CBCS SCHEME

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BCS401

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2024
Analysis and Design of Algorithms

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	What is an algorithm? Explain the fundamentals of algorithmic problem solving.	10	L2	CO1
	b.	Develop an algorithm to search an element in an array using sequential search. Calculate the best case, worst case and average case efficiency of this algorithm.	10	L3	CO1
		OR			
Q.2	a.	Explain asymptotic notations with example.	10	L2	CO1
	b.	Give the general plan for analyzing the efficiency of the recursive algorithm. Develop recursive algorithm for computing factorial of a positive number. Calculate the efficiency in terms of order of growth.	10	L3	CO1
		Module – 2			
Q.3	a.	Explain Strassen's matrix multiplication approach with example and derive its time complexity.	10	L3	CO2
	b.	What is divide and conquer? Develop the quick sort algorithm and write its best case. Make use of this algorithm to sort the list of characters: E, X, A, M, P, L, E.	10	L2	CO2
		OR			
Q.4	a.	Distinguish between decrease & conquer and divide & conquer algorithm design techniques with block diagram. Develop insertion sort algorithm to sort a list of integers and estimate the efficiency.	10	L3	CO2
	b.	Define topological sorting. List the two approaches of topological sorting and illustrate with examples.	10	L2	CO2
		Module – 3			
Q.5	a.	Define AVL tree with an example. Give worst case efficiency of operations on AVL tree. Construct an AVL tree of the list of keys: 5, 6, 8, 3, 2, 4, 7 indicating each step of key insertion and rotation.	10	L3	CO3
	b.	Define Heap. Explain the bottom-up heap construction algorithm. Apply heap sort to sort the list of numbers 2, 9, 7, 6, 5, 8 in ascending order using array representation.	10	L3	CO3
		OR			
Q.6	a.	Define 2-3 tree. Give the worst case efficiency of operations on 2-3 tree. Build 2-3 tree for the list of keys 9, 5, 8, 3, 2, 4, 7 by indicating each step of key insertion and node splits.	10	L3	CO3
	b.	Design Horspool algorithm for string matching. Apply this algorithm to find the pattern BARBER in the text: JIM_SAW_ME_IN_A_BARBERSHOP	10	L3	CO3
		Module – 4			
Q.7	a.	Apply Dijkstra's algorithm to find the single source shortest path for given graph [Fig.Q7(a)] by considering 's' as source vertex. Illustrate each step.	10	L3	CO4

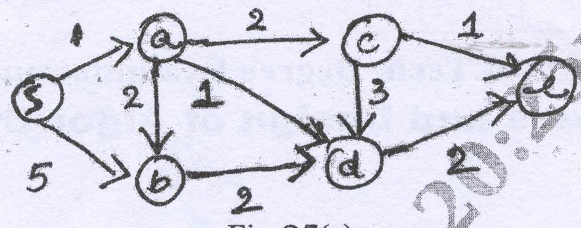

Fig.Q7(a)

b. Define transitive closure. Write Warshall's algorithm to compute transitive closure. Illustrate using the following directed graph. — 10 | L3 | CO4

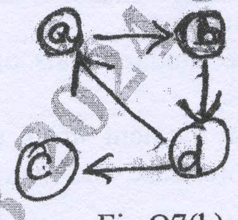

Fig.Q7(b)

OR

Q.8 a. Define minimum spanning tree. Write Kruskal's algorithm to find minimum spanning tree. Illustrate with the following undirected graph. — 10 | L3 | CO4

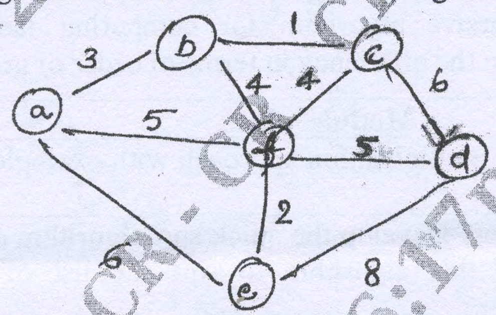

Fig.Q8(a)

b. Construct Huffman Tree and resulting code for the following: — 10 | L3 | CO4

Character	A	B	C	D	-
Probability	0.4	0.1	0.2	0.15	0.15

(i) Encode the text : ABACABAD

(ii) Decode the text : 100010111001010

Module – 5

Q.9 a. Explain n-Queen's problem with example using backtracking approach. — 10 | L2 | CO5

b. Solve the following instance of the knapsack problem by the branch-and-bound algorithm. Construct state-space tree. — 10 | L3 | CO5

Item	Weight	Value
1	4	$ 40
2	7	$ 42
3	5	$ 25
4	3	$ 12

The knapsack's capacity W is 10.

