CBCS SCHEME

USN | | | | | | | | | | |

18EE63

Sixth Semester B.E. Degree Examination, July/August 2022
Digital Signal Processing

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Compute 4-point DFT of casual three sample sequence given by

$$x(n) = \frac{1}{3};\ 0 \le n \le 2$$
$$= 0\ ;\ \text{else.}$$ **(06 Marks)**

b. State and prove linearity property of DFT. **(06 Marks)**

c. Find the circular convolution of two finite duration sequences $x_1(n)$ and $x_2(n)$ using concentric circle method. Where $x_1(n)$ and $x_2(n)$ are given by

$x_1(n) = \{1, -1, -2, 3, -1\}$

$x_2(n) = \{1, 2, 3\}$. **(08 Marks)**

OR

2 a. Compute circular convolution using Stockham's method for following sequences:
$x_1(n) = \{2, 3, 1, 1\}$ and $x_2(n) = \{1, 3, 5, 3\}$. **(10 Marks)**

b. Find the output y(n) of a filter whose impulse response h(n) = (1, 2) and input signal x(n) = {1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1} using overlap save method. Use block length of N = 4. **(10 Marks)**

Module-2

3 a. Develop decimation in time algorithm for finding FFT. Draw signal flow graph for N = 8 for DIT algorithm. **(10 Marks)**

b. Find the 8 point DFT of sequence x(n) = {1, 1, 0, 0, -1, -1, 0, 0} using DIT FFT algorithm. Draw signal flow graph. **(10 Marks)**

OR

4 a. Develop a decimation in frequency FFT algorithm for N = 8. Draw signal flow graph. **(10 Marks)**

b. The DFT X(k) of sequence is given as, $X(k) = \{0, 2\sqrt{2}(1-j), 0, 0, 0, 0, 0, 2\sqrt{2}(1+j)\}$. Determine the corresponding time sequence x(n) using DIF-FFT algorithm. Write its signal flow graph. **(10 Marks)**

Module-3

5 a. A system function of the normalized lowpass filter is given below:

$H(s) = \dfrac{1}{s^2 + \sqrt{2}s + 1}$. Determine H(z) using impulse invariant transformation. Consider T = 1sec. **(08 Marks)**

b. Design an analog filter with maximally flat response in the passband and an acceptable attenuation of -2dB at 20radians/second. The attenuation in the stop band should be more than 10dB beyond 30 radian/second. **(12 Marks)**

OR

6 a. Transform the analog filter $H(s)=\dfrac{s+0.1}{(s+0.1)^2+9}$ into a digital filter using bilinear transformation. The digital filter should have resonant frequency $w_r = \pi/4$. **(05 Marks)**

b. Design an analog Chebyshev filter with the following specifications:
Passband ripple: 1dB for $0 \le \Omega \le$ rad/sec.
Stopband attenuation : -60dB for $\Omega \ge 50$ rad/sec. **(10 Marks)**

c. Let $H(s)=\dfrac{1}{s^2+\sqrt{2}s+1}$ represent the transfer function of a lowpass filter with a passband of 1 rad/sec. Use frequency transformation to find the transfer functions of the following analog filters.
i) A lowpass filter with pass band of 10rad/sec
ii) A high pass filter with cut-off frequency of rad/sec. **(05 Marks)**

Module-4

7 a. Compare Butterworth and Chebyshev filter approximations. **(05 Marks)**

b. Design a digital low pass filter to satisfy the following pass band ripple $1 \le H(j\Omega) \le 0$, for $0 \le \Omega \le 1404\pi$ rad/sec and stop band attenuation $|H(\Omega)| > 60\text{dB}$ for $\Omega \ge 8268\ \pi$ rad/sec sampling interval $T_s = \dfrac{1}{10^4}$ sec. Use BLT for designing. **(15 Marks)**

OR

8 A discrete time system H(z) is expressed as

$$H(z)=\frac{10\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{2}{3}z^{-1}\right)\left(1+2z^{-1}\right)}{\left(1-\frac{3}{4}z^{-1}\right)\left(1-\frac{1}{8}z^{-1}\right)\left[1-\left(\frac{1}{2}+j\frac{1}{2}\right)z^{-1}\right]\left[1-\left(\frac{1}{2}-j\frac{1}{2}\right)z^{-1}\right]}.$$

a. For the discrete time system defined by H(z), find the difference equation of the system. **(02 Marks)**

b. For the discrete time system, H(z) realize the system in direct form-I and II. **(08 Marks)**

c. For the discrete time system H(z), realize parallel and cascade forms using second order sections. **(10 Marks)**

Module-5

9 a. The desired frequency response of the low pass filter is given by

$$H_d(e^{jw}) = H_d(w) = \begin{cases} e^{-j3w}; & |w| < 3\pi/4 \\ 0; & 3\pi/4 < |w| < \pi \end{cases}$$

Determine the frequency response of FIR filter if the hamming window is used, with N = 7. **(08 Marks)**

b. Design an ideal band pass filter with frequency response.
$H_d(e^{jw}) = 1$, for $\dfrac{\pi}{4} \le |w| \le \dfrac{3\pi}{4}$. Use rectangular window with N = 11 in the design. **(12 Marks)**

OR

10 a. Determine the impulse response h(n) of a filter having desired frequency response.

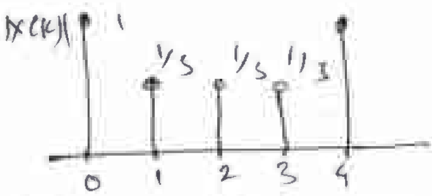
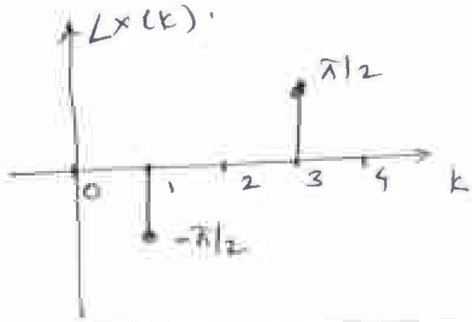
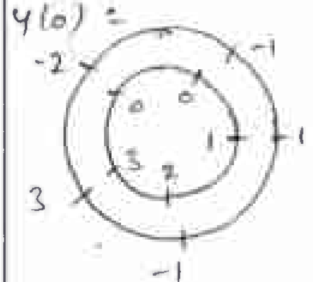
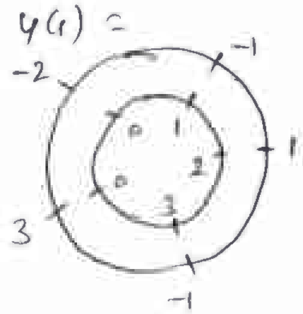
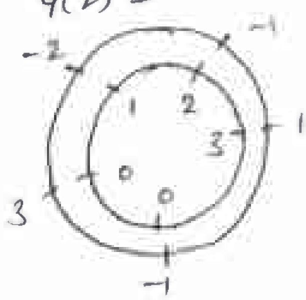
$$H_d(e^{jw}) = \begin{cases} e^{-j(N-1)w/2} & \text{for } 0 \le |w| \le \pi/2 \\ 0 & \text{for } \frac{\pi}{2} \le |w| \le \pi \end{cases}$$

