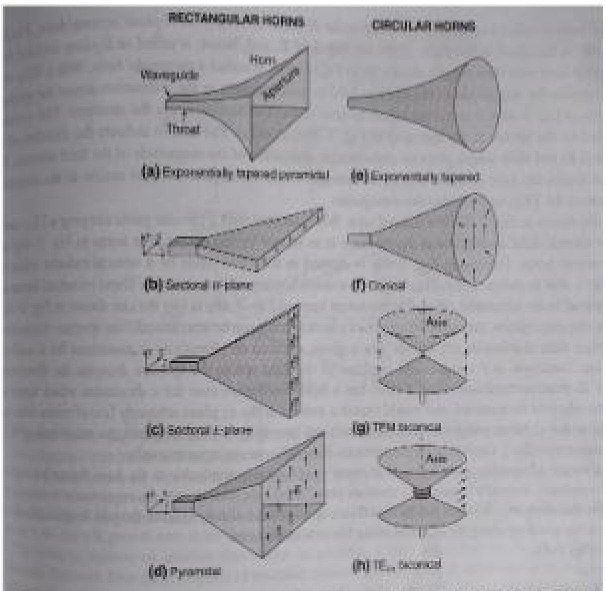


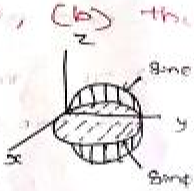
Figure 7-40 Types of rectangular and circular horn antennas. Arrows indicate E-field directions.

Horn antennas

- A horn antenna may be regarded as a flared-out (or opened-out) waveguide.
- The function of the horn is to produce a uniform phase front with a larger aperture than that of the waveguide and hence greater directivity.
- Several types of antennas are illustrated in Fig 7.40.
- Rectangular horns are energized from rectangular waveguides.
- To minimise reflections of the guided wave, the transition region or horn between the waveguide at the throat and free space at the aperture could be given a gradual exponential taper as shown in the Fig 7.40 a or e.

8b

Find (a) the exact directivity, (b) the approximate directivity, (c) the decibel difference



(a) $$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega}$$

Consider $$\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\, d\Omega$$

$$= \int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} [E_n(\theta,\phi)]^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} [\sin\theta \cdot \sin\phi]^2 \sin\theta\, d\theta\, d\phi$$

$$= \int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} \sin^3\theta \sin^2\phi\, d\theta\, d\phi$$

$$= \int_{\phi=0}^{\pi} \sin^2\phi\, d\phi \int_{\theta=0}^{\pi} \sin^3\theta\, d\theta$$

$$= I_1 \times I_2 \quad (1)$$

$$I_1 = \int_{\phi=0}^{\pi} \sin^2\phi\, d\phi$$

$$= \int_{\phi=0}^{\pi} \frac{1-\cos 2\phi}{2}\, d\phi$$

$$= \frac{1}{2}\left\{ \phi\Big]_{\phi=0}^{\pi} - \frac{\sin 2\phi}{2}\Big]_{\phi=0}^{\pi} \right\}$$

$$= \frac{1}{2}(\pi - 0) = \frac{\pi}{2}$$

$$I_2 = \int_{\theta=0}^{\pi} \sin^3\theta\, d\theta$$

[Note: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$]

$$= \int_{\theta=0}^{\pi} \frac{1}{4}\left[3\sin\theta - \sin 3\theta\right] d\theta$$

$$= \frac{1}{4}\left\{3\left(-\cos\theta\right]_{\theta=0}^{\pi} + \frac{\cos 3\theta}{3}\right]_{\theta=0}^{\pi}\right\}$$

$$= \frac{1}{4}\left[-3\left(-1-1\right) + \frac{1}{3}\left(-1-1\right)\right]$$

$$= \frac{1}{4}\left(6 - \frac{2}{3}\right) = \frac{1}{4} \times \frac{16}{3} = \frac{4}{3}$$

$\therefore$ Equation (1) becomes,

$$\int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) = \frac{\pi}{2} \times \frac{4}{3} = \frac{2\pi}{3}.$$

$$D = \frac{4\pi}{2\pi/3} = 6$$