OR

Q.10 a. Differentiate between Branch and Bound technique and Backtracking. Apply backtracking to solve the following instance of subset-sum problem S = {3, 5, 6, 7} and d = 15. Construct a state space tree. — 10 | L3 | CO5

b. Explain greedy approximation algorithm to solve discrete knapsack problem. — 10 | L2 | CO5

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Subject Title : Analysis & Design of Algorithms **Subject Code :** BCS401

Question Number	Solution	Marks Allocated
1.a.	Algorithm Definition →	1M
	Diagram :	

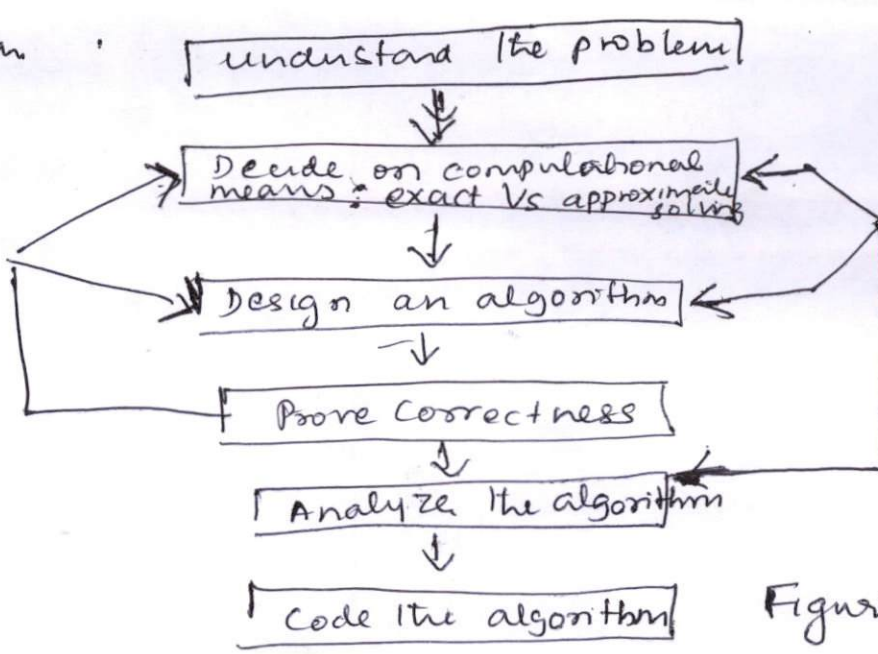

	Figure →	3M
	explanation of each step →	6M
	total →	10M
1.b.	Sequential Search Algorithm →	~~4~~M
	efficiencies - best case $C_{best}(n) = 1$ →	1M
	worst case $C_{worst}(n) = n$ →	2M
	average case efficiency ~~average~~ case:	
	$C_{avg}(n) = [1 \cdot P/n + 2 \cdot P/n + \dots + n \, P/n] + n(1-P)$	
	$= P/n \, [1 + 2 + \dots + n] + n(1-P)$	

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Question Number	Solution	Marks Allocated
	$= \frac{P}{n} \frac{n(n+1)}{2} + n(1-P) = \frac{P(n+1)}{2} + n(1-P)$	1M
	for successful search if $P = 1$ ~~$C_{avg} = \frac{(n+1)}{2}$~~ $C_{avg}(n) = \frac{n+1}{2}$ →	1M
	for unsuccessful search if $P = 0$ $C_{avg}(n) = n$ →	1M
	total	10M
2.a.	Asymptotic notations: Big-oh (O) → definition, figure & example →	3M
	Big-omega (Ω) - definition, figure & example →	3M
	Big-theta (θ) - definition, figure & example	4M
	total →	10M
2.b.	General plans listing: → 1. Decide on a parameter indicating an input size. 2. Identify the algorithm's basic operation 3. Check whether the no. of times the basic operation executed can vary on different i/ps of same size.	2M
	4. Set up a Recurrence relation with an 5. initial condition. 5. Solve the recurrence, ascertain the order of growth	1M