N = 7, use frequency sampling approach. **(10 Marks)**

b. Realize the following system function $H(z) = 1 + \frac{3}{4}z^{-1} + \frac{17}{8}z^{-2} + \frac{3}{4}z^{-3} + z^{-4}$ in
i) Direct form ii) Cascaded form. **(10 Marks)**

Scheme & Solutions

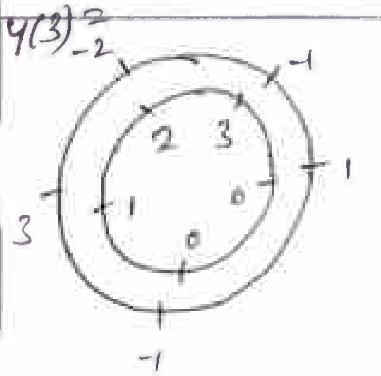
Subject Title : Digital Signal Processing **Subject Code :** 18EE63.

Question Number	Solution	Marks Allocated
Ans. 1a	$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$, Here $N=4$ ∴ 4 point DFT is.	1
	$X(k) = \sum_{n=0}^{3} x(n) e^{-j2\pi kn/4} = \frac{1}{3} + \frac{1}{3} e^{-j\pi k/2} + \frac{1}{3} e^{-j\pi k} + 0$.	1
	$X(k) = \frac{1}{3}\left[1 + \cos\frac{\pi k}{2} - j\sin\frac{\pi k}{2} + \cos\pi k - j\sin\pi k\right]$.	2
	$k = 0, 1, 2, 3$. $X(k) = \{1\angle 0, \frac{1}{3}\angle -\pi/2, \frac{1}{3}\angle 0, \frac{1}{3}\angle \pi/2\}$.  Fig magnitude response $X(k)$ for $N=4$. 	2 1+1+2+2 = 6
Ans 1b	statement of linearity property → 2 Proof of linearity property → 4	2+4 = 6
Ans 1c	$x_1(n) = \{1, -1, -2, 3, -1\}$; $x_2(n) = \{1, 2, 3, 0, 0\}$	1
	$y(0) =$  $y(0) = 8$. $y(1) =$  $y(1) = -2$ $y(2) =$  $y(2) = -1$	

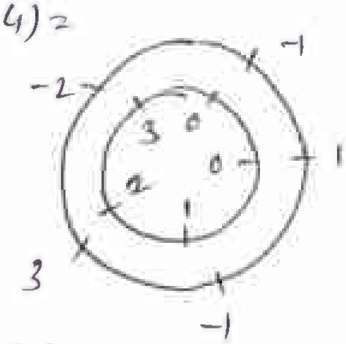
Subject Title : Digital Signal Processing Subject Code : 18EE63

Question Number	Solution	Marks Allocated

$y(3) =$



$y(4) =$



(6)

$y(3) = -4$ $\qquad$ $y(4) = -1$

$y(n) = \{8, -2, -4, -1\}$ (1)

1+6+1 = 8

Ans 2a

DFT of $x_1(n) = \{2, 3, 1, 1\}$ is

$X_1(k) = \{7, 1-j2, -1, 1+j2\}$ (3)

DFT of $x_2(n) = \{1, 3, 5, 3\}$ is (3)

$X_2(k) = \{12, -4, 0, -4\}$

$$X_1(k)X_2(k) = \begin{bmatrix} 7 \\ 1-j2 \\ -1 \\ 1+j2 \end{bmatrix} \begin{bmatrix} 12 \\ -4 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 84 \\ -4+j8 \\ 0 \\ -4-j8 \end{bmatrix}$$ (1)

Taking IDFT of $X_1(k) \cdot X_2(k)$ (3)

$x_1(n) \circledast x_2(n) = [19, 17, 23, 25]$

3+3+1+3 = 10

Ans 2b

Here $h(n) = \{1, 2\}$ and (1)

$x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$

$N = 4$, $\therefore N = M + L - 1 \Rightarrow 4 = 2 + L - 1 \Rightarrow L = 3$.

$\therefore M = 2, N = 4, L = 3$.

$x_1(n) = \{0, 1, 2, -1\}$; $x_2(n) = \{-1, 2, 3, -2\}$.

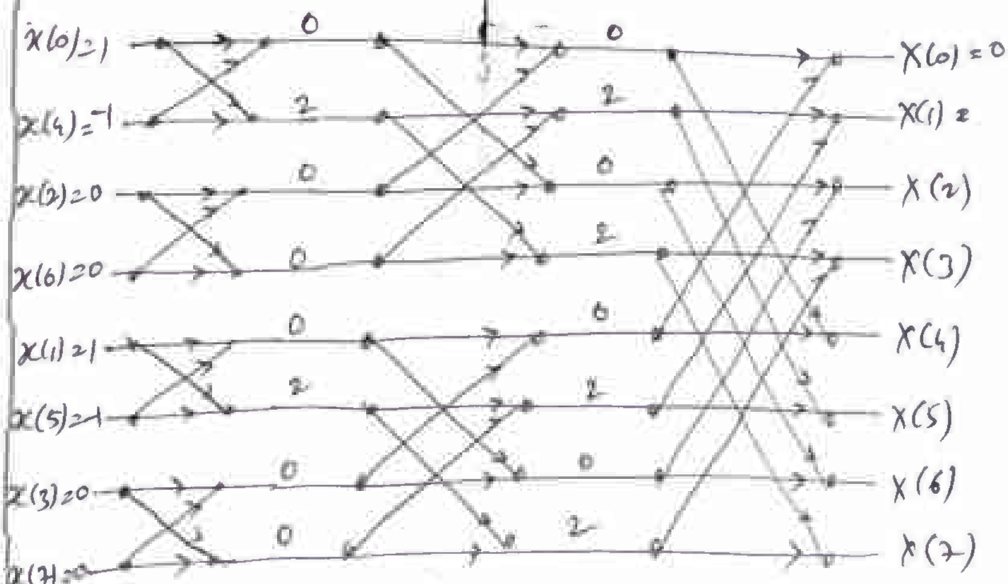
$x_3(n) = \{-2, -3, -1, 1\}$; $x_4(n) = \{1, 1, 2, -1\}$; (2)