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Question Number	Solution	Marks Allocated
	~~Alg.~~ Recursive algorithm for computing factorial number Algorithm F(n) if n=0 return 1 else return F(n-1)* n	2M
	Recurrence relation : $M(n) = M(n-1) + 1$ for n>0 $M(0) = 0$	2M
	Solving recurrence relation using bwd substitutions	3M
3.a.	Strassen's Matrix Multiplication formula: $\begin{bmatrix} c_{00} & c_{01} \\ c_{10} & c_{11} \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} * \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{bmatrix}$ $= \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix}$ where $m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$ $m_2 = (a_{10} + a_{11}) \times b_{00}$ $m_3 = a_{00} \times (b_{01} - b_{11})$ $m_4 = a_{11} \times (b_{10} - b_{00})$ $m_5 = (a_{00} + a_{01}) \times b_{11}$ $m_6 = (a_{10} - a_{00}) \times (b_{00} + b_{01})$ $m_7 = (a_{01} - a_{11}) \times (b_{10} + b_{11})$	3M

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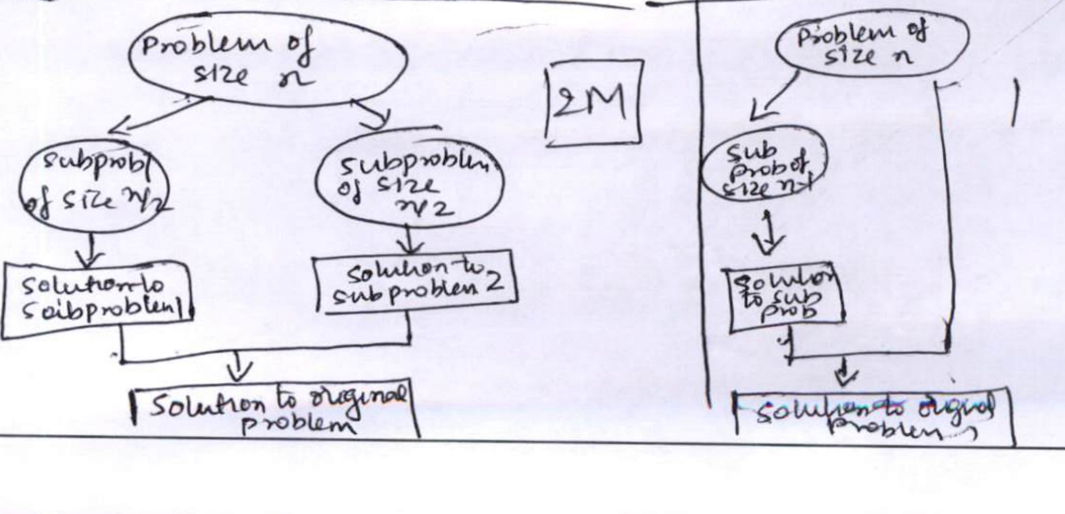
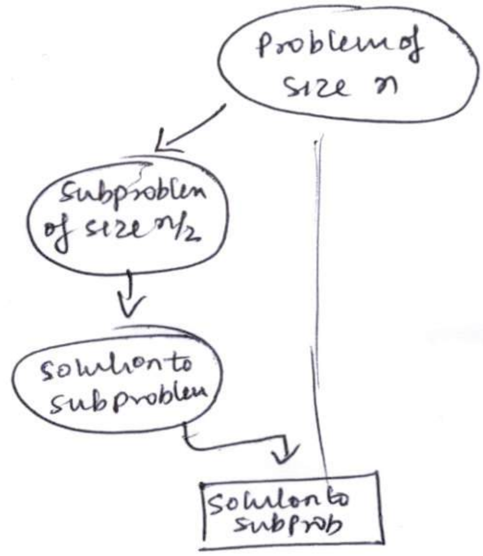
Question Number	Solution	Marks Allocated
	explanation ⟶	2M
	Recurrence relation of strassen's matrix multiplication $M(n) = 7M(n/2)$ for $n>1$ $M(1) = 1$	2M
	derivation with solution $M(n) = 7^{\log_2 n} = n^{\log_2 7} \simeq n^{2.807}$	3M
3.b.	Definition of Divide and Conquer ⟶	1M
	(or) quicksort algo : ⟶	4M
	Sorting the list of characters ⟶	4M
	Quicksort best case efficiency ⟶ $\Theta(n \log n)$	1M
	Total ⟶	10M
4.a.	Divide and Conquer technique :	2M
	Decrease by one and conquer technique	2M



Question Number	Solution	Marks Allocated
	Decrease by half conquer technique:	~~2M~~



~~2M~~

Insertion sort algorithm ⟶ 4M

```
for i ← 1 to n-1 do
    v ← A[i]
    j ← i-1
    while j ≥ 0 and A[j] > v do
        A[j+1] ← A[j]
        j ← j-1
    A[j+1] ← v
```

$$C_{worst}(n) = \Theta(n^2)$$

$$C_{best}(n) = \Theta(n)$$

$$C_{avg}(n) = \frac{n^2}{4} \in \Theta(n^2)$$

3M

Question Number	Solution	Marks Allocated
4.b.	Definition of topological sorting : ⟶	1M
	~~List~~ 2 approaches → 1) Source removal 2) DFS based	2M
	Illustration with example (2 approaches)	7M

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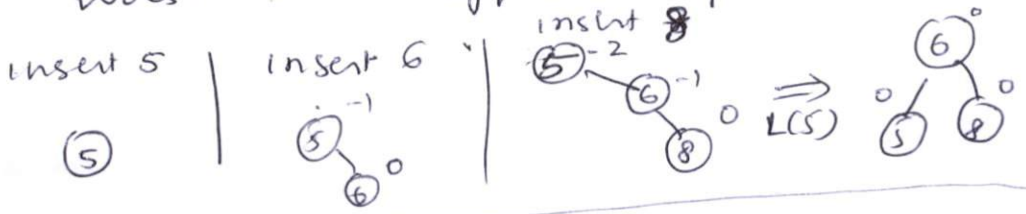
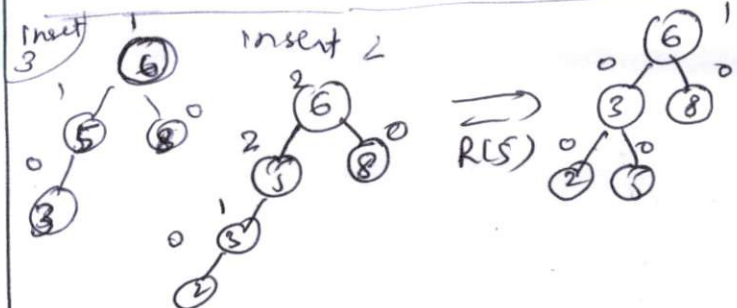
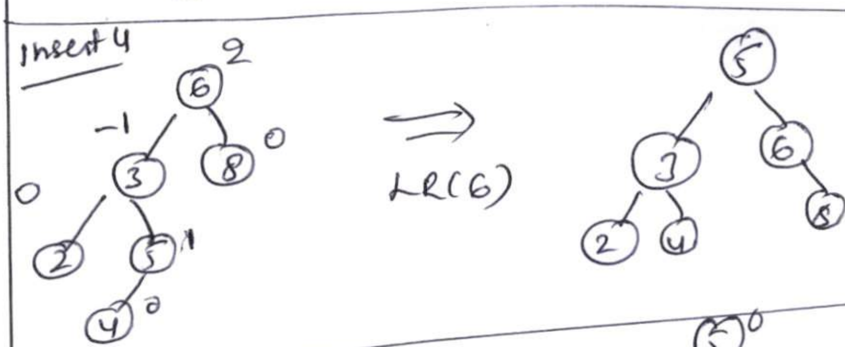
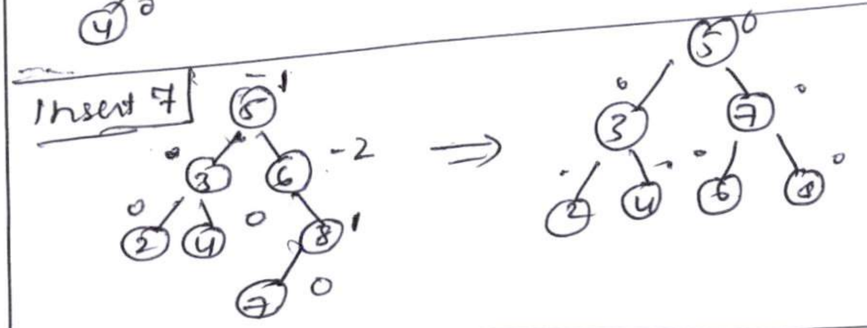
Question Number	Solution	Marks Allocated
5.a.	AVL definition with example →	2M
	worst case efficiency → $\Theta(\log n)$	1M
		1M
		2M
		2M
		2M
	total	10
5.b.	Heap definition & ~~example~~ →	1M
	bottomup construction algorithm (next page) →	3M
	2,9,7,6,5,8 → heap construction (in array rep)	3M
	Sorting the numbers (array rep)	3M