$x_5(n) = \{-1, 0, 0, 0\}$.

$y_1(n) = x_1(n) \circledast_4 h(n) = \{-2, 1, 4, 3\}$.

Subject Title : Digital Signal Processing Subject Code : 18EE63

Question Number	Solution	Marks Allocated
	$y_2(n) = x_2(n)$ ④ $h(n) = \{-5, 0, 7, 4\}$	
	$y_3(n) = \{ x_3(n)$ ④ $h(n) = \{0, -7, -7, -1\}$.	
	$y_4(n) = x_4(n)$ ④ $h(n) = \{-1, 3, 4, 3\}$.	
	$y_5(n) = x_5(n)$ ④ $h(n) = \{-1, -2, 0, 0\}$ 5	
	$y_1(n) \Rightarrow -2\ 1\ 3\ 3$	$1+2+5$
	$y_2(n) \Rightarrow -5\ 0\ 7\ 4$	$+2$
	$y_3(n) \Rightarrow 0\ -7\ -7\ -1$ 2	$=10$
	$y_4(n) \Rightarrow -1\ 3\ 4\ 3$	
	$y_5(n) \Rightarrow -1\ -2\ 0\ 0$	
	$y(n) = 1\ 4\ 3\ 0\ 7\ 4\ -7\ -7\ -1\ 3\ 4\ 3\ -2\ \cancel{0\ 0}$	
	$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$.	
Ans 3a)	DFT is given by $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$.	
	Using even and odd separation of n. 2	
	$X(k) = \sum_{n=0}^{(N/2)-1} x(2n) W_N^{2nk} + \sum_{n=0}^{(N/2)-1} x(2n+1) W_N^{(2n+1)k}$	
	Note that $W_N^2 = \exp(-2j(2\pi/N)) = \exp[-j(2\pi/(N/2)] = W_{N/2}$	
	$\therefore X(k) = \sum_{n=0}^{(N/2-1)} x(2n) W_{N/2}^{nk} + W_N^k \sum_{n=0}^{(N/2)-1} x(2n+1) W_{N/2}^{nk}$ 1	
	Defining.	
	$X_{m-1,1}(k) = \sum_{n=0}^{(N/2)-1} x(2n) W_{N/2}^{nk}$.	
	$X_{m-1,2}(k) = \sum_{n=0}^{(N/2)-1} x(2n+1) W_{N/2}^{nk}$ 2	
	$X_m(k) = X_{m-1,1}(k) + W_N^k X_{m-1,2}(k) \xrightarrow[\text{Eq (1)}]{}$ ①	
	The N-point $X_{m-1,p}(k)$, $p = 1,2$ are separated into sets with each set representing N/2 point DFTs. The point DFT is periodic with period N. ∴ N/2 point DFTs have period N/2, that is	
	$X_{m-1,p}(k+N/2) = X_{m-1,p}(k)$.	

Question Number	Solution	Marks Allocated
	Eqn ① can be used directly when $0 \le k \le N/2-1$. For $N/2 \le k \le N-1$, it should be modified to account for the periodicity of the $N/2$ point DFTs $X_{m-1,1}(k)$ and $X_{m-1,2}(k)$. The resulting modification is ② $X_m(k+N/2) = X_{m-1,1}(k+N/2) + W_N^{k+N/2} X_{m-1,2}(k+N/2)$ $= X_{m-1,1}(k) - W_N^k X_{m-1,2}(k)$. N=8 point signal flow graph ③	2+1+2 +2+3 =10
Ans 3b)	$N=8$, $W_8^0 = 1$, $W_8^1 = 0.707 - j0.707$, $W_8^2 = -j$, $W_8^3 = -0.707 - j0.707$.  $X(k) = \{0, 3.41 - j1.41, 0, 0.586 - j1.41, 0, 0.586 + j1.41, 0, 3.41 + j1.41\}$.	② ⑧ 2+8 =10
Ans 4a	Decimation in frequency algorithm ⑦ Signal flow graph for DIF ③	7+3 =10

Subject Title : Digital Signal Processing **Subject Code :** 18EE63.

Question Number	Solution	Marks Allocated

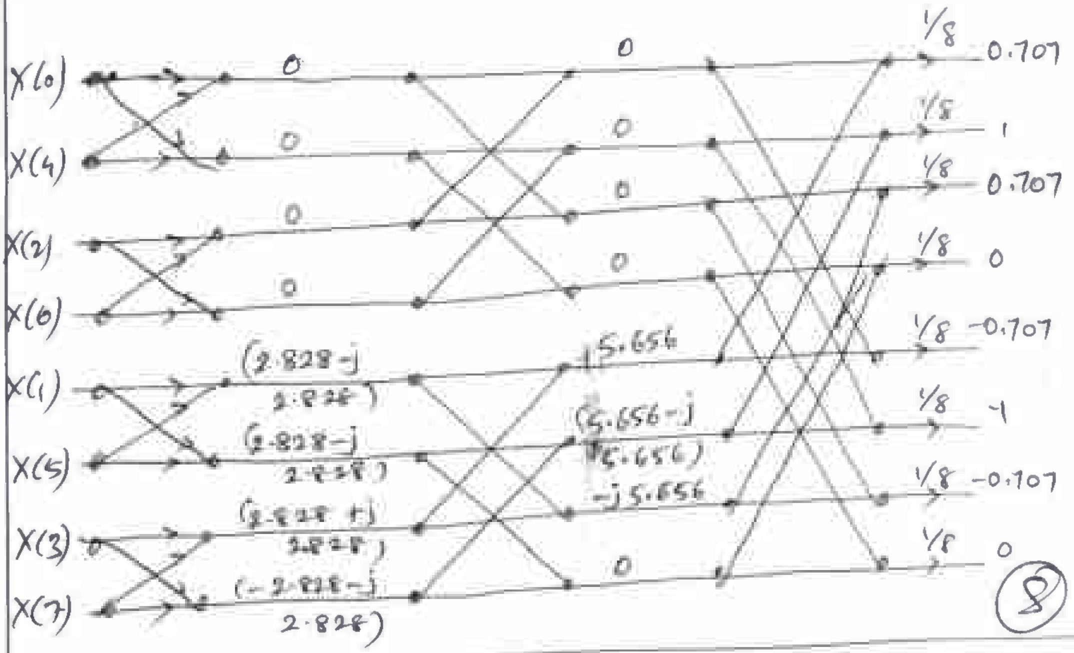
Ans 4b.

$X(0)=0,\ X(1)=2\sqrt{2}(1-j)=2.828-j2.828,$

$X(2)=0,\ X(3)=0,\ X(4)=0,\ X(5)=0,\ X(6)=0,$

$X(7)=2\sqrt{2}(1+j)=2.828+j\,2.828.$ (2)

$W_8^0=1,\ W_8^{-1}=0.707+j0.707,\ W_8^{-2}=j,\ W_8^{-3}=-0.707+j0.707.$



(8) 2+8 =10

Ans 5a To obtain $h(t)$ from $H(s)$

$$H(s)=\frac{1}{s^2+\sqrt{2}s+1}=\frac{1}{\left(s+\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}=\sqrt{2}\left[\frac{1/\sqrt{2}}{\left(s+\frac{1}{\sqrt{2}}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}\right]$$ (1)

Using standard laplace transform relation to obtain

$h(t)\cdot \mathcal{L}[e^{-at}\sin\omega t]=\frac{\omega}{(s+a)^2+\omega^2}$ (2)

Then $h(t)=\sqrt{2}\cdot e^{-\frac{1}{\sqrt{2}}t}\sin\left(\frac{1}{\sqrt{2}}t\right)u(t).$

Substitute ~~h(nT)=~~ $t=nT$ $\therefore h(nT)=\sqrt{2}e^{-\frac{1}{\sqrt{2}}nT}\sin\left(\frac{1}{\sqrt{2}}nT\right)$

since $T=1$, $h(n)=\sqrt{2}e^{-\frac{1}{\sqrt{2}}n}\sin\left(\frac{1}{\sqrt{2}}n\right)$. $\longrightarrow$ (A)

Consider $a^n\sin(\omega_0 n)u(n)\xrightarrow{z}\frac{\left(\frac{z}{a}\right)^{-1}\sin\omega_0}{1-2\left(\frac{z}{a}\right)^{-1}\cos\omega_0+\left(\frac{z}{a}\right)^{-2}}$ (2)

let $a=e^{-1/\sqrt{2}}$ and $\omega_0=1/\sqrt{2}$.

z-transform of eqn (A) will be (3)

$$H(z)=\sqrt{2}\cdot\frac{\left(\frac{z}{e^{-1/\sqrt{2}}}\right)^{-1}\sin\left(\frac{1}{\sqrt{2}}\right)}{1-2\left(\frac{z}{e^{-1/\sqrt{2}}}\right)^{-1}\cos\left(\frac{1}{\sqrt{2}}\right)+\left(e^{z\cdot 1/\sqrt{2}}\right)^{-2}}=\frac{0.453z^{-1}}{1-0.752z^{-1}+0.243z^{-2}}$$

1+2+2+3 = 8

Subject Title : Digital Signal Processing Subject Code : 18EE63.

Question Number	Solution	Marks Allocated
5b)	$A_p = 2\,dB$, $\Omega_p = 20$ rad/sec, $A_s = 10\,dB$, $\Omega_s = 30$ rad/sec. $N = \dfrac{\log\sqrt{\dfrac{10^{0.1A_s}-1}{10^{0.1A_p}-1}}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)} = 3.37 \approx 4.$ (1) $\Omega_c = \dfrac{1}{2}\left\{\dfrac{\Omega_p}{(10^{0.1A_p dB}-1)^{\frac{1}{2N}}} + \dfrac{\Omega_s}{(10^{0.1A_s dB}-1)^{1/2N}}\right\} = 22$ rad/sec (3) $s_1 = -8.419 + j20.325$, $s_1^* = -8.419 - j20.325$ (5) $s_2 = -20.325 + j8.419$, $s_2^* = -20.325 - j8.419$. $H_a(s) = \dfrac{\Omega_c^N}{(s-s_1)(s-s_1^*)(s-s_2)(s-s_2^*)}$ (3) $H_a(s) = \dfrac{(22)^4}{(s^2+16.83s+484)(s^2+40.65s+484)}.$	1+3+5+3 =12
6a.	To obtain poles: $(s+0.1)^2 + 9 = (s+0.1-j3)(s+0.1+j3)$ poles are at $s = -0.1 \pm j3$. (1) $\sigma = -0.1$, and $\Omega = \pm 3$. To obtain T: $\Omega = \dfrac{2}{T}\tan\dfrac{\omega}{2}$ or $T = \dfrac{2}{\Omega}\tan\dfrac{\omega}{2} = 0.2761$ (1) Bilinear transformation: (1) $s = \dfrac{2}{T}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right) = 3.621\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)$ $H(z) = H_a(s)\Big\rvert_{s=3.621\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \dfrac{3.621\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1}{\left[3.621\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1\right]^2+9}$ $H(z) = \dfrac{0.1629(1-0.0537z^{-1}-0.946z^{-2})}{1-0.359z^{-1}+0.937z^{-2}}$ (2)	1+1+1 +2 =5

Subject Title: Digital Signal Processing **Subject Code:** 18EE63

Question Number	Solution	Marks Allocated

Ans 6b) Order of Chebyshev filter

$A_p = 1\,dB,\ \Omega_p = 10\ rad/sec$

$A_s = 60\,dB,\ \Omega_s = 50\ rad/sec$

$\varepsilon = \sqrt{10^{0.1A_p} - 1} = 0.509,\ \mu = \dfrac{1+\sqrt{1+\varepsilon^2}}{\varepsilon} = 4.17$

$a = \Omega_p \left[\dfrac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] = 3.65;\ b = \Omega_p \left[\dfrac{\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}}{2}\right] = 10.64$ (2)

for $N = 4$, $\phi_k = \dfrac{2k+5}{8}$, $k = 0, 1, 2, 3$.