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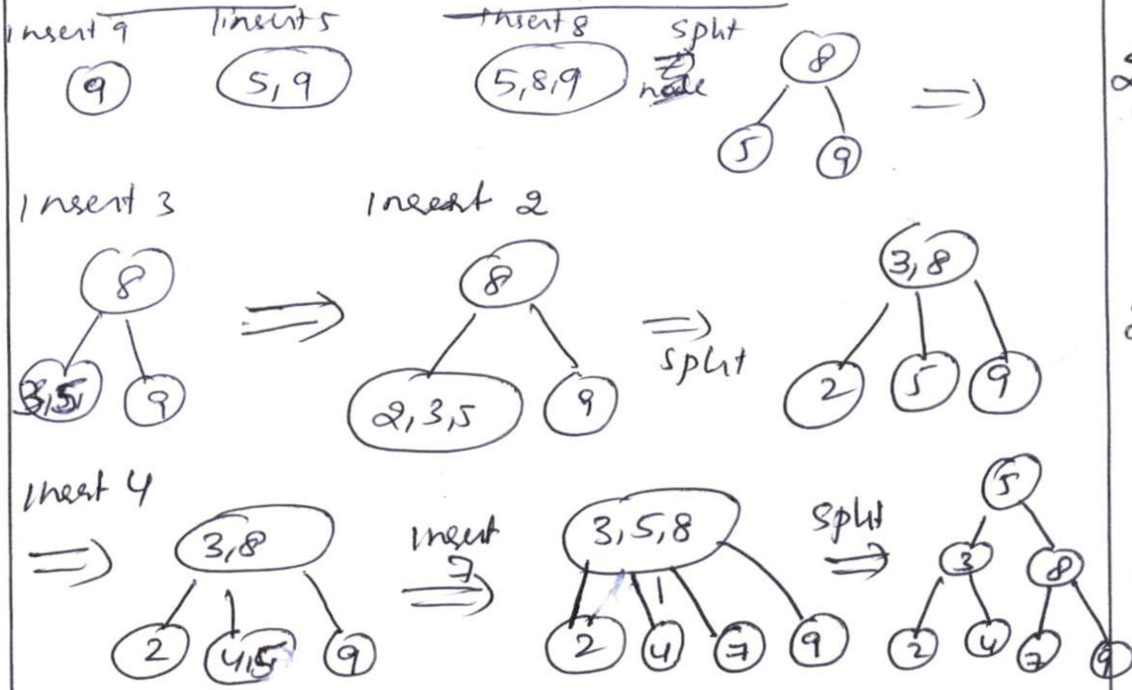
Question Number	Solution	Marks Allocated
5.b cont.	**Bottom up heap construction algorithm:**	

```
for i ← ⌊n/2⌋ downto 1 do
    k ← i   v ← H[k]
    heap ← false
while not heap and 2*k ≤ n
    j ← 2*k
    if j < n
        if H[j] < H[j+1]  j ← j+1
    if v ≥ H[j]
        heap ← true
    else H[k] ← H[j] ; k ← j
H[k] ← v
```

<u>10</u>

Question Number	Solution	Marks Allocated
6.a	Definition 2-3 tree & ~~example~~ →	2M
	worst case efficiency →	1M
	construction 2-3 tree 9, 5, 8, 3, 2, 4, 7	



2M

2M

3M

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Question Number	Solution	Marks Allocated
6. b.	Horspool's algorithm	4M

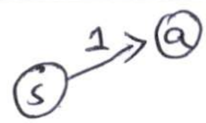
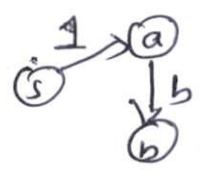
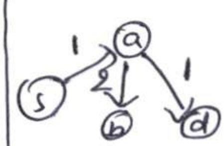
Shift table for BARBER — 2M

A	B	C	D	E	F	......	R	...	Z	_
4	2	6	6	1	6	6	3	6 ..	6	6

actual search — 4M, 4M

```
T: JIM_SAW_ME_IN_A_BARBERSHOP
P: BARBER
         BARBER
           BARBER
                BARBER
                  BARBER
                     BARBER
                     matched.
```

7.a.