Complex conjugate poles are:

$s_1 = -1.4 + j9.83$ and $s_1^* = -1.4 - j9.83$

$s_2 = -3.37 + j4.1$ and $s_2^* = -3.37 - j4.1$ (5)

$$H_a(s) = \frac{k}{(s-s_1)(s-s_1^*)(s-s_2)(s-s_2^*)}$$

$$H_a(s) = \frac{k}{(s^2+2.8s+98.59)(s^2+6.74s+28.17)}$$ (2)

Since 'N' is even,

$$k = \frac{b_0}{\sqrt{1+\varepsilon^2}} = \frac{98.59 \times 28.17}{\sqrt{1+0.509^2}} = 2475.1$$ (1)

$$H_a(s) = \frac{2475.1}{(s^2+2.8s+98.54)(s^2+6.74s+28.17)}$$

Marks Allocated: 2+5+2+1=10

Ans 6c) (i) To obtain lowpass filter with $\Omega_{LP} = 10\ rad/sec$. Lowpass to low pass transformation is given as, $s \to \dfrac{\Omega_p}{\Omega_{LP}} s$. $s \to \dfrac{s}{10}$ (2)

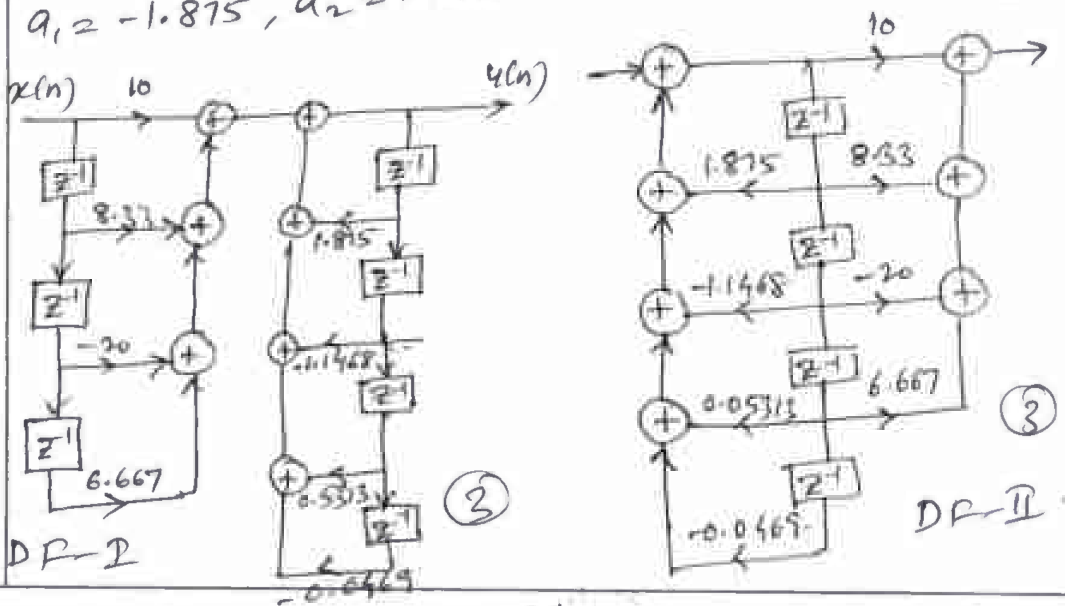
$$\therefore H_a(s) = H_a(s)\big|_{s=\frac{s}{10}} = \frac{1}{\frac{s^2}{100} + \sqrt{2}\frac{s}{10} + 1} = \frac{100}{s^2 + 10\sqrt{2}s + 100}$$

(ii) To obtain highpass filter with cut off frequency 10 rad/sec. $s \to \dfrac{\Omega_p \Omega_{HP}}{s}$ $\therefore s \to 10/s$.

$\cancel{H_a(s) = H_a(s)\big|_{s=\frac{10}{s}} = \frac{1}{\frac{100}{s}}}$

Subject Title : Digital Signal Processing Subject Code : 18EE63.

Question Number	Solution	Marks Allocated
	$H_a(s) = H_a(s)\big\vert_{s=\frac{10}{s}}$ (3) $= \dfrac{1}{\frac{100}{s^2} + \frac{10\sqrt{2}}{s} + 1} = \dfrac{s^2}{s^2 + 10\sqrt{2}s + 100}$	2+3 =5
Ans. 7a	~~Ans~~ 5 comparisons between Butterworth filter and Chebyshev filter.	5×1 =5
Ans7b.	$A_p = 1dB$, $\Omega_p = 1404\pi$ rad/sec, $A_s = 60dB$, $\Omega_s = 8268\pi$ rad/sec. Since ripple in passband is given, this is chebyshev filter. (1) Specification of digital filter $\omega_p = \Omega_p T_s = 0.1404\pi$, $\omega_s = \Omega_s T_s = 0.8268\pi$ (1) Prewarping for bilinear transform (1) $\Omega = \frac{2}{T}\tan\frac{\omega}{2} = \tan\frac{\omega}{2}$. $\therefore \Omega_p = \tan\frac{\omega_p}{2} = 0.224$ rad/sec ; $\Omega_s = \tan\frac{\omega_s}{2} = 3.58$ rad/sec Order of Chebyshev filter ~~$N \geq \cosh^{-1}\sqrt{\frac{10^{0.1A_s}-1}{10^{0.1A_p}-1}}$~~ $N = \dfrac{\cosh^{-1}\sqrt{\frac{10^{0.1A_s}-1}{10^{0.1A_p}-1}}}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)} \approx 3.$ (1) Pole of Chebyshev filter ~~$s_1 = -0.11$, $s_2 = -0.055 + j0.216$ and $s_2^* = -0.055 - j0.216$~~ $A_p = 1dB$, $\varepsilon = \sqrt{10^{0.1A_p} - 1} = 0.508$. $\mu = \dfrac{1 + \sqrt{1+\varepsilon^2}}{\varepsilon} = 4.176$; $a = \Omega_p\left[\dfrac{\mu^{\frac{1}{N}} - \mu^{-\frac{1}{N}}}{2}\right] = 4.176$. $b = \Omega_p\left[\dfrac{\mu^{\frac{1}{N}} + \mu^{-\frac{1}{N}}}{2}\right] = 0.224\left[\dfrac{4.176^{\frac{1}{3}} + 4.176^{-\frac{1}{3}}}{2}\right] 0.25$ $\phi_k = \dfrac{(2k+4)\pi}{6}$, $k = 0, 1, 2 \ldots$ (5) complex conjugate poles are $s_1 = -0.11$, $s_2 = -0.055 + 0.216$ and $s_2^* = -0.055 - j0.216$	

Question Number	Solution	Marks Allocated
	System function Ha(s)	
	$Ha(s) = \frac{k}{(s-s_1)(s-s_2)(s-s_2^*)} = \frac{k}{(s+0.11)(s^2+0.11s+0.05)}$	
	For odd N, $k = b_0 = 0.11 \times 0.05 = 0.0055$. (3)	1+1+1+1 +5+3+3 =15
	$\therefore \; Ha(s) = \frac{0.0055}{(s+0.11)(s^2+0.11s+0.05)}$	
	To obtain H(z) using bilinear transformation	
	$H(z) = Ha(s)\Big\|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = Ha(s)\Big\|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$, since $\frac{2}{T}=1$	
	$H(z) = \frac{0.0043(1+z^{-1})^3}{(1-0.8z^{-1})(1-1.64z^{-1}+0.81z^{-2})}$ (3)	
Ans 8a	$\frac{Y(z)}{H(z)} = \frac{10+8.33z^{-1}-20z^{-2}+6.667z^{-3}}{1-1.875z^{-1}+1.14688z^{-2}-0.5313z^{-3}+0.0469z^{-4}}$ → (1)	1+1=2
	Taking inverse z-transform.	
	$y(n) = 1.875y(n-1) - 1.14688y(n-2) + 0.5313y(n-3) - 0.0469y(n-4) + 10x(n) + 8.33x(n-1) - 20x(n-2)$ → (1)	
	$+ 6.667x(n-3)$.	
Ans 8b	we have:	
	$b_0 = 10,\ b_1 = 8.33,\ b_2 = -20,\ b_3 = 6.667$.	
	$a_1 = -1.875,\ a_2 = 1.1468,\ a_3 = -0.5313,\ a_4 = 0.0469$. (2)	
		2+3+3 =8

Subject Title: Digital Signal Processing Subject Code: 18EE63.

Question Number	Solution	Marks Allocated

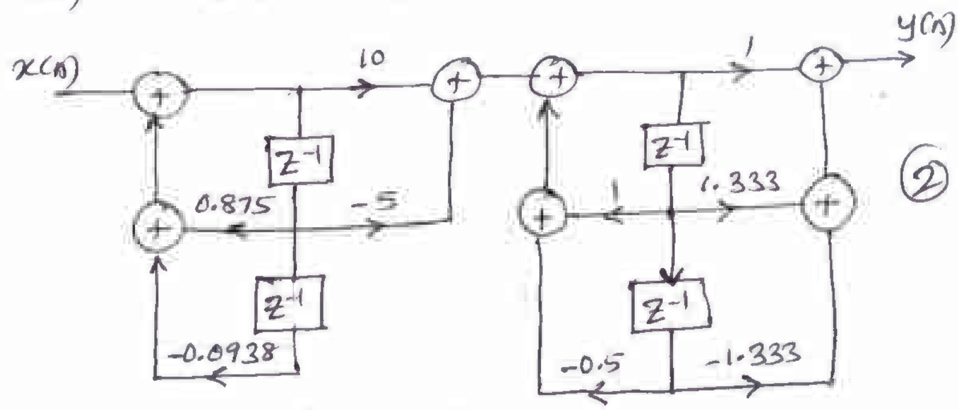
Q8c. The given function can be expressed as

$$H(z) = \frac{10z(z-0.5)(z-0.6667)(z+2)}{(z-0.75)(z-0.125)[z-(0.5+j0.5)][z-(0.5-j0.5)]}$$

$$= \frac{10z(z-0.5)}{(z-0.75)(z-0.125)} \cdot \frac{(z-0.6667)(z+2)}{[z-(0.5+j0.5)][z-(0.5-j0.5)]}$$

$$= \frac{10-5z^{-1}}{1-0.875z^{-1}+0.0938z^{-2}} \cdot \frac{1+1.333z^{-1}-1.333z^{-2}}{1-z^{-1}+0.5z^{-2}} \quad (2)$$

$H(z) = H_1(z) \cdot H_2(z)$



(2)

Fig Cascade realization

The system function is expressed as a rational function i.e

$$H(z) = \frac{10+8.33z^{-1}-20z^{-2}+6.667z^{-3}}{1-1.875z^{-1}+1.1468z^{-2}-0.5313z^{-3}+0.0469z^{-4}}$$

$$= \frac{10z^4+8.33z^3-20z^2+6.667z}{z^4-1.875z^3+1.1468z^2-0.5313z+0.0469} \quad (1)$$

$$\frac{H(z)}{z} = \frac{10z^3+8.33z^2-20z+6.667}{z^4-1.875z^3+1.1468z^2-0.5313z+0.0469}$$