Tree vertices	Remaining Vertices	Illustrate	Marks Allocated
s(-,0)	a(s,1) b(s,5) c(-,∞) d(-,∞) e(-,∞)		1M
a(s,1)	b(a,3) c(a,3) d(a,2) e(-,∞)		2M
~~b(a,3)~~	(b or c) any one can be selected to proceed as both are having same distance		
b(a,3)	c(a,3) d(a,2) e(-,∞)		2M

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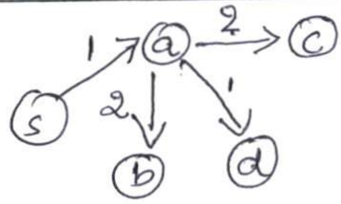
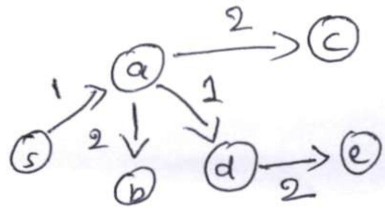
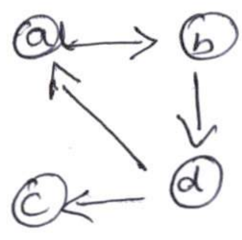
Question Number	Solution	Marks Allocated
	d(a,2) | <u>c(a,3)</u> e(d,4) 	2M
	c(a,3) | e(d,4) 	2M
	e(d,4)	1M
	Shortest path $s \to a$ is 1 $s \to a - b$ is 3 $s \to a \to c$ is 3 | $s \to a - d$ is 2 $s \to a \to d \to e$ is 4	1M
7.b.	Definition of transitive closure →	1M
	~~transit~~ Warshall's algorithm →	3M
	Adjacency matrix  → $\begin{bmatrix} 0&1&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 1&0&1&0 \end{bmatrix}$	1M
	$R^{(0)} = \begin{bmatrix} 0&1&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 1&0&1&0 \end{bmatrix}$ $R^{(1)} = \begin{bmatrix} 0&1&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 1&1&1&0 \end{bmatrix}$	2M
	$R^{(2)} = \begin{bmatrix} 0&1&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \\ 1&1&1&1 \end{bmatrix}$ $R^{(3)} = \begin{bmatrix} 0&1&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \\ 1&1&1&1 \end{bmatrix}$	2M

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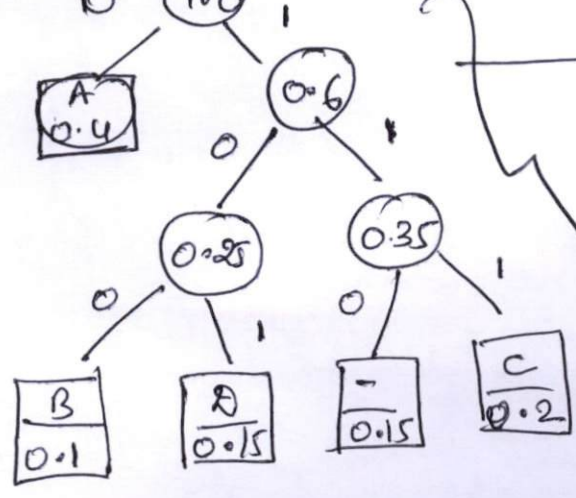
Question Number	Solution	Marks Allocated
	$R^{(4)}: \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	2M
	$R^{(4)}$ is the transitive closure.	
	~~Efficiency is $O(n^3)$~~	
8.a.	Definition of MST ⟶	1M
	algorithm ⟶	3M
	Steps to derive MST 5 steps ⟶ (bc, ef, ab, bf, df edges)	6M
8.b.	~~Final~~ Stepwise presentation of Huffman tree	
	 Code ⟶	4M
	A → 0 B → 100 C → 111 D → 101 – → 110	2M