Expanding above equation in partial fractions

$$\frac{H(z)}{z} = \frac{9.2979}{z-1.2813} + \frac{13.4141}{z-0.11} + \frac{6.7431-j12}{z-(0.2419+j0.5238)}$$

$$+ \frac{6.7431+j12}{z-(0.2419-j0.5238)} \quad (1)$$

Question Number	Solution	Marks Allocated

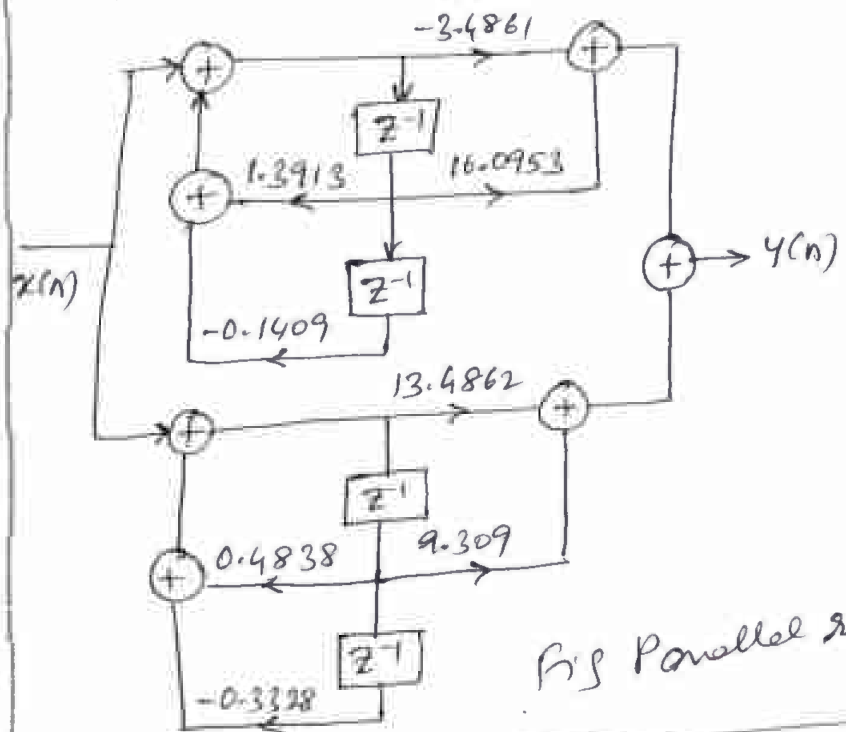
Combine the complex conjugate poles and convert to second order sections:

$$\frac{H(z)}{z} = \frac{-3.4861z+16.0953}{z^2-1.3913z+0.1409} + \frac{13.4862z+9.309}{z^2-0.4838z+0.3328}$$ (2)

$$\therefore H(z) = \frac{-3.4861+16.0953z^{-1}}{1-1.3913z^{-1}+0.1409z^{-2}} + \frac{13.4862+9.304z^{-1}}{1-0.4838z^{-1}+0.3328z^{-2}}$$

2+2+1 +1+2 +2=10

$$H(z) = H_1(z) + H_2(z)$$



(2)

7

Fig Parallel realization

Ans 9a

* To obtain $h_d(n)$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)e^{j\omega n}\,d\omega = \frac{1}{2\pi}\int_{-\frac{3\pi}{4}}^{3\pi/4} e^{-j3\omega}e^{j\omega n}\,d\omega$$ (2)

$$h_d(n) = \frac{\sin\frac{3\pi}{4}(n-3)}{\pi(n-3)}$$

for $n=3$, $h_d(n) = \frac{1}{2\pi}\int_{-3\pi/4}^{3\pi/4} d\omega = 0.75$.

Thus $h_d(n) = \begin{cases} \frac{\sin 0.75\pi(n-3)}{\pi(n-3)} & \text{for } n \neq 3. \\ 0.75 & \text{for } n=3. \end{cases}$

Bos

Subject Title : Digital Signal Processing | Subject Code : 18EE63

Question Number	Solution	Marks Allocated
	To perform windowing and obtain $h(n)$, the hamming window is given: $w_H(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$ for $n=0,1,\ldots M-1$. For $M=7$, $w_H(n) = 0.54 - 0.46\cos\left(\frac{\pi n}{3}\right)$ for $n=0,1,\ldots 6$ (4)	

Sl. No	$h_d(n) = \begin{cases} \frac{\sin 0.75\pi(n-3)}{\pi(n-3)} & \text{for } n\neq 3 \\ 0.75 & \text{for } n=3 \end{cases}$	$w_H(n) = 0.54 - 0.46\cos \pi n/6$
0	0.075	0.08
1	-0.159	0.31
2	0.225	0.77
3	0.75	1.00
4	0.225	0.77
5	-0.159	0.31
6	0.075	0.08

$h(n) = h_d(n)\cdot w_H(n) = \{0.006, -0.04929, 0.17325, 0.75, 0.17325, -0.04929, 0.006\}$ (2)

2+4+2 = 8

7

To obtain frequency response

$$|H(\omega)| = h\left(\frac{M-1}{2}\right) + 2\sum_{n=0}^{(M-3)/2} h(n)\cos\omega\left(n - \frac{M-1}{2}\right)$$

$$= h(3) + 2\sum_{n=0}^{2} h(n)\cos\omega(n-3)$$

$$= 0.75 + 2[0.006\cos 3\omega - 0.04929\cos 2\omega + 0.17325\cos\omega]$$

Ans 9 b

The given bandpass filter has a passband from $\omega_{c1} = \pi/4$ to $\omega_{c2} = 3\pi/4$ rad/sample. The desired unit sample response of the ideal bandpass filter

$$h_d(n) = \begin{cases} \frac{\sin\omega_{c2}(n-\tau) - \sin\omega_{c1}(n-\tau)}{\pi(n-\tau)} & \text{for } n\neq\tau \\ \frac{\omega_{c2}-\omega_{c1}}{\pi} & \text{for } n=\tau \end{cases}$$ (2)

Subject Title : Digital Signal Processing Subject Code : 18EE63.

Question Number	Solution	Marks Allocated

Here $\tau = \frac{M-1}{2} = \frac{11-1}{2} = 5$, Putting values in above equation, (3)

$$h_d(n) = \begin{cases} \dfrac{\sin\left[\frac{3\pi(n-5)}{4}\right] - \sin\left[\frac{\pi(n-5)}{4}\right]}{\pi(n-5)} & \text{for } n \neq 5 \\ \dfrac{\frac{3\pi}{4} - \frac{\pi}{4}}{\pi} = \frac{1}{5} & \text{for } n = 5 \end{cases}$$

(ii) To obtain $h(n)$ by windowing (Rectangular)

$h(n) = h_d(n)$ for $0 \le n \le M-1$ (1)

ie $h(n) = h_d(n)$ for $0 \le n \le 10$

The value of $h(n)$ are:

$h(0) = 0$, $h(1) = 0$, $h(2) = 0$, $h(3) = -0.3183$, $h(4) = 0$

$h(5) = 1/5$, $h(6) = 0$, $h(7) = -0.3183$, $h(8) = 0$,

$h(9) = 0$, $h(10) = 0$. (6)

2+3+1+6=12

Ans 10(a) Desired frequency response:

putting N=7. $H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & \text{for } 0 \le |\omega| \le \pi/2 \\ 0 & \pi/2 \le |\omega| \le \pi \end{cases}$ (1)

To sample $H_d(e^{j\omega})$

put $\omega = \frac{2\pi k}{N}$, $k = 0, 1, \dots N-1$. For N=7, $\omega = \frac{2\pi k}{N}$, (1)

$k = 0, 1, 2, \dots 6$.