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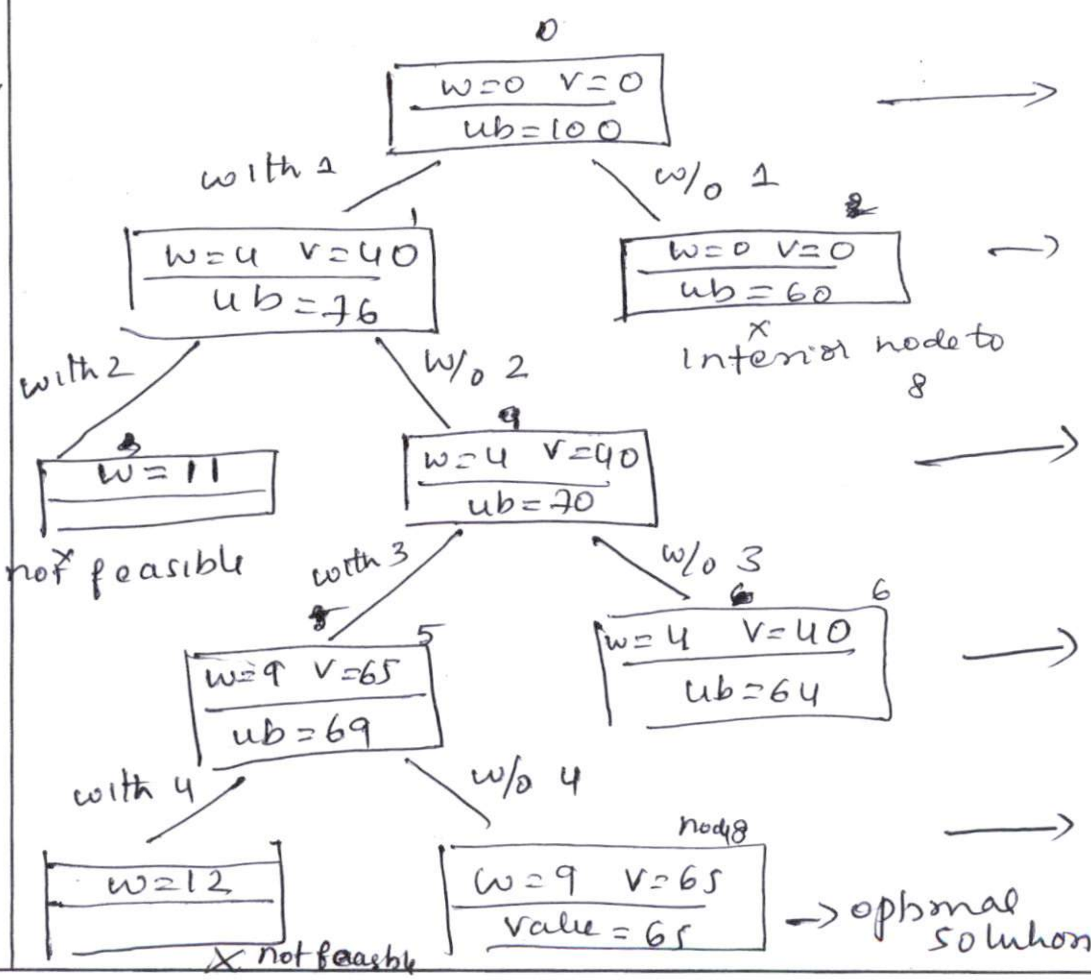
Question Number	Solution	Marks Allocated
	~~Avg # of bits per symbol = 2.2 bits~~	~~1M~~
	~~Fixed length encoding require = 3 bits~~	~~1M~~
	~~Compression ratio = $\left(\frac{3-2.2}{3}\right)\times 1$~~ ~~$= \left(\frac{3-2.2}{3}\right)\times 100\%$~~ ~~$= 26.67\%$~~	~~2M~~
	Encode text ⟶	2M
	Decode text ⟶	2M
9.a.	n-queen's problem explanation —	4M
	example with statespace tree —	6M
9.b.		2M 3M 2M 2M 1M

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Question Number	Solution	Marks Allocated
10.a	Difference → Any 2 × 1M	2M
	State space tree for subset sum problem →	8M
10.b	discrete knapsack problem Algorithm	4M
	Explanation	6M