$\therefore H_d(e^{j\omega})$ becomes,

$$H(k) = \begin{cases} e^{-j\frac{2\pi k}{7}\cdot 3} & \text{for } 0 \le \frac{2\pi k}{7} \le \frac{\pi}{2} \\ 0 & \text{for } \frac{\pi}{2} \le \frac{2\pi k}{7} \le \pi \end{cases} = \begin{cases} e^{-j\frac{6\pi k}{7}} & \text{for } 0 \le k \le 7/4 \\ 0 & \text{for } \frac{7}{4} \le k \le 7/2 \end{cases}$$

The range of k in above equation can be written in nearest integers as follows: $H(k) = \begin{cases} e^{-j\frac{6\pi k}{7}} & \text{for } 0 \le k \le 2 \\ 0 & \text{for } 2 \le k \le 4 \end{cases}$ (2)

To obtain $h(n)$

$$h(n) = \frac{1}{M}\left\{H(0) + 2\sum_{k=1}^{p} Re\left[H(k)e^{j2\pi kn/M}\right]\right\}.$$ (2)

Here $H(0) = 1$ for $k = 0$, $M = N = 7$ and

$p = \frac{M-1}{2} = \frac{7-1}{2} = 3$. Hence above equation becomes,

~~h(n)=~~

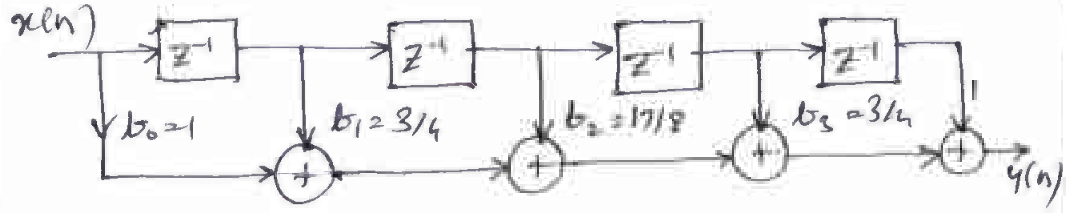
Subject Title : Digital Signal Processing **Subject Code :** 18EE63 .

Question Number	Solution	Marks Allocated
	$h(n) = \frac{1}{7}\left\{1 + 2\sum_{k=1}^{2} Re\left[e^{-j\frac{6\pi k}{7}} \cdot e^{j2\pi kn/7}\right]\right\}$. (4) $h(n) = \frac{1}{7}\left\{1 + 2\sum_{k=1}^{2} \cos\left[\frac{2\pi k(3-n)}{7}\right]\right\}$, $n = 0, 1, 2, \ldots 6$. here $Re[e^{j\theta}]$ is $\cos\theta$. The above equation is the unit sample response of the required filter.	1+1+2+2 +4=10

Ans 10b.

(i) Direct form. (4)

$b_0 = 1$, $b_1 = 3/4$, $b_2 = 17/8$, $b_3 = 3/4$, $b_4 = 1$



(ii) Cascade form structure.

$H(Z) = H_1(Z) \cdot H_2(Z)$

$H(Z) = (b_{10} + b_{11}Z^{-1} + b_{12}Z^{-2})(b_{20} + b_{21}Z^{-1} + b_{22}Z^{-2})$

$= b_{10}b_{20} + (b_{10}b_{21} + b_{11}b_{20})Z^{-1} + (b_{10}b_{22} + b_{11}b_{21} + b_{12}b_{20})Z^{-2} + (b_{11}b_{22} + b_{12}b_{21})Z^{-3} + b_{12}b_{22}Z^{-4}$

$b_{10}b_{20} = 1$; $b_{10}b_{21} + b_{11}b_{20} = 3/4$; $b_{10}b_{22} + b_{11}b_{21} + b_{12}b_{20} = \frac{17}{8}$;

$b_{11}b_{22} + b_{12}b_{21} = 3/4$; $b_{12}b_{22} = 1$.

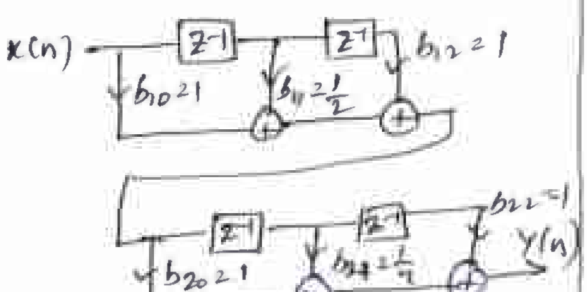
On solving the above eqn we get

$b_{10} = 1$, $b_{11} = \frac{1}{2}$, $b_{12} = 1$, $b_{20} = 1$, $b_{21} = \frac{1}{4}$, $b_{22} = 1$.

$H(z) = \left(1 + \frac{1}{2}Z^{-1} + Z^{-2}\right)\left(1 + \frac{1}{4}Z^{-1} + Z^{-2}\right)$

$H_1(Z) = 1 + \frac{1}{2}Z^{-1} + Z^{-2}$

$H_2(Z) = 1 + \frac{1}{4}Z^{-1} + Z^{-2}$



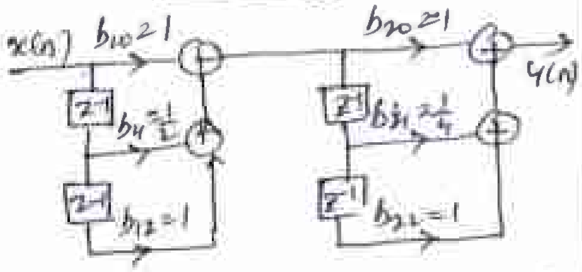


Fig Two type cascade